Homework 2 for CMSC 858E

Due 09/25/2018

1 Problem 1

Consider the following variant of Traveling Salesman Problem: find the shortest path that starts from a specific vertex, and visit every vertex in the graph exactly once (note for original TSP, you need to come back to the starting vertex, but this variant does not require it).

Please please give an 1.5-approximation algorithm for this variant, and prove its approximation ratio.

2 Problem 2

Consider a set of costs $c_{ij} \geq 0$ on the edges of a complete graph K_n with n nodes, and let $S \subseteq \{v_1, \ldots, v_n\}$. We wish to find a subgraph of K_n that has odd degree at nodes in S, an even degree (possibly zero) at all other nodes, and as low cost as possible. Show that this is a minimum weight matching problem. (Min weight matching can be solved in polynomial time in graphs that are not necessarily bipartite.)

3 Problem 3

We are given a connected graph $G = (V, E)$ and a cost function $c : E \to Z^+$. We wish to find a walk, traversing each edge of E at least once, such that the total cost of the walk (with multiple traversals of an edge charged multiply) is as small as possible. Give a polynomial time algorithm for this problem. (You may assume a solution to problem (2) even if you did not solve problem (2).)

4 Problem 4

Consider the following approximation algorithm for unweighted vertex cover. Run DFS in the graph (assume it is connected) to obtain a DFS tree T. Now pick all the non-leaf vertices in the vertex cover. Prove that this is a 2-approximation. Why does this form a vertex cover? (Hint: Any matching in G is a lower bound on the size of a vertex cover.)