Closures
(Implementing Higher Order Functions)
Returning Functions as Results

- In OCaml you can pass functions as arguments
  - to `map`, `fold`, etc.
- and you can return functions as results

```ocaml
# let pick_fn n =
  let plus_three x = x + 3 in
  let plus_four x = x + 4 in
  if n > 0 then plus_three else plus_four
val pick_fn : int -> (int->int) = <fun>
```

- Here, `pick_fn` takes an `int` argument, and returns a function

```ocaml
# let g = pick_fn 2;;
val g : int -> int = <fun>
# g 4;;  (* evaluates to 7 *)
```
Multi-argument Functions

- Consider a rewriting of the prior code (above)

```ocaml
let pick_fn n =  
    if n > 0 then (fun x->x+3) else (fun x->x+4)
```

- Here’s another version

```ocaml
let pick_fn n =  
    (fun x -> if n > 0 then x+3 else x+4)
```

- ... the shorthand for which is just

```ocaml
let pick_fn n x =  
    if n > 0 then x+3 else x+4
```

I.e., a multi-argument function!
Currying

- We just saw a way for a function to take multiple arguments!
  - I.e., no separate concept of multi-argument functions – can encode one as a function that takes a single argument and returns a function that takes the rest.

- This encoding is called currying the function
  - Named after the logician Haskell B. Curry
  - But Schönfinkel and Frege discovered it
    - So maybe it should be called Schönfinkelizing or Fregging
Curried Functions In OCaml

- OCaml syntax defaults to currying. E.g.,

  ```ocaml
  let add x y = x + y
  ```

  is identical to all of the following:

  ```ocaml
  let add = (fun x -> (fun y -> x + y))
  let add = (fun x y -> x + y)
  let add x = (fun y -> x+y)
  ```

- Thus:
  
  - `add` has type `int -> (int -> int)`
  - `add 3` has type `int -> int`
    - `add 3` is a function that adds 3 to its argument
  - `(add 3) 4` = 7

- This works for any number of arguments
Syntax Conventions for Currying

Because currying is so common, OCaml uses the following conventions:

• \(\rightarrow\) associates from the right
  
  ➢ Thus \(\text{int} \rightarrow \text{int} \rightarrow \text{int}\) is the same as
  
  ➢ \(\text{int} \rightarrow (\text{int} \rightarrow \text{int})\)

• function application associates from the left
  
  ➢ Thus \(\text{add } 3 \ 4\) is the same as
  
  ➢ \((\text{add } 3) \ 4\)
Quiz 1: Which f definition is equivalent?

```ocaml
let f a b = a / b;;
```

A. let f b = fun a -> a / b;;
B. let f = fun a | b -> a / b;;
C. let f (a, b) = a / b;;
D. let f = fun a -> (fun b -> a / b);;
Quiz 1: Which f definition is equivalent?

```
let f a b = a / b;;
```

A. `let f b = fun a -> a / b;;`
B. `let f = fun a | b -> a / b;;`
C. `let f (a, b) = a / b;;`
D. `let f = fun a -> (fun b -> a / b);;`
Quiz 2: What is enabled by currying?

A. Passing functions as arguments
B. Passing only a portion of the expected arguments
C. Naming arguments
D. Recursive functions
Quiz 2: What is enabled by currying?

A. Passing functions as arguments
B. Passing only a portion of the expected arguments
C. Naming arguments
D. Recursive functions
Another way you could encode support for multiple arguments is using tuples

- `let f (a, b) = a / b (* int*int -> int *)`
- `let f a b = a / b (* int-> int-> int *)`

Is there a benefit to using currying instead?

- Supports **partial application** – useful when you want to provide some arguments now, the rest later

  - `let add a b = a + b;;`
  - `let addthree = add 3;;`
  - `addthree 4;; (* evaluates to 7 *)`
Currying is Standard In OCaml

- Pretty much all functions are curried
  - Like the standard library `map`, `fold`, etc.
  - See `/usr/local/ocaml/lib/ocaml` on Grace
    - In particular, look at the file `list.ml` for standard list functions
    - Access these functions using `List.<fn name>`
    - E.g., `List.hd`, `List.length`, `List.map`

- OCaml works hard to make currying efficient
  - Because otherwise it would do a lot of useless allocation and destruction of closures
  - What are those, you ask? Let’s see …
How Do We Implement Currying?

- Implementing currying is tricky. Consider:

```ocaml
let addN n l = let add x = n + x in map add l
```

- (Equivalent to...)

```ocaml
let addN n = (fun l -> map (fun x -> n + x) l)
```

- When the anonymous function is called by map, \( n \) may not be on the stack any more!
  - We need some way to keep \( n \) around after \addN\ returns
The Call Stack in C/Java/etc.

```c
void f(void) {
    int x;
    x = g(3);
}

int g(int x) {
    int y;
    y = h(x);
    return y;
}

int h (int z) {
    return z + 1;
}

int main(){
    f();
    return 0;
}
```

```
| x   | 4 |
| x   | 3 |
| y   | 4 |
| z   | 3 |
```

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Now Consider Returning Functions

```ocaml
let map f n = match n with
  [] -> []
| (h::t) -> (f h)::(map f t)

let addN n l =
  let add x = n + x in
  map add l

addN 3 [1; 2; 3]
```

- Uh oh...how does `add` know the value of `n`?
  - OCaml does *not* read it off the stack
    - The language could do this, but can be confusing (see above)
  - OCaml uses **static scoping** like C, C++, Java, and Ruby
Static Scoping (aka Lexical Scoping)

In static or lexical scoping, (nonlocal) names refer to their nearest binding in the program text

- Going from inner to outer scope
- In our example, `add` refers to `addN`’s `n`
- C example:

```
int x;
void f() { x = 3; }
void g() { char *x = "hello"; f(); }
```

Refers to the `x` at file scope – that’s the nearest `x` going from inner scope to outer scope in the source code
Closures Implement Static Scoping

- An environment is a mapping from variable names to values
  - Just like a stack frame

- A closure is a pair \((f, e)\) consisting of function code \(f\) and an environment \(e\)

- When you invoke a closure, \(f\) is evaluated using \(e\) to look up variable bindings
Example – Closure 1

```ml
let add x = (fun y -> x + y)
```

```
(add 3) 4 → <cl> 4 → 3 + 4 → 7
```

Diagram:
- Function: `fun y -> x + y`
- Environment: `x = 3`
- Closure
Example – Closure 2

```ocaml
let mult_sum (x, y) =
  let z = x + y in
  fun w -> w * z
```

\[(\text{mult\_sum } (3, 4))\ 5\ \rightarrow \ <\text{cl}>\ 5\ \rightarrow\ 5\ *\ 7\ \rightarrow\ 35\]
Example – Closure 3

```
let twice (n, y) =
  let f x = x + n in
  f (f y)
```

twice (3, 4) → <cl> (<cl> 4) → <cl> 7 → 10
Example – Closure 4

```plaintext
let add x = (fun y -> (fun z -> x + y + z))
```

**add() took 3 arguments?** The compiler figures this out and avoids making closures

```
(((add 1) 2) 3) → ((<cl> 2) 3) → (<cl> 3) → 1+2+3
```
Quiz 3: What is x?

```plaintext
let a = 1;;
let a = 0;;
let b = 10;;
let f () = a + b;;
let b = 5;;
let x = f ();;
```

A. 10
B. 1
C. 15
D. Error - variable name conflicts
Quiz 3: What is x?

```
let a = 1;;
let a = 0;;
let b = 10;;
let f () = a + b;;
let b = 5;;
let x = f ();;
```

A. 10
B. 1
C. 15
D. Error - variable name conflicts
Quiz 4: What is \( z \)?

\[
\begin{align*}
\text{let } f \ x & = \text{fun } y \to x - y \text{ in} \\
\text{let } g & = f \ 2 \text{ in} \\
\text{let } x & = 3 \text{ in} \\
\text{let } z & = g \ 4 \text{ in} \\
z & ; ;
\end{align*}
\]

A. 7  
B. -2  
C. -1  
D. Type Error – insufficient arguments
Quiz 4: What is \( z \)?

\[
\begin{align*}
\text{let } f \ x &= \ \text{fun } y \to x - y \ \text{in} \\
\text{let } g &= f 2 \ \text{in} \\
\text{let } x &= 3 \ \text{in} \\
\text{let } z &= g 4 \ \text{in} \\
z &;;
\end{align*}
\]

A. 7
B. -2
C. -1
D. Type Error – insufficient arguments
Quiz 5: What does this evaluate to?

```ocaml
let f x = x+1 in
let g = f in
g (fun i -> i+1) 1
```

A. Type Error  
B. 1  
C. 2  
D. 3
Quiz 5: What does this evaluate to?

```
let f x = x+1 in
let g = f in
(g (fun i -> i+1)) 1
```

A. Type Error – Too many arguments passed to g (application is left associative)

B. 1
C. 2
D. 3
Higher-Order Functions in C

- C supports function pointers

```c
typedef int (*int_func)(int);
void app(int_func f, int *a, int n) {
    for (int i = 0; i < n; i++)
        a[i] = f(a[i]);
}
int add_one(int x) { return x + 1; }
int main() {
    int a[] = {5, 6, 7};
    app(add_one, a, 3);
}
```
C does not support closures
  • Since no nested functions allowed
  • Unbound symbols always in global scope

```
int y = 1;
void app(int(*f)(int), n) {
    return f(n);
}
int add_y(int x) {
    return x + y;
}
int main() {
    app(add_y, 2);
}
```
Higher-Order Functions in C (cont.)

- Cannot access non-local variables in C
- OCaml code

```ocaml
let add x y = x + y
```

- Equivalent code in C is illegal

```c
int (* add(int x))(int) {
    return add_y;
}
int add_y(int y) {
    return x + y; /* error: x undefined */
}
```
Higher-Order Functions in C (cont.)

- OCaml code
  
  ```ocaml
  let add x y = x + y
  ```

- Works if C supports nested functions
  - Not in ISO C, but in gcc; but not allowed to return them
    
    ```c
    int (* add(int x))(int) {
    int add_y(int y) {
      return x + y;
    }
    return add_y; }
    ```

  - Does not allocate closure, so x popped from stack and add_y will get garbage (potentially) when called
Java 8 Supports Lambda Expressions

- Ocaml’s
  ```
  fun (a, b) -> a + b
  ```
- Is like the following in Java 8
  ```
  (a, b) -> a + b
  ```
- Java 8 supports closures, and variations on this syntax
Java 8 Example

```java
public class Calculator {
    interface IntegerMath { int operation(int a, int b);  }
    public int operateBinary(int a, int b, IntegerMath op) {
        return op.operation(a, b);
    }
    public static void main(String... args) {
        Calculator myApp = new Calculator();
        IntegerMath addition = (a, b) -> a + b;
        IntegerMath subtraction = (a, b) -> a - b;
        System.out.println("40 + 2 = " +
            myApp.operateBinary(40, 2, addition));
        System.out.println("20 - 10 = " +
            myApp.operateBinary(20, 10, subtraction));
    }
}
```