Problem 1. Use mathematical induction to show the following:

(a) \[ \sum_{i=1}^{n} i(i + 1) = \frac{n(n + 1)(n + 2)}{3} \]

(b) \[ \sum_{i=0}^{n} 2^i = 2^{n+1} - 1 \]

Problem 2.

(a) Assume \( b^x = a \). What is \( x \) (in terms of \( a \) and \( b \))?

(b) Using only part (a), show that \( \log_c(ab) = \log_c a + \log_c b \).

(c) Show that \( a^{\log_b n} = n^{\log_b a} \).

Problem 3. Differentiate the following functions:

(a) \( \ln(x^2 + 5) \)

(b) \( \lg(x^2 + 5) \) \[ NOTE: In Computer Science we use \( \lg x \) to mean \( \log_2 x \). \]

(c) \( \frac{1}{\ln(x^2 + 5)} \)

Problem 4. Integrate the following functions:

(a) \( \int \frac{1}{x} \)

(b) \( \int \frac{1}{x+3} \)

(c) \( \ln x \) \[ HINT: Use integration by parts. \]

(d) \( x \ln x \) \[ HINT: Use integration by parts. \]

(e) \( x \lg x \)

Problem 5. Consider the formula \( 3n^4 + 7n^3 \log n + 2n^2 \).

(a) What is the high order term?

(b) What is the second order term?

(c) Write the formula in \( \Theta \) notation (in simplest form).