The following exercises are designed to test your understanding of recursion. The functions are defined using a variant of LISP known as meta-LISP. In order to aid your understanding, the function defined in problem 1 is identical to the one below:

\[
\text{drop}(x) = \begin{cases} 
  \text{nil} & \text{if } \text{null } x \\
  \text{car } x \text{ cons } \text{drop}(\text{cdr } x) & \text{else}
\end{cases}
\]

The idea is that
\begin{align*}
  a &= \text{car } x \\
  d &= \text{cdr } x \\
  n &= \text{null } x \\
  at &= \text{atom } x \\
  a.b &= a \text{ cons } b \\
  <a> &= a \text{ cons } \text{nil} \text{ = a list whose single element is a} \\
  a*b &= \text{concatenate lists } a \text{ and } b \text{ (i.e. append list } b \text{ to list } a) \\
  \text{reverse}[x] &= \text{reverses the top level list } x. \text{ For example } \text{reverse}[(A \ B \ C)] = (C \ B \ A). \text{ But } \text{reverse}[((A \ B \ C)(D \ E))] = ((D \ E)(A \ B \ C)).
\end{align*}

1. Consider the function \text{drop} defined by
   \[
   \text{drop}[x] \leftarrow \begin{cases} 
  \text{nil} & \text{if } \text{n x} \\
  \text{[a x].drop[d x]} & \text{else}
\end{cases}
   \]
   Compute (by hand) \text{drop}[(A \ B \ C)]. What does drop do to lists in general?

2. What does the function \text{r2} do to lists of lists? How about
   \[
   \text{r3}[x] \leftarrow \begin{cases} 
  \text{reverse}[x] & \text{if } \text{at } x \\
  \text{reverse[\text{r4}[x]]} & \text{else}
\end{cases}
   \]
   \[
   \text{r4}[x] \leftarrow \begin{cases} 
  \text{nil} & \text{if } \text{n x} \\
  \text{\text{r3}[a x].r4[d x]} & \text{else}
\end{cases}
   \]

3. Compare the following function with the function \text{r3} of the preceding example:
   \[
   \text{r3'}[x] \leftarrow \begin{cases} 
  \text{\text{r3'}[d x]*\text{r3'}[a x]} & \text{if } \text{at } x
\end{cases}
   \]

4. Consider \text{r5} defined by
   \[
   \text{r5}[x] \leftarrow \begin{cases} 
  x & \text{if } \text{n x} \lor \text{n d x} \\
  \text{[a \text{r5}[d x]].r5[a x . \text{r5[r5[d x]]}] & \text{else}
\end{cases}
   \]
   Compute \text{r5}[(A \ B \ C \ D)]. What does r5 do in general. Needless to say, this is not a good way of computing this function even though it involves no auxiliary functions.