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Find a linear hyperplane (decision boundary) that will separate the data.
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One possible solution
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Other possible solutions
Which one is better? $B_1$ or $B_2$? 
How do you define better?
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Find hyperplane **maximizes** the margin $\rightarrow B_1$ is better than $B_2$
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\[ \vec{w} \cdot \vec{x} + b = 0 \]
\[ \vec{w} \cdot \vec{x} + b = -1 \]
\[ \vec{w} \cdot \vec{x} + b = +1 \]

\[ f(x) = \begin{cases} 
1 & \text{if } \vec{w} \cdot \vec{x} + b \geq 1 \\
0 & \text{if } \vec{w} \cdot \vec{x} + b \leq -1 
\end{cases} \]

Margin = \( \frac{2}{||\vec{w}||^2} \)
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We want to maximize:

\[ \text{Margin} = \frac{2}{||\vec{w}||^2} \]

Which is equivalent to minimizing:

\[ L(w) = \frac{||\vec{w}||^2}{2} \]

But subject to the following constraints:

\[ \vec{w} \cdot \vec{x} + b \geq 1 \text{ if } y_i = 1 \]
\[ \vec{w} \cdot \vec{x} + b \leq -1 \text{ if } y_i = -1 \]

This is a constrained optimization problem

- Numerical approaches to solve it (e.g., quadratic programming)
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What if the problem is not linearly separable?

Apply some sort of penalty:

$$\frac{\xi_i}{\|w\|}$$
What if the problem is not linearly separable?

- Introduce slack variables
- Need to minimize:

\[
L(w) = \frac{||w||^2}{2} + C \left( \sum_{i=1}^{N} \xi_i^K \right)
\]

- Subject to:

\[
\begin{align*}
\vec{w} \cdot \vec{x} + b & \geq 1 - \xi_i \text{ if } y_i = 1 \\
\vec{w} \cdot \vec{x} + b & \leq -1 + \xi_i \text{ if } y_i = -1
\end{align*}
\]
What if the decision boundary is not linear?
NONLINEAR SUPPORT VECTOR MACHINES

Transform data into higher dimensional space
from sklearn import svm

# Fit a default SVM classifier to fake data
X = [[0, 0], [1, 1]]
y = [0, 1]
clf = svm.SVC()
clf.fit(X, y)

SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0,
decision_function_shape=None, degree=3, gamma='auto',
kernel='rbf', max_iter=-1, probability=False,
random_state=None, shrinking=True, tol=0.001,
verbose=False)

Lots of defaults used for hyperparameters – can use cross validation to search for good ones
from sklearn.model_selection import train_test_split
from sklearn.model_selection import GridSearchCV
from sklearn.metrics import classification_report

# ... Load some raw data into X and y ...
# Split the dataset in two equal parts
X_train, X_test, y_train, y_test = \
    train_test_split(X, y, test_size=0.5, random_state=0)

# Pick values of hyperparameters you want to consider
tuned_parameters = [\n    {'kernel': ['rbf'],
     'gamma': [1e-3, 1e-4],
     'C': [1, 10, 100, 1000]},
    {'kernel': ['linear'],
     'C': [1, 10, 100, 1000]}\n]
MODEL SELECTION IN SCIKIT-LEARN

# Perform a complete grid search + cross validation
# for each of the hyperparameter vectors
clf = GridSearchCV(SVC(C=1),
                   tuned_parameters,
                   cv=5,
                   scoring='precision')
clf.fit(X_train, y_train)

# Now that you’ve selected good hyperparameters via CV,
# and trained a model on your training data, get an
# estimate of the “true error” on your test set
y_true, y_pred = y_test, clf.predict(X_test)
print(classification_report(y_true, y_pred))