Modeling of String Instruments

Matthew Adams

October 6, 2014

Motivation

String Physics

- string under tension
- one-dimensional wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Assumptions:

- amplitude $<<$ wavelength
  - each point only moves perpendicular to velocity
- uniform string
- neglect gravity

String Physics

- Differential equation is linear!
  - Linear combinations of solutions are also solutions
  - Superposition

A solution:

$$y(x, t) = A \sin \left[\frac{2\pi}{\lambda} (x \pm vt) \right]$$

String Physics

- Wave amplitude and direction gets reversed at boundaries
- Waves traveling down and being reflected back interfere constructively
- Can be simply represented by two wave equations

$$y_1(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x+vt)\right]$$
$$y_2(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x-vt)\right]$$

Standing Waves

- When a string is plucked, waves at resonant frequencies f_n remain much longer
- Higher frequency waves get dampened out more quickly, due to frequency dependence of energy

$$f_n = n\frac{v}{\lambda} \qquad n \in \mathbb{N}$$

- Fundamental: f_1 (the wavelength $=$ 2$\times$string length)

How to model?

- Additive synthesis: combine many sine waves
- Subtractive synthesis: filter out frequencies from a signal
- There is a simpler, more efficient method

Digital Waveguides

- Leverage the two interfering equations of a standing wave
- Simulate a wave by continually dampening and reflecting the two equations

Karplus-Strong: Classic Wavetable Synthesis

- Specific type of digital waveguide, discovered earlier

$$Y_t = Y_{t-p}$$

where Y_t is the value of the tth sample and p is the period of the resulting tone (in number of samples)

- Load table with samples
- Run through the samples repeatedly, at some frequency

Karplus-Strong

- Big idea: Modify the wave table, instead of shaping the sounds after the fact
- Averaging successive samples simulates dampening of the high frequencies

$$Y_t = \frac{1}{2}(Y_{t-p} + Y_{t-p-1})$$

Karplus-Strong: Results

- Highly efficient- the basic model only requires adding and shifting operations
- Preloading the wavetable with different signals offers flexibility
  - Typically seeded with random values, potentially modified by a filter
  - Not as generalizable as other methods

Karplus-Strong: Extensions

- Changing p produces drum-like sounds
- Sympathetic strings (Jaffe and Smith)
  - Parallel Karplus-Strong instances that bleed with some probability
  - Multiple strings (like a mandolin) should be done without bleed, because beating informs timbre
- Damping simulated explicitly at ends, with the string being lossless
- Changing the attack
- Body models
  - Filter-based methods are computationally expensive
  - Karjalainen et al.
- Adding longitudinal forces to the model
  - Breaks linearity of differential equation

Recent Work

- Exploring essentially new methods
- Refining simulation of different types of attacks and interaction with the body
- Handling nonlinearity

Works Cited

Giancoli, Douglas C. Physics for Scientists and Engineers. 4th ed. Ch 15-16. Pearson Prentice Hall, NJ. 2009.

Land, Bruce. Lecture on Aug 28, 2012. Cornell University. youtube.com/watch?v=QHo1cArGyVE

Kevin Karplus, Alex Strong (1983). "Digital Synthesis of Plucked String and Drum Timbres". Computer Music Journal (MIT Press) 7 (2): 4355

D. Jaffe and J. Smith. Extensions of the Karplus-Strong Plucked-String Algorithm. Computer Music Journal. Vol. 7, No. 2, 1983. MIT Press.

Works Cited

Matti Karjalainen, Vesa Välimäki, and Zoltàn Jànosy. Towards High-Quality Sound Synthesis of the Guitar and String Instruments. International Computer Music Conference, September 10-15, 1993, Tokyo, Japan.

Sound Sources:

- en.wikipedia.org/wiki/File:Karplus-strong-A2.ogg
- Led Zeppelin, Stairway to Heaven