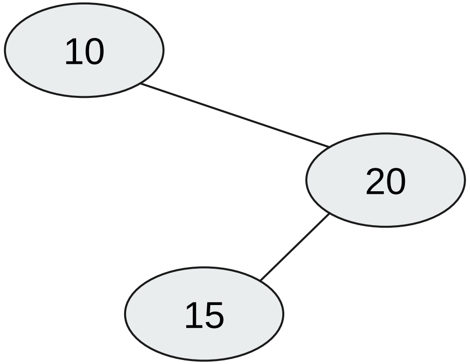
Advanced Tree Structures

CMSC132

Degenerate Search Trees

- Standard BST only as good as its insertion order
- Very easy to make "degenerate"



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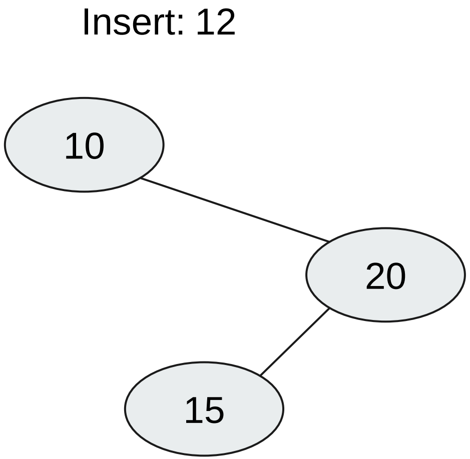

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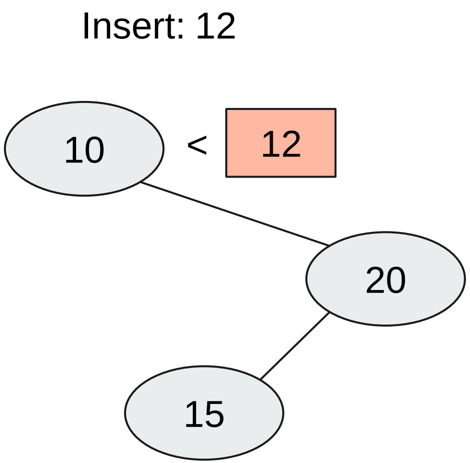

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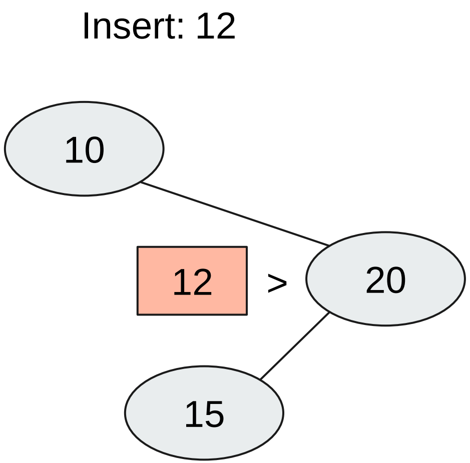

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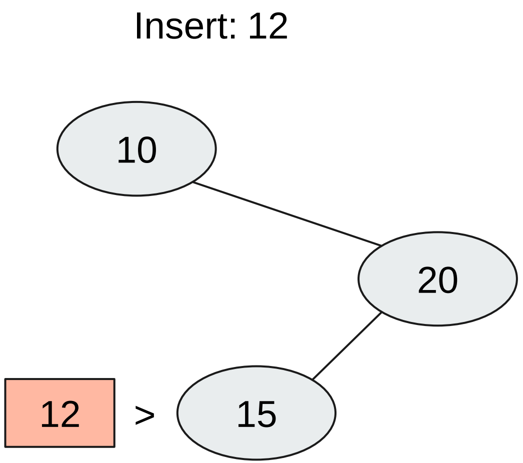
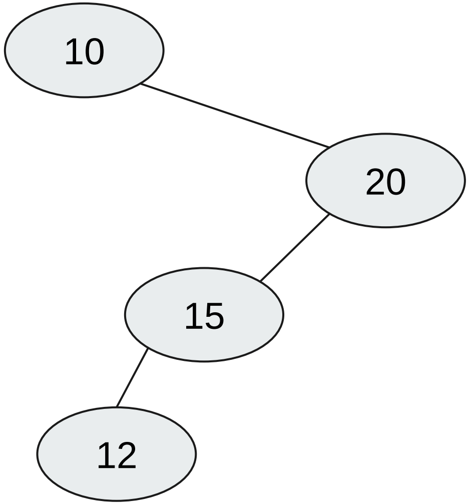


Degenerate Search Trees

- Standard BST only as good as its insertion order
- Very easy to make "degenerate"
- Congratulations, you have a linked list
- How long to find/insert/delete?



Modified Binary Search Trees

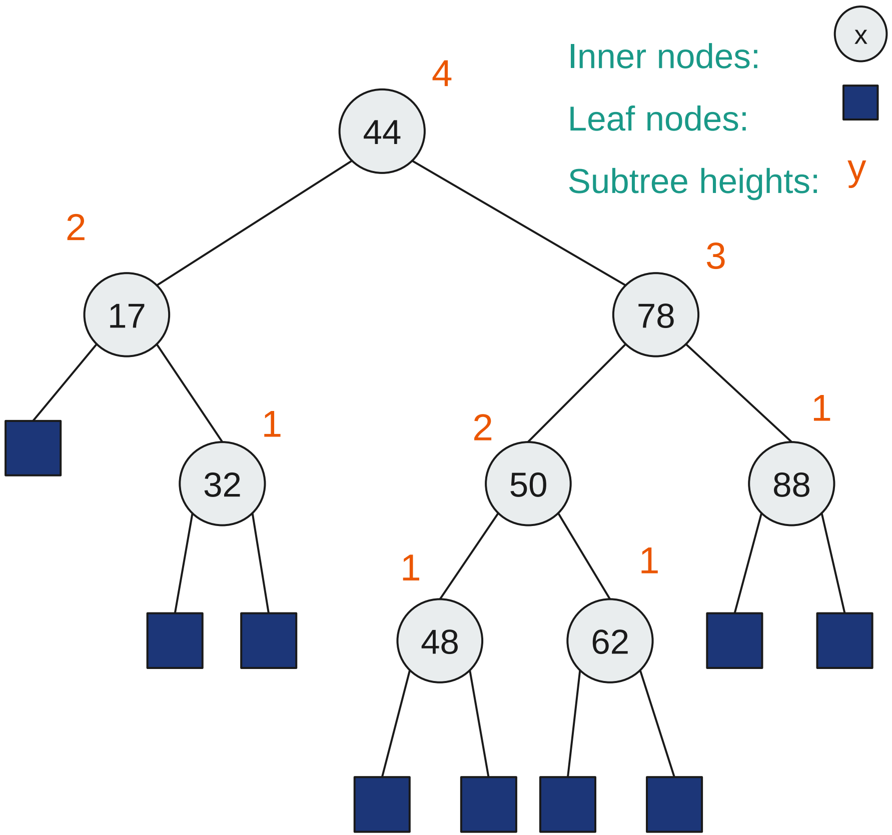
The real world often doesn't use stock BSTs

Strategy	Tree Type	Use case	Tree Type
Rotation	AVL trees, red-black trees	Strings	Tries
Multi-way	2-3 trees	File systems/db	B/B+ trees
Rebuild	Scapegoat trees	Usage frequency	Splay trees
Randomization	Treaps, Skiplists	Multi-dimensional	K-d trees

Many of these covered in detail in CMSC420: Advanced Data Structures

AVL Trees

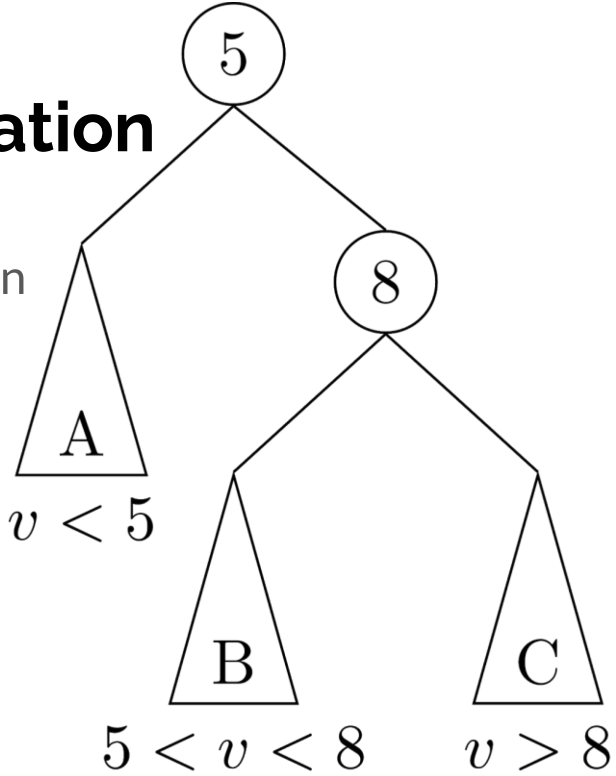
- Invented by **A**delson-**V**elsky and **L**andis in 1962
- *Approach*: keep height of subtrees roughly equal
- *Strategy*: when unequal, rebalance tree with rotations
- *Outcome*: worst case O(logn) performance



Left Rotation

A moves down

C moves up



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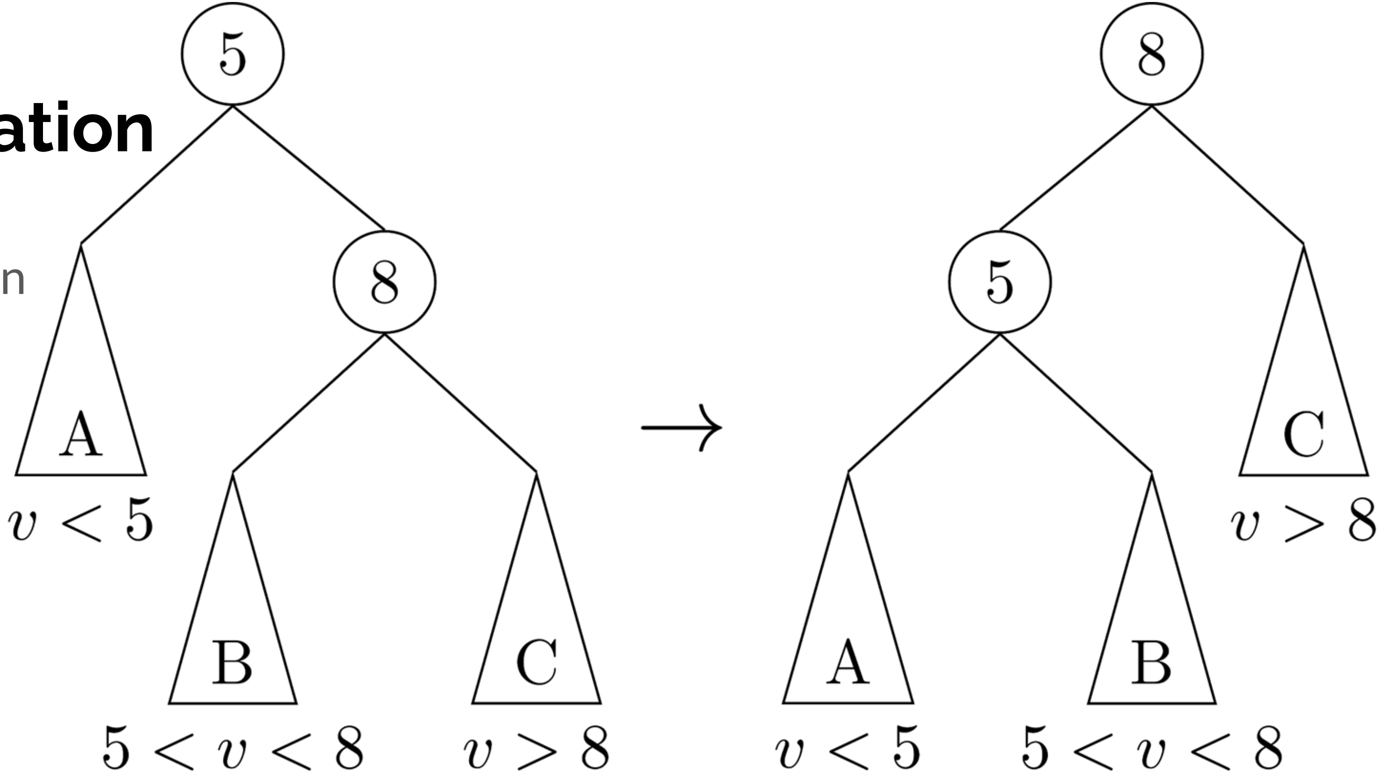


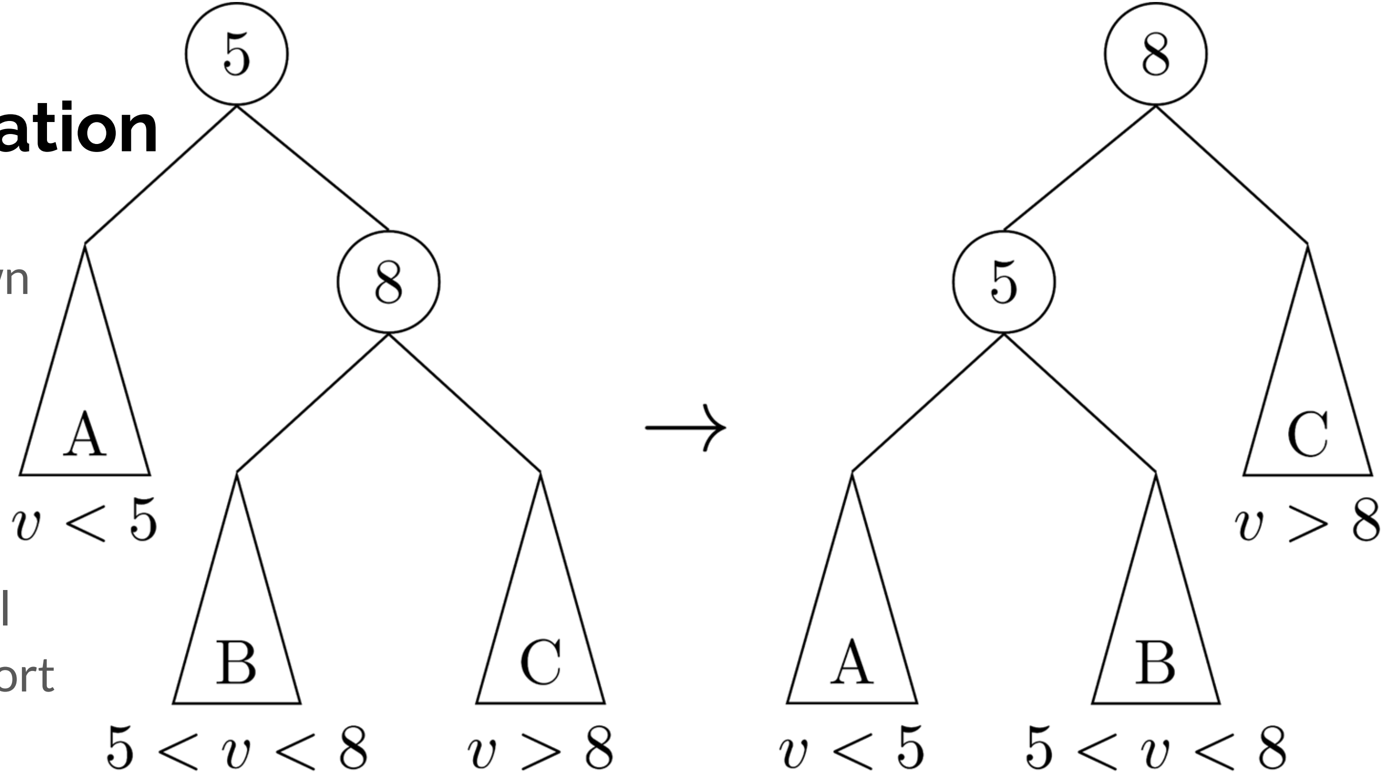
Left Rotation

A moves down

C moves up

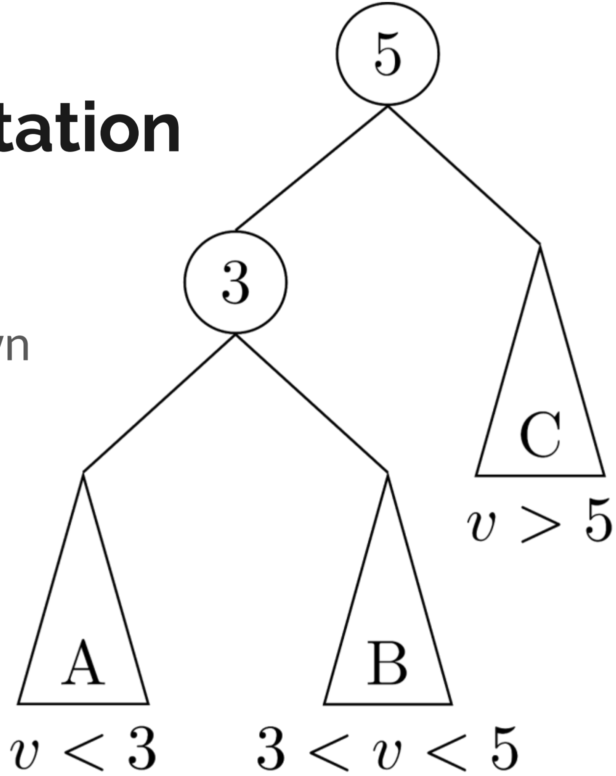
Fixes:

- C too tall
- A too short



Right rotation

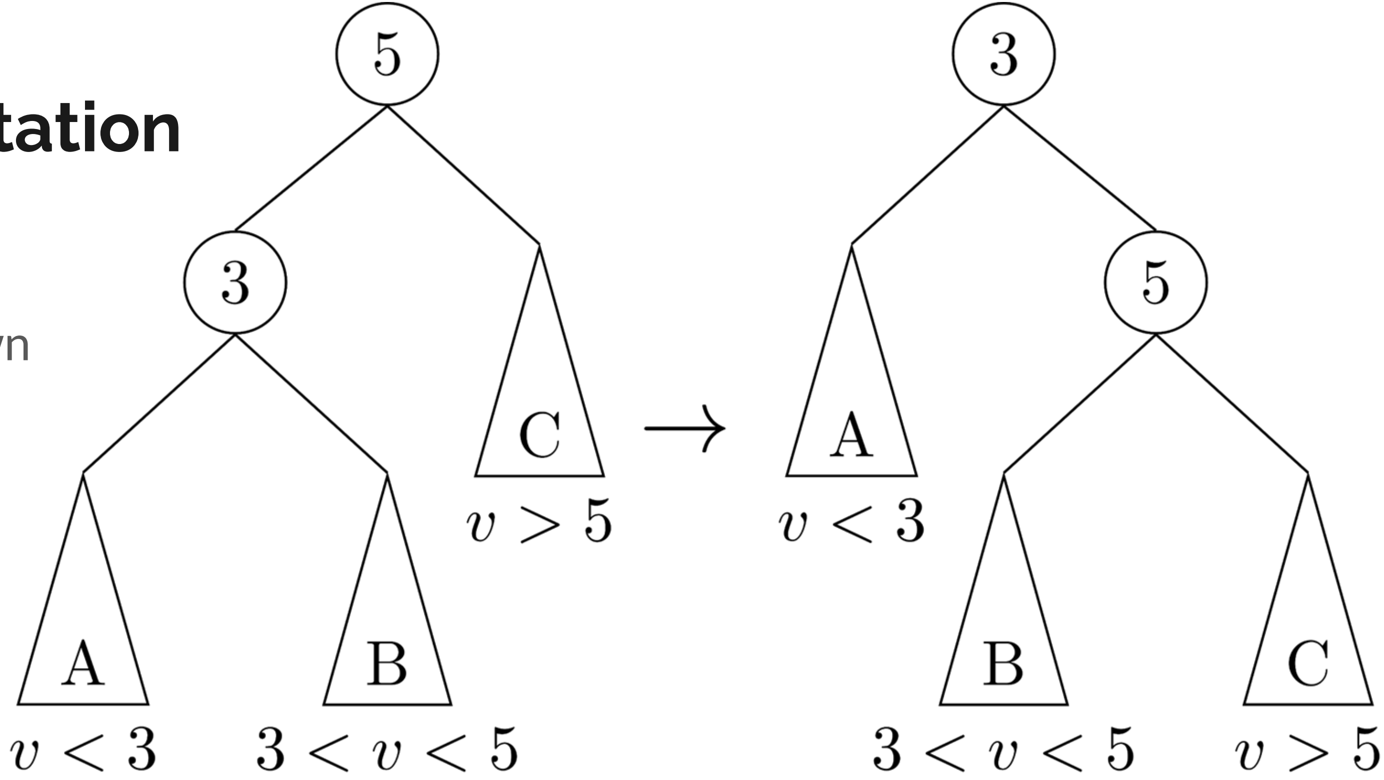
A moves up
C moves down



Right rotation

A moves up

C moves down



Right rotation

A moves up
C moves down
Fixes:

- C too short
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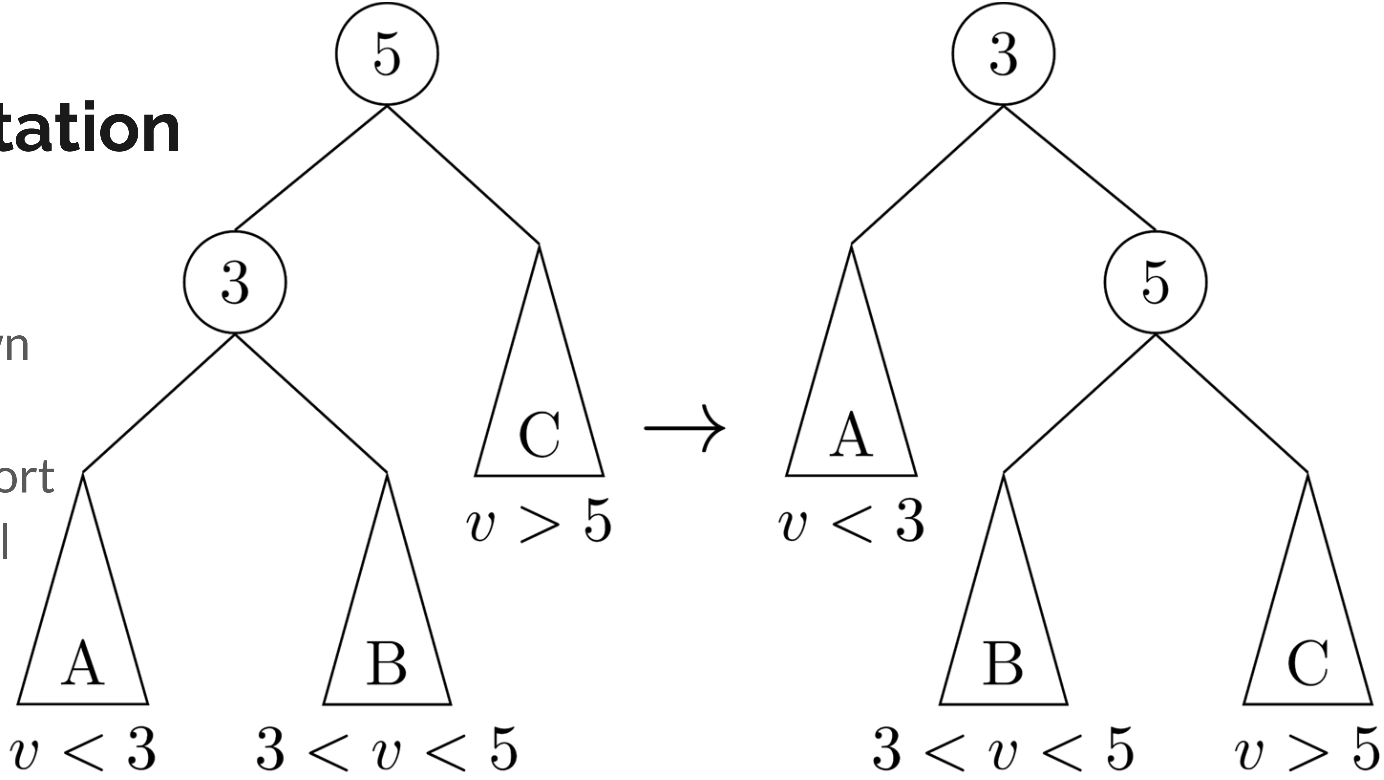
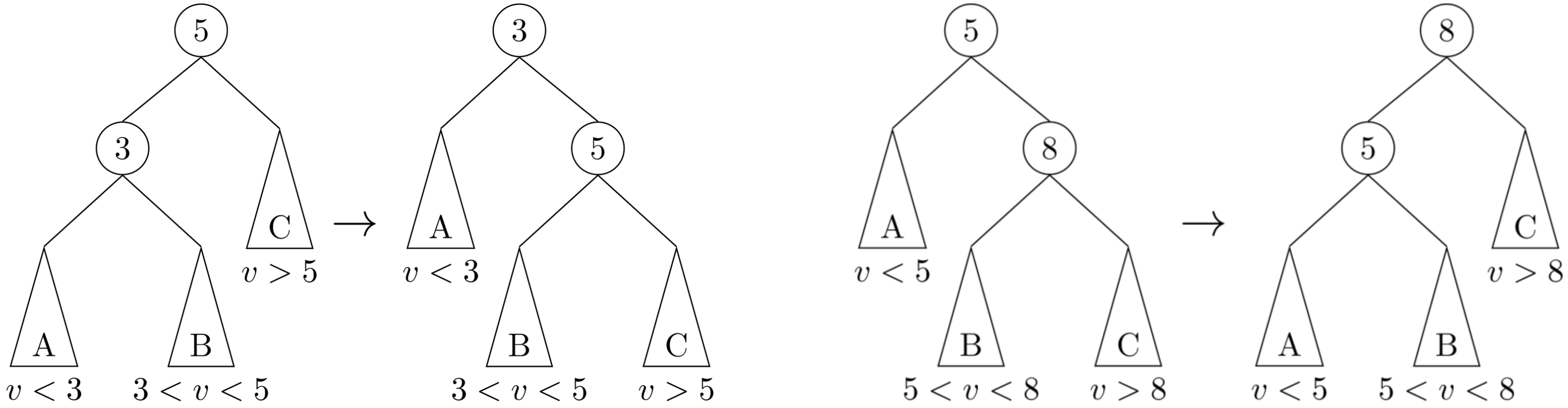


Problem

Left and right rotation move A & C up & down, but what about B?



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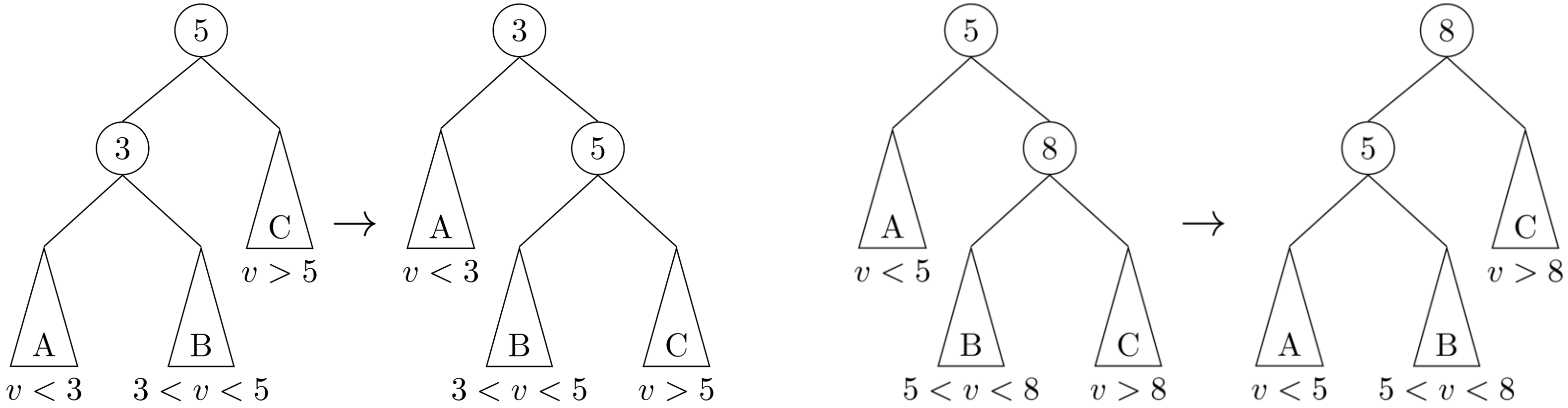
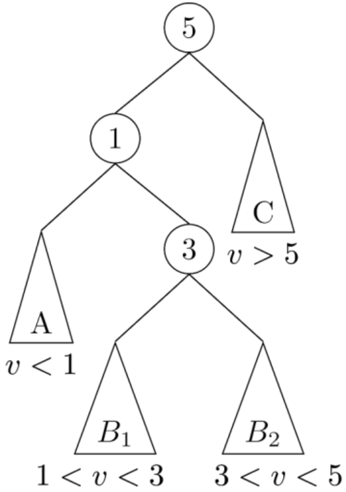


Rotate twice!

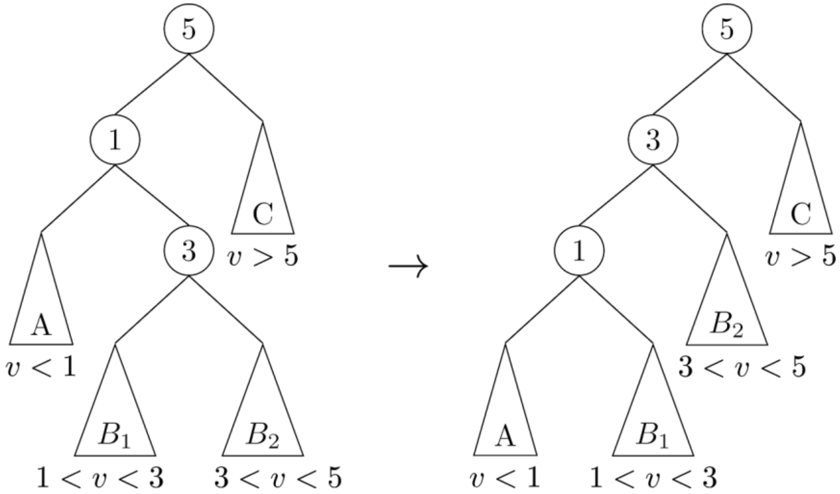
Left-Right Rotation

Let's fix the height of B1/B2 - We've already seen this can't be done with a single rotation



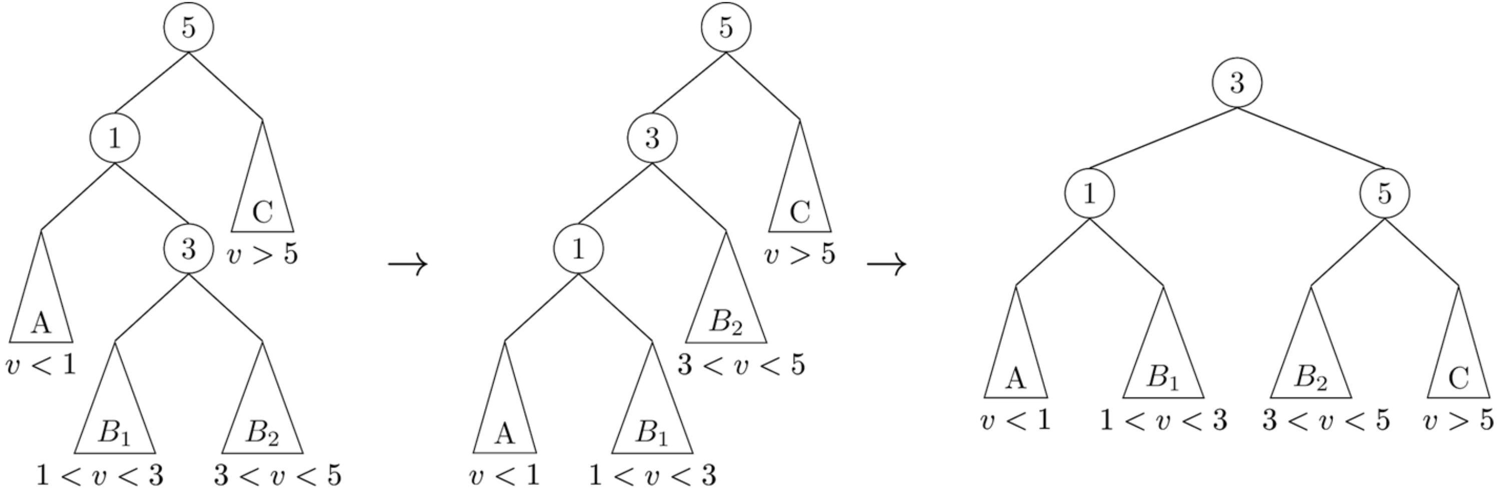
Left-Right Rotation

First, rotate left so that B2 moves up and A moves down



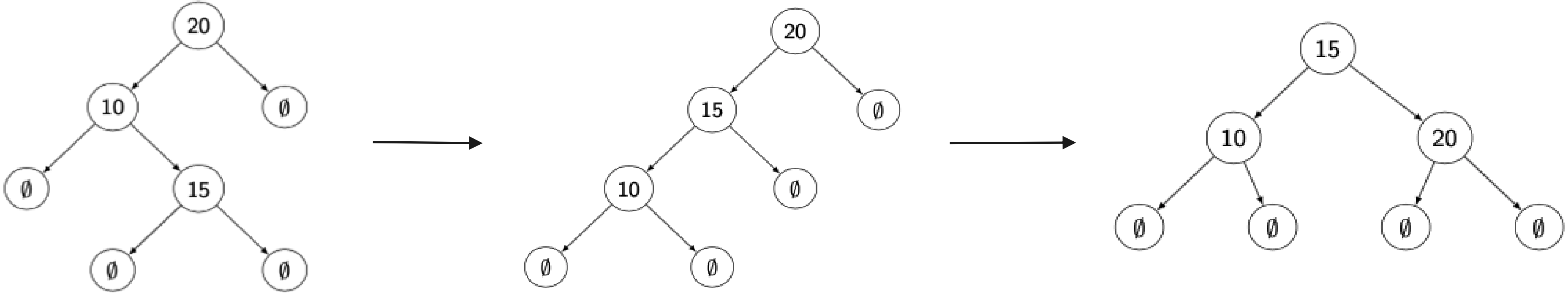
Left-Right Rotation

First, rotate left so that B2 moves up and A moves down
Then, rotate right so that C moves down and the subtree rooted at 1 moves up



Specific Left-Right Rotation Example

- Insert 20, then 10, then 15 into an empty tree.
- Insertion of 15 unbalances the tree, so we must perform a LR rotation.



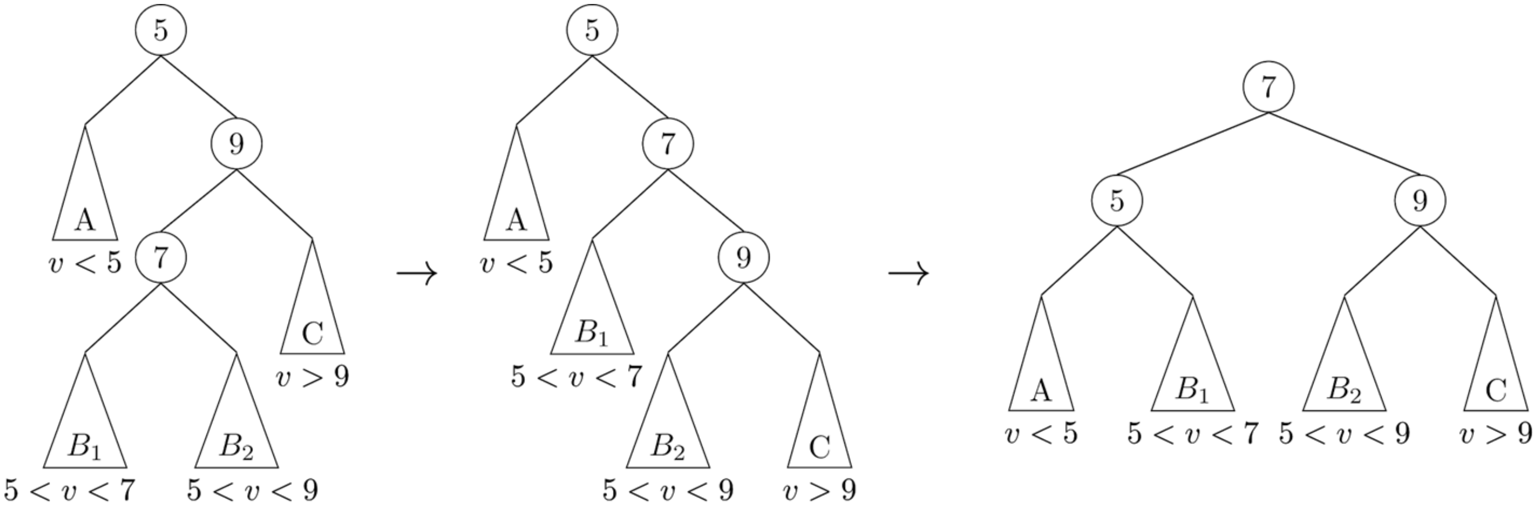
Starting with unbalanced tree

Rotated left around “10”

Rotated right around “15” to restore balance

Right-Left Rotation

The same, but in reverse



Rotation Rules

- These 4 rotations allow an AVL tree to *self* rebalance
- Rotate based off of which grandchild is too tall

1. Left-left: right rotation
2. Left-right: left-right rotation
3. Right-left: right-left rotation
4. Right-right: left rotation

Implementation Details

Code for AVL trees is “relatively” simple:

1. Add extra field for keeping track of height in Node class
2. After modification, update appropriate height fields
3. After modification, rebalance at each level if needed
4. Key search as normal

Extra functions: rebalance, updateHeight, and rotations

Result

- Other trees can be more involved
- AVL isn’t perfect - Java uses red-black trees
  - AVL provides faster lookup and slower insert/remove
  - R-B provides faster insertion/removal and slower lookup
  - R-B uses slightly less storage
  - R-B is harder to do in a 30 minute presentation
- AVL Demo: https://www.cs.usfca.edu/~galles/visualization/AVLtree.html

Resources

These are the notes from Professor David Mount's CMSC420 class from Fall 2020.

http://www.cs.umd.edu/class/fall2020/cmsc420-0201/Lects/lect05-avl.pdf

http://www.cs.umd.edu/class/fall2020/cmsc420-0201/Slides/lect05-avl-slides.pdf

AVL trees on Wikipedia:

https://en.wikipedia.org/wiki/AVL_tree

References

1. Adel'son-Vel'skii, George M., and Evgenii Mikhailovich Landis. "An algorithm for organization of information." In *Doklady Akademii Nauk*, vol. 146, no. 2, pp. 263-266. Russian Academy of Sciences, 1962.
2. Cormen, Thomas H., Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein. *Introduction to algorithms*. MIT press, 2009.

End of presentation

(Subsequent slides include some notes for TAs & extra material on keeping track of node heights)

Keeping track of height

```
class Node {
    T data;
    Node left;
    Node right;
    int height;
}
```

```
int height(Node node) {
    if (node == null) return 0;
    return Math.max(height(node.left),
height(node.right)) + 1;
}
```

Keeping track of height

```java
class Node {
    T data;
    Node left;
    Node right;
    int height;
}
```

```java
void updateHeight(Node node) {
    if (node == null) return;
    node.height = Math.max(height(node.left),
height(node.right)) + 1;
}

int height(Node node) {
    return node == null ? 0 : node.height;
}
```

TAs:

- Subtree heights and the height difference of subtrees is favored over the terminology "balance factor"
- TAs can theme the presentation however they wish, but colors of diagrams may be adversely affected