

CMSC330 – Organization of Programming Languages
Fall 2022
Exam 2

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University of Maryland
Department of Computer Science

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- Do not remove the staple from your exam packet.
- Do not any individual sheets from the exam packet
- Write your name and UID in the header of each page.
- Refrain from bending or folding the exam in any place except near the staple, this helps us when scanning your exams.
- Read *all* questions carefully before starting.

NAME: Solutions.

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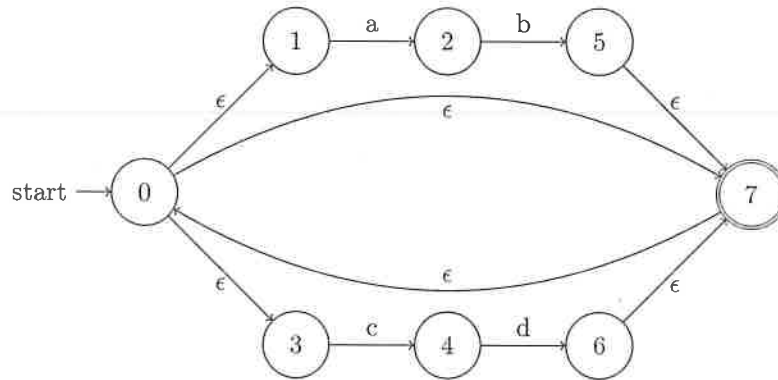
| Question | Points |
|----------|--------|
| 1 | 10 |
| 2 | 10 |
| 3 | 10 |
| 4 | 6 |
| 5 | 15 |
| 6 | 10 |
| Total: | 61 |

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1. Non-deterministic Finite Automata

(a) 8 points Convert the following NFA into a DFA:



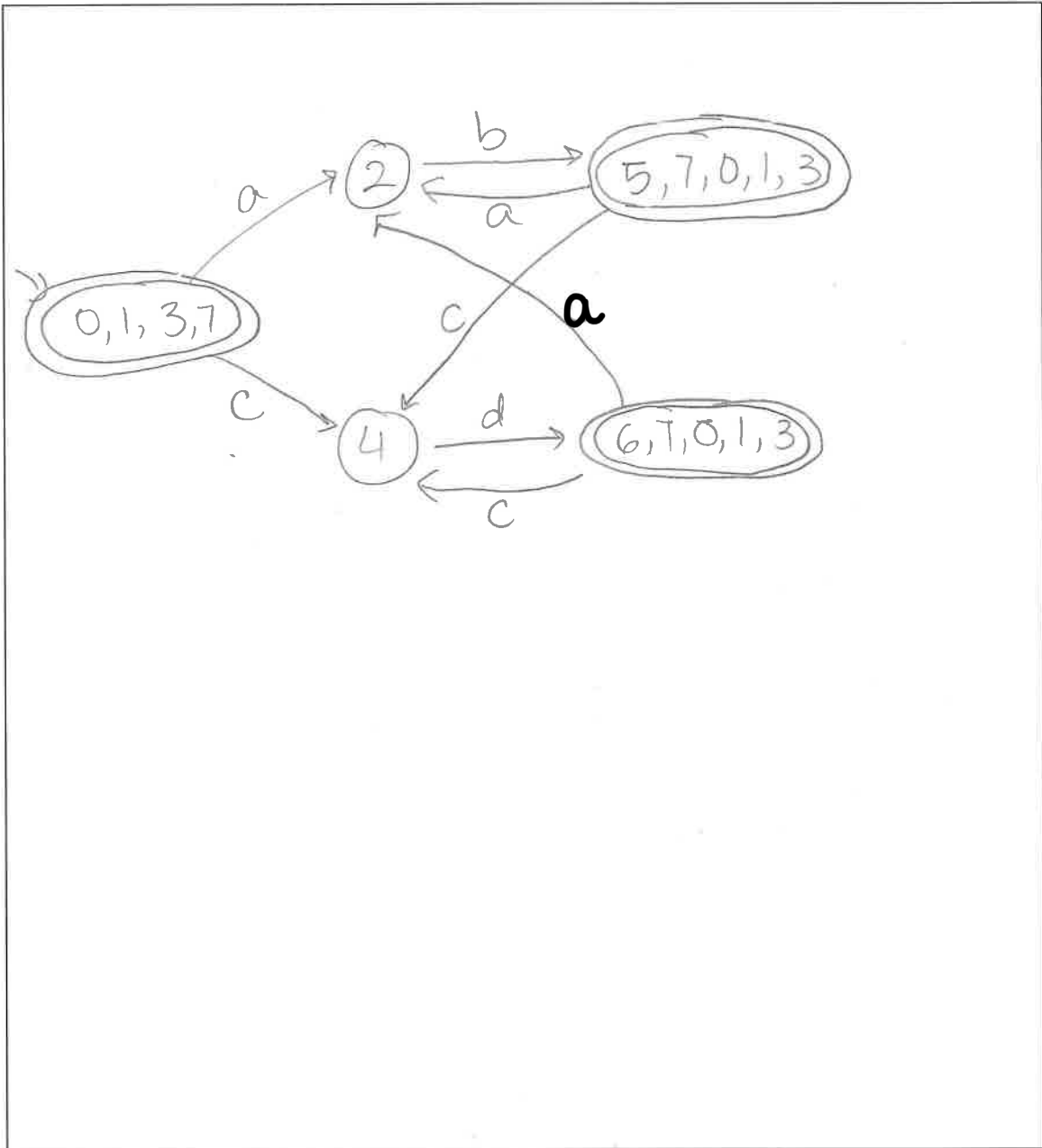
Scratch space for Question 1:

| States | a | b | c | d |
|---------------|-------------|---------------|-------------|---------------|
| 0, 1, 3, 7 | 2 | \emptyset | 4 | \emptyset |
| 2 | \emptyset | 5, 7, 0, 1, 3 | \emptyset | \emptyset |
| 4 | \emptyset | \emptyset | \emptyset | 6, 7, 0, 1, 3 |
| 5, 7, 0, 1, 3 | 2 | \emptyset | 4 | \emptyset |
| 6, 7, 0, 1, 3 | 2 | \emptyset | 4 | \emptyset |

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Your answer here (There is scratch space on the previous page):



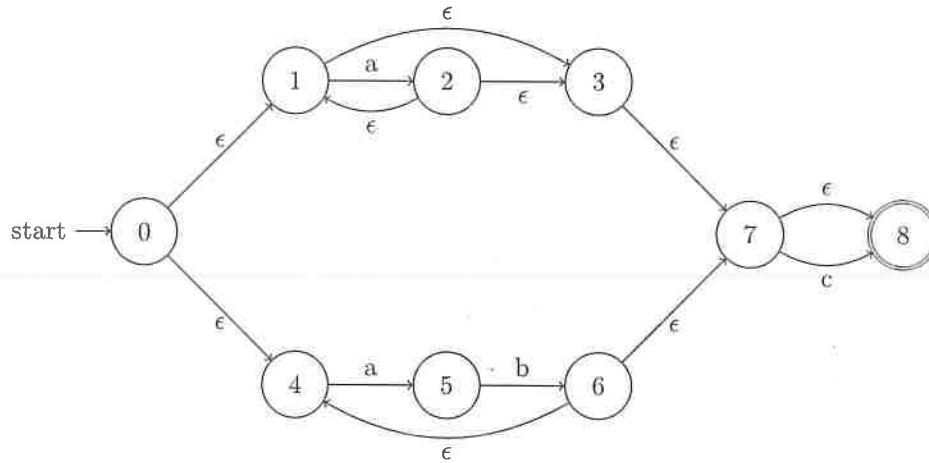
(b) 2 points What is an equivalent Regular Expression for the NFA in Question 1(a)?

$(ab|cd)^*$

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2. Consider the following NFA:



(a) 6 points Which of the following strings does the NFA accept? Mark all that apply.

- "aaaaac"
- <empty string>
- "abbc"
- "ababac"
- "ababab"
- "ababc"

(b) 4 points What is an equivalent Regular Expression for the Question 2 NFA?

$(a^* | (ab)^+)c?$

Scratch space to use as you please:

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3. Context Free Grammars

- (a) 4 points Write a Regular Expression that describes the same set of strings as the following grammar, if it cannot be converted to a regex, explain why:

$$\begin{aligned} S &\rightarrow aSU \mid T \\ T &\rightarrow bTU \mid \epsilon \\ U &\rightarrow cU \mid c \end{aligned}$$

Cannot be converted to Regex - the number of a's and number of c's are related and this cannot be done with regex.

- (b) 6 points Consider the following grammar:

$$\begin{aligned} S &\rightarrow bSc \mid AS \mid c \mid \epsilon \\ A &\rightarrow bA \mid \epsilon \end{aligned}$$

- i. Provide the derivation of an example string of your own invention (i.e. you cannot use the string from the next part) that shows that the grammar is ambiguous.

"bc"
 $S \rightarrow bSc \rightarrow bEc \rightarrow bc$
 $S \rightarrow AS \rightarrow bAS \rightarrow bES \rightarrow bS \rightarrow bc$

- ii. Given the string "bbc", either draw two ASTs or show two derivations that prove the grammar is ambiguous:

$$\begin{aligned} S &\rightarrow bSc \rightarrow bASc \rightarrow bbASc \rightarrow bbESc \rightarrow bbSc \rightarrow bbEc \rightarrow bbc \\ S &\rightarrow AS \rightarrow bAS \rightarrow bbAS \rightarrow bbES \rightarrow bbS \rightarrow bbc \end{aligned}$$

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4. Parsing

- (a) 3 points Can the following grammar be parsed by a recursive-decent parser? Select the option that is the most accurate.

$$\begin{aligned} S &\rightarrow bS \mid C \\ C &\rightarrow cC \mid \epsilon \end{aligned}$$

- Yes
 No, because the grammar is ambiguous
 No, because the grammar is left recursive
 No, because the grammar is ambiguous *and* left recursive
- (b) 3 points You are using a programming language with the grammar provided below. In your editor, you write the program “1 2 + *” and try to run the program. At which stage of the implementation will there be an error, if any?

$$\begin{aligned} S &\rightarrow M + S \mid M - S \mid M \\ M &\rightarrow N * S \mid N / S \mid N \\ N &\rightarrow n \end{aligned}$$

- Lexer
 Parser
 Interpreter
 There will be no error, that is a valid program.

5. Semantics

Take note of the following Operational Semantics for a simple language with let-bindings, conditionals, addition, and comparison of numbers. Each semantic rule is labelled with a name to its left (e.g. the top-left-most rule is labelled ‘num’).

$$\begin{array}{l} \text{num} \frac{}{A; n \rightarrow n} \quad \text{lookup} \frac{A(x) = v}{A; x \rightarrow v} \quad \text{let} \frac{A; e_1 \rightarrow v_1 \quad A; x : v_1; e_2 \rightarrow v_2}{A; \text{let } x = e_1 \text{ in } e_2 \rightarrow v_2} \\ \\ \text{add} \frac{A; e_1 \rightarrow v_1 \quad A; e_2 \rightarrow v_2 \quad v_3 \text{ is } v_1 + v_2}{A; e_1 + e_2 \rightarrow v_3} \\ \\ \text{gt-true} \frac{A; e_1 \rightarrow n_1 \quad A; e_2 \rightarrow n_2 \quad n_1 > n_2}{A; e_1 > e_2 \rightarrow \text{true}} \quad \text{gt-false} \frac{A; e_1 \rightarrow n_1 \quad A; e_2 \rightarrow n_2 \quad n_1 \leq n_2}{A; e_1 > e_2 \rightarrow \text{false}} \\ \\ \text{if-true} \frac{A; e_1 \rightarrow \text{true} \quad A; e_2 \rightarrow v}{A; \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \rightarrow v} \quad \text{if-false} \frac{A; e_1 \rightarrow \text{false} \quad A; e_3 \rightarrow v}{A; \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \rightarrow v} \end{array}$$

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(a) 1 point What is the term for a conclusion that automatically holds (i.e. there is nothing to show in order for us to accept it as true)?

- Predicate
- Syllogism
- Axiom
- Premise

(b) 1 point Which rules, if any, from the Operational Semantics above are an instance of your answer to Question 5(a)?

$$\text{num} \frac{}{A; n \rightarrow n}$$

(c) 3 points The semantics, as given, do not define how to evaluate booleans, even though they are used in the semantics! To understand this, look at the fact that there's no rule where *only* true or *only* false are evaluated. Add your own rule(s) that fix this issue.

$$\text{true} \frac{}{A; \text{true} \rightarrow \text{true}} \qquad \text{false} \frac{}{A; \text{false} \rightarrow \text{false}}$$

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- (d) 10 points Construct the full proof for the following proposition (you should not use your new rule):

$$\begin{array}{c} \frac{\frac{A; 21 \rightarrow 21 \quad A; 21 \rightarrow 21 \quad 42 \text{ is } 21+21}{A; 21+21 \rightarrow 42} \quad \frac{\frac{A, a:42; 42 \rightarrow 42 \quad A, a:42; 42 \rightarrow 42 \quad 84 \text{ is } 42+42}{A, a:42; (42+42) \rightarrow 84} \quad \frac{A, a:42; a \rightarrow 42 \quad 84 > 42}{A, a:42; (42+42) > a \rightarrow \text{true}}}{A; \text{let } a = 21 + 21 \text{ in } (42 + 42) > a \rightarrow \text{true}} \quad \begin{array}{l} A(a) = 42 \\ \uparrow \end{array} \end{array}$$

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6. Lambda Calculus

- (a) Consider the following lambda expression:

$$\lambda j.k \ i \ \lambda h.e \ \lambda k.h \ g \ \lambda f.j \ k$$

Circle the *free* variables:

$$\lambda j.\textcircled{k} \ \textcircled{i} \ \lambda h.\textcircled{e} \ \lambda k.h \ \textcircled{g} \ \lambda f.j \ k$$

Circle the *bound* variables:

$$\lambda j.k \ i \ \lambda h.e \ \lambda k.\textcircled{h} \ g \ \lambda f.\textcircled{j} \ \textcircled{k}$$

- (b) Evaluate the following lambda expression as much as possible:

$$(\lambda m.\lambda n.n \ m) (\lambda f.\lambda x.f \ (f \ x)) (\lambda f.\lambda x.f \ x)$$

Result:

$$(\lambda x.\lambda a.x \ (x \ a))$$

Scratch space for Question 6(b):

$$\begin{aligned} & (\lambda m.\lambda n.n \ m) (\lambda f.\lambda x.f \ (f \ x)) (\lambda f.\lambda x.f \ x) \\ \Rightarrow & (\lambda n.n \ (\lambda f.\lambda x.f \ (f \ x))) (\lambda f.\lambda x.f \ x) \\ \Rightarrow & (\lambda f.\lambda x.f \ x) (\lambda f.\lambda x.f \ (f \ x)) \\ \Rightarrow & \lambda x. (\lambda f.\lambda x.f \ (f \ x)) \ x \\ \Rightarrow & (\lambda x. (\lambda f.\lambda a.f \ (f \ a)) \ x) \\ \Rightarrow & \lambda x. (\lambda a.x \ (x \ a)) \end{aligned}$$

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General Scratch Space
