

CMSC 838B & 498Z: Differentiable Programming

Backpropagation

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Office Hours: After Class or By Appointment

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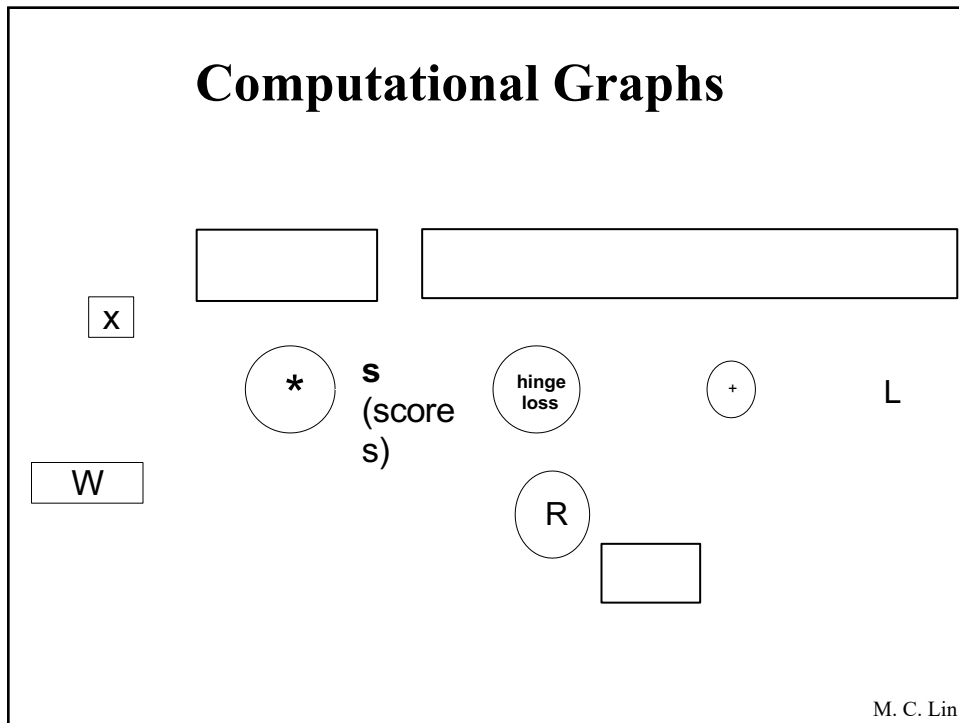
Backpropagation

- Widely used for training feed-forward neural networks and generalized for ANNs & functions
- It computes the gradient of the loss function w.r.t. the weights of the network for a single I/O example and does it very efficiently
- Its efficiency makes it possible for training multilayer networks & updating weights to minimize losses
- Computing the gradient of the loss function w.r.t. each weight by the chain rule, computing the gradient one layer at a time, iterating backward from the last layer to avoid redundant calculations of intermediate terms in the chain rule

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Computational Graphs



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Computational Graphs

- Every operation in the computational graph given its inputs can immediately compute two things:
 - its output value
 - the *local* gradient of its inputs w.r.t its output value
- The chain rule tells us literally that each operation should take its local gradients and multiply them by the gradient that *flows* backwards into it
 - *This is backpropagation!!!*

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Unintuitive Effects of Backprop: Multiplication

- Consider multiplication op: $f(a, b) = a \times b$
- The gradients are clearly $\partial f / \partial b = a$ and $\partial f / \partial a = b$.
 - in a computational graph these would be local gradients w.r.t inputs
- If a is large and b is tiny, then gradient assigned to b will be large, and the gradient to a would be small
- This has implications: e.g. linear classifiers ($w^T x_i$) where you perform many multiplications
 - the magnitude of the gradient is directly proportional to the magnitude of the data
 - multiply x_i by 1000, and the gradients also increase by 1000
 - if you don't lower the learning rate to compensate your model might not learn
- **Need to always pay attention to data normalization**

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Unintuitive Effects of Backprop: *vanishing gradients of the sigmoid*

- Popular to use sigmoids (or tanh) in hidden layers...

Gradient of $\sigma(x) = \sigma(x)(1 - \sigma(x))$

- As part of a larger network where this is local gradient, if x is large (+ve or -ve), then all gradients backwards from this point will be zero due to multiplication of chain rule
 - Why might x be large?
- Maximum gradient is achieved when $x = 0$ ($\sigma(x) = 0.5$, $dx = 0.25$). i.e. the maximum gradient that can flow out of a sigmoid will be a quarter of input gradient
 - What's the implication of this in a deep network with sigmoid activations?

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Working Examples

See the attached worksheet

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