

CMSC 451: Homework 3, Spring 2002

Due at the beginning of class on March 12 (**please note the somewhat unusual due date**)

If you cannot come up with algorithms that run in the required time, then provide (correct) slower algorithms for partial credit. Write your answers using *pseudo-code* in the same style as the textbook. These make the algorithm description precise, and easy to read (as opposed to code in C or some other language).

1. Let $G = (V, E)$ be a directed graph with a non-negative weight specified for each edge. Give an $O(|V|^3)$ time algorithm to find a cycle in G of minimum weight.
2. Let $G = (V, E)$ be a directed graph with a non-negative weight $w(v)$ specified for each *vertex* v . The cost of a directed path $u, x_1, x_2, \dots, x_i, v$ from u to v is defined as follows. If $i = 0$ (i.e., if the path is an edge from u to v), then the cost of this path is zero; else if $i \geq 1$, the cost of the path is $w(x_1) + w(x_2) + \dots + w(x_i)$. Given a vertex $s \in V$, give an efficient algorithm to find a minimum-cost path from s to all other vertices; also prove that the algorithm is correct.
3. We are given an undirected binary tree T with a weight specified for each edge. The *diameter* of T is the maximum shortest-path distance between any two vertices of T . Give a linear-time algorithm to find the diameter of T . (Hint: try to do a divide-and-conquer by solving a similar problem on the left and right subtrees of T).
4. **For graduate students only.** We are given an array A of n real numbers. Design an $O(n)$ time algorithm to find a subsequence $A[i], A[i+1], \dots, A[j]$ of consecutive elements in A , such that the product of these elements is the maximum over all consecutive subsequences. (The product of the empty subsequence is defined to be 1.) For instance, if the array has the five elements 2, 0.01, -3, 0.5, -4, then the consecutive subsequence that has the maximum product is the (-3, 0.5, -4). (Hint: do an appropriate divide-and-conquer on the first and second halves of the array.)