CMSC 451: Mid-Term Exam, Spring 2002

11AM – 12:15PM, March 14th 2002

If you cannot come up with complete solutions but have some partial idea that looks promising, please write that down clearly. Write your answers using *pseudo-code* in the same style as the textbook.

Undergraduate students: please do problems 1, 2, 3. Graduate students: please do problems 1, 2, 4.

1. Let $G = (V, E)$ be an undirected graph represented in **adjacency matrix** format. Give as fast an implementation as you can, of depth-first search. (If you prefer, you can just say how to modify the **adjacency list** version given in the textbook.) Spell out, with proof, the worst-case running time of your algorithm. **(10 points)**

2. Show that there is a directed graph $G = (V, E)$, vertices $s, t \in V$, and a weight for each edge in E, such that the following holds:

   > There exists a path P such that P is now a shortest path from s to t in G. However, there is some value α such that if we increase the weight of each edge by the same value α, then P is no longer a shortest path from s to t in G.

   [**Hint:** There is such a graph G with very few vertices.] **(5 points)**

3. **(For undergraduate students only.)** You are given an undirected graph $G = (V, E)$ in adjacency list format. You are also given that G is a simple cycle: i.e., we can order the vertices as $v_0, v_1, \ldots, v_{n-1}$ (where $n = |V|$) such that the set of edges E is exactly the set

   $$\{(v_0, v_1), (v_1, v_2), \ldots, (v_{n-1}, v_n), (v_n, v_0)\}.$$

   Given a weight for each edge of G, give an $O(V)$ time algorithm to find a minimum spanning tree in G. [**Hint:** Make use of the fact that G is a simple cycle.] **(5 points)**

4. **(For graduate students only.)** You are given an undirected binary tree $T = (V, E)$ in adjacency list format; each edge has a given weight. The *distance* between any two vertices is the total weight of the unique path that connects them. Give an $O(V^2)$ time algorithm to print out the distance between every pair of vertices in V. Justify why the running time is $O(V^2)$. **(10 points)**