

CMSC 451: Homework 1, Spring 2004

Due at the beginning of class on February 12.

Warning: some of the problems require thought - do not wait until the last day to start working on them!

If you cannot come up with algorithms that run in the required time, then provide (correct) slower algorithms for partial credit. Write your answers using *pseudo-code* in the same style as the textbook. These make the algorithm description precise, and easy to read (as opposed to code in C or some other language).

1. We are given an undirected graph G in *adjacency matrix* format. Describe how to implement breadth-first-search on G , and analyze the running time of your algorithm.
2. Suppose u and v are different vertices of a directed graph G . Show that in a DFS of G , v is a descendant of u if and only if v is discovered when u is gray.
3. We are given a directed graph $G = (V, E)$. An *induced subgraph* of G is obtained by choosing an arbitrary subset S of V as a new set of vertices, and retaining only edges that go between elements of S . Prove the following: G has a valid topological sort if and only if every possible induced subgraph H of G has a vertex with no outgoing edges.
4. **(Only for graduate students)** Consider the procedure $DFS(G)$ from the textbook. Consider the following property: “no matter in what order procedure $DFS(G)$ looks at the vertices u in line 5 of its code (in page 541 of the textbook), only one DFS tree is produced”. Show that this property holds if and only if G is strongly connected.