Compression & Huffman Codes

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Compression

- **Definition**
  - Reduce size of data
    (number of bits needed to represent data)
- **Benefits**
  - Reduce storage needed
  - Reduce transmission cost / latency / bandwidth

Compression Examples

- Tools
  - winzip, pkzip, compress, gzip
- Formats
  - Images
    - .jpg, .gif
  - Audio
    - .mp3, .wav
  - Video
    - mpeg1 (VCD), mpeg2 (DVD), mpeg4 (Divx)
  - General
    - .zip, .gz

Sources of Compressibility

- Redundancy
  - Recognize repeating patterns
  - Exploit using
    - Dictionary
    - Variable length encoding
- Human perception
  - Less sensitive to some information
  - Can discard less important data

Types of Compression

- Lossless
  - Preserves all information
  - Exploits redundancy in data
  - Applied to general data
- Lossy
  - May lose some information
  - Exploits redundancy & human perception
  - Applied to audio, image, video

Effectiveness of Compression

- Metrics
  - Bits per byte (8 bits)
    - 2 bits / byte ⇒ ¼ original size
    - 8 bits / byte ⇒ no compression
  - Percentage
    - 75% compression ⇒ ¼ original size

Effectiveness of Compression

- Depends on data
  - Random data ⇒ hard
    - Example: 1001110100 ⇒ ?
  - Organized data ⇒ easy
    - Example: 1111111111 ⇒ 1×10
- Corollary
  - No universally best compression algorithm

Effectiveness of Compression

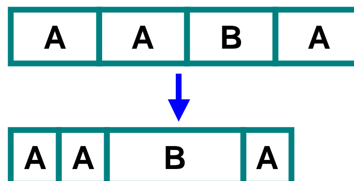
- Compression is not guaranteed
  - Pigeonhole principle
    - Reduce size 1 bit ⇒ can only store ½ of data
    - Example
      - 000, 001, 010, 011, 100, 101, 110, 111 ⇒ 00, 01, 10, 11
  - If compression is always possible (alternative view)
    1. Compress file (reduce size by 1 bit)
    2. Recompress output
    3. Repeat (until we can store data with 0 bits)

Lossless Compression Techniques

- LZW (Lempel-Ziv-Welch) compression
  - Build pattern dictionary
  - Replace patterns with index into dictionary
- Burrows-Wheeler transform
  - Block sort data to improve compression
- Run length encoding
  - Find & compress repetitive sequences
- Huffman code
  - Use variable length codes based on frequency

Huffman Code

- Approach
  - Variable length encoding of symbols
  - Exploit statistical frequency of symbols
  - Efficient when symbol probabilities vary widely
- Principle
  - Use fewer bits to represent frequent symbols
  - Use more bits to represent infrequent symbols



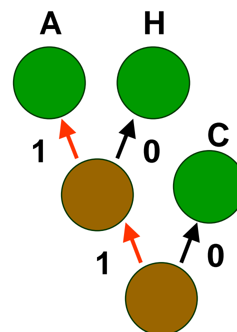
Huffman Code Example

Symbol	Dog	Cat	Bird	Fish
Frequency	1/8	1/4	1/2	1/8
Original Encoding	00	01	10	11
	2 bits	2 bits	2 bits	2 bits
Huffman Encoding	110	10	0	111
	3 bits	2 bits	1 bit	3 bits

- Expected size
  - Original ⇒ 1/8×2 + 1/4×2 + 1/2×2 + 1/8×2 = 2 bits / symbol
  - Huffman ⇒ 1/8×3 + 1/4×2 + 1/2×1 + 1/8×3 = 1.75 bits / symbol

Huffman Code Data Structures

- Binary (Huffman) tree
  - Represents Huffman code
  - Edge ⇒ code (0 or 1)
  - Leaf ⇒ symbol
  - Path to leaf ⇒ encoding
  - Example
    - A = “11”, H = “10”, C = “0”
- Priority queue
  - To efficiently build binary tree



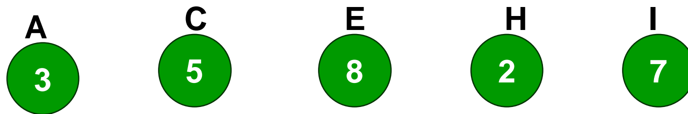
Huffman Code Algorithm Overview

- Encoding
  1. Calculate frequency of symbols in file
  2. Create binary tree representing “best” encoding
  3. Use binary tree to encode compressed file
     - For each symbol, output path from root to leaf
     - Size of encoding = length of path
  4. Save binary tree

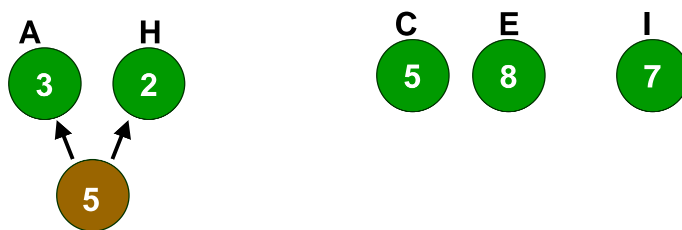
Huffman Code – Creating Tree

- Algorithm
  1. Place each symbol in leaf
     - Weight of leaf = symbol frequency
  2. Select two trees L and R (initially leafs)
     - Such that L, R have lowest frequencies in tree
  3. Create new (internal) node
     - Left child ⇒ L
     - Right child ⇒ R
     - New frequency ⇒ frequency(L) + frequency(R)
  4. Repeat until all nodes merged into one tree

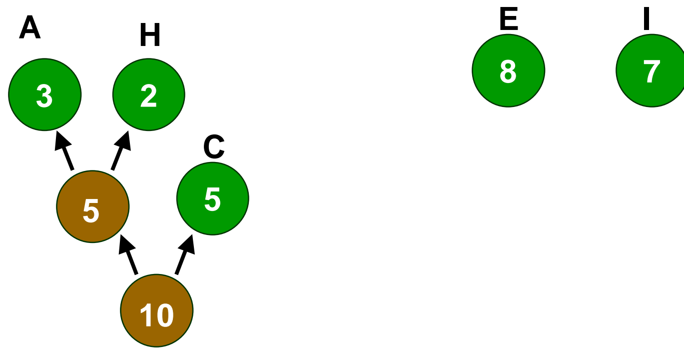
Huffman Tree Construction 1



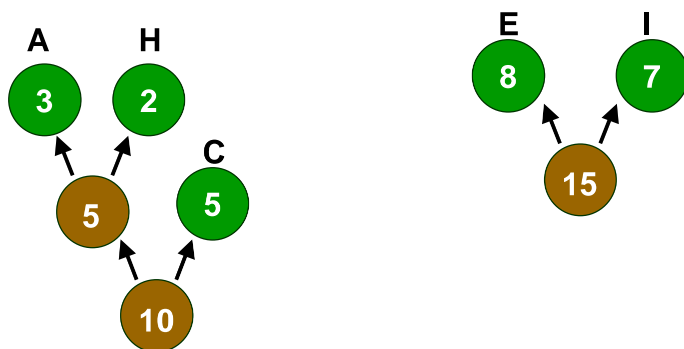
Huffman Tree Construction 2



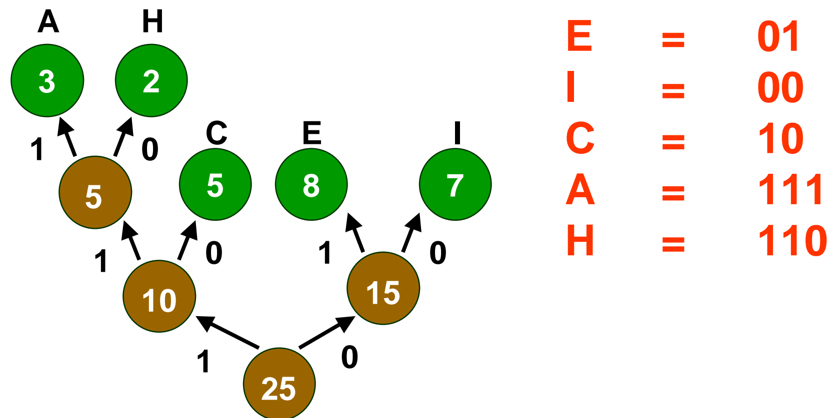
Huffman Tree Construction 3



Huffman Tree Construction 4



Huffman Tree Construction 5



Huffman Coding Example

- **Huffman code**

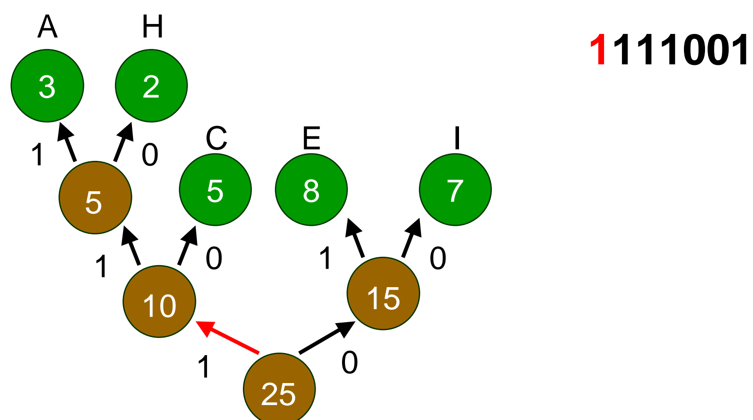
E	=	01
I	=	00
C	=	10
A	=	111
H	=	110

- **Input**
  - ACE
- **Output**
  - (111)(10)(01) = 1111001

Huffman Code Algorithm Overview

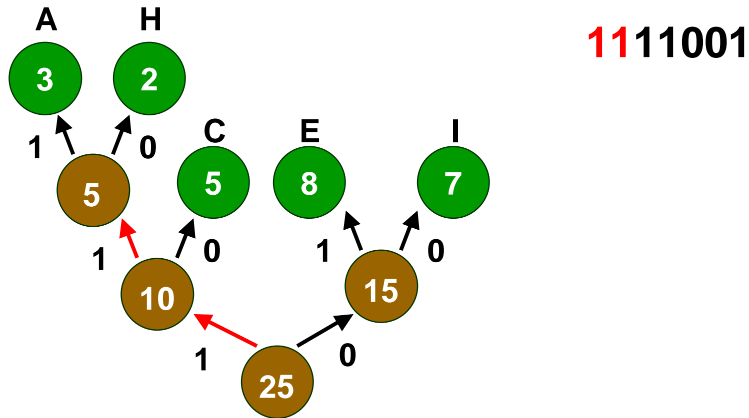
- **Decoding**
  1. **Read compressed file & binary tree**
  2. **Use binary tree to decode file**
     - **Follow path from root to leaf**

Huffman Decoding 1



1111001

Huffman Decoding 2



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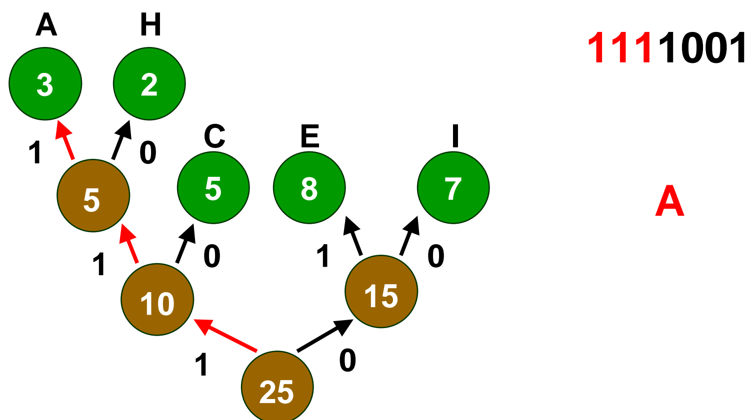
Huffman Decoding 3



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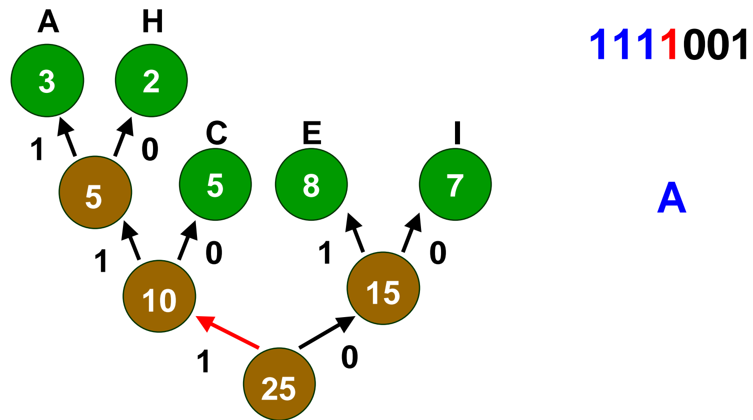
A

Huffman Decoding 4



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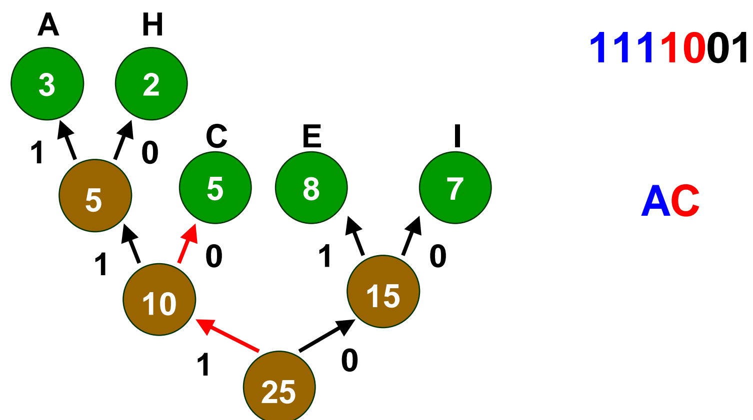
A

Huffman Decoding 5



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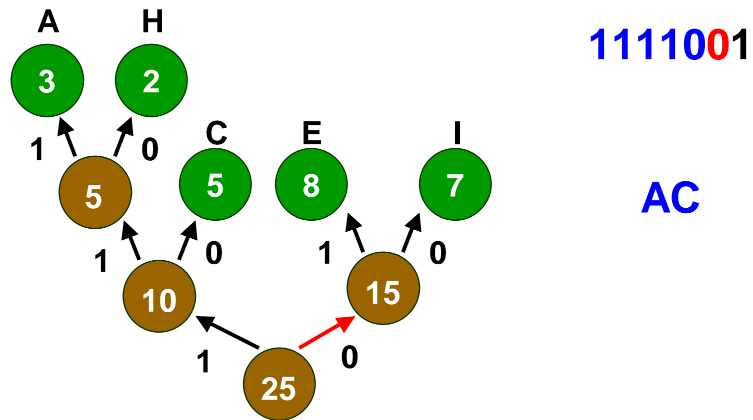
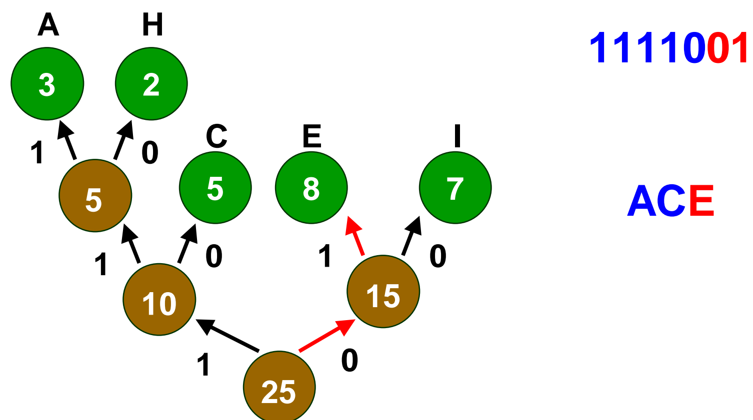
AC

Huffman Decoding 6



Huffman Decoding 7



Huffman Code Properties

- Prefix code
  - No code is a prefix of another code
  - Example
    - Huffman("dog") ⇒ ab
    - Huffman("cat") ⇒ abc // not legal prefix code
  - Can stop as soon as complete code found
  - No need for end-of-code marker
- Nondeterministic
  - Multiple Huffman coding possible for same input
  - If more than two trees with same minimal weight

Huffman Code Properties

- Greedy algorithm
  - Chooses best local solution at each step
  - Combines 2 trees with lowest frequency
- Still yields overall best solution
  - Optimal prefix code
  - Based on statistical frequency
- Better compression possible (depends on data)
  - Using other approaches (e.g., pattern dictionary)