Computational Logic Lecture 1

Introduction

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Human Logic

Fragments of Information

The red block is on the green block.
The green block is somewhere ***above*** *the blue block.*
The green block is ***not*** *on the blue block.*
The yellow block is on the green block ***or*** *the blue block.*
There is ***some*** *block on the black block.*

Conclusions

The red block is on the green block.
The green block is on the yellow block.
The yellow block is on the blue block.
The blue block is on the black block.
The black block is directly on the table.

Proof

We are told that the yellow block is on the green block or the blue block. We are also told that the red block is on the green block. Given the assumption that there can be only one block on another and that a block cannot be two colors at once, we can conclude that the yellow block is not on the green block. But then, by elimination, the yellow block must be on the blue block.

Reasoning by Pattern

All Accords are Hondas.
All Hondas are Japanese.
Therefore, all Accords are Japanese.

All borogoves are slithy toves.
All slithy toves are mimsy.
Therefore, all borogoves are mimsy.

All x are y.
All y are z.
Therefore, all x are z.

Questions

Which patterns are correct?

How many patterns are enough?

Unsound Patterns

Pattern

All x are y.
Some y are z.
Therefore, some x are z.

Good Instance

All Toyotas are Japanese cars.
Some Japanese cars are made in America.
Therefore, some Toyotas are made in America.

Not-So-Good Instance

All Toyotas are cars.
Some cars are Porsches.
Therefore, some Toyotas are Porsches.

Induction - Unsound

I have seen 1000 black ravens.
I have never seen a raven that is not black.
Therefore, every raven is black.
Now try red Hondas.

Abduction - Unsound

If there is no fuel, the car will not start.
If there is no spark, the car will not start.
There is spark.
The car will not start.
Therefore, there is no fuel.
What if the car is in a vacuum chamber?

Deduction - Sound

Logical Entailment/Deduction:

Does not say that conclusion is true in general

Conclusion true *whenever* premises are true

Leibnitz: *The intellect is freed of all conception of the objects involved, and yet the computation yields the correct result.*

Russell: *Math may be defined as the subject in which we never know what we are talking about nor whether what we are saying is true.*

Formal Logic

Formal Mathematics

Algebra

1. Formal language for encoding information
2. Legal transformations

Logic

1. Formal language for encoding information
2. Legal transformations

Algebra Problem

Xavier is three times as old as Yolanda. Xavier's age and Yolanda's age add up to twelve. How old are Xavier and Yolanda?

$$\begin{array}{r} x - 3y = 0 \\ \underline{x + y = 12} \\ -4y = -12 \end{array}$$

$$y = 3$$

$$x = 9$$

Logic Problem

If Mary loves Pat, then Mary loves Quincy. If it is Monday, then Mary loves Pat or Quincy. If it is Monday, does Mary love Quincy?

If it is Monday, does Mary love Pat?

Mary loves only one person at a time. If it is Monday, does Mary love Pat?

Formalization

Simple Sentences:

Mary loves Pat. p

Mary loves Quincy. q

It is Monday. m

Premises:

If Mary loves pat, Mary loves Quincy. $p \Rightarrow q$

If it Monday, Mary loves Pat or Quincy. $m \Rightarrow p \vee q$

Mary loves one person at a time. $p \wedge q \Rightarrow$

Questions:

Does Mary love Pat? $\Rightarrow p$

Does Mary love Qunicy? $\Rightarrow q$

Rule of Inference

Propositional Resolution

$$\frac{\begin{array}{ccc} p_1 \wedge \ldots \wedge p_k & \Rightarrow & q_1 \vee \ldots \vee q_l \\ r_1 \wedge \ldots \wedge r_m & \Rightarrow & s_1 \vee \ldots \vee s_n \end{array}}{p_1 \wedge \ldots \wedge p_k \wedge r_1 \wedge \ldots \wedge r_m \Rightarrow q_1 \vee \ldots \vee q_l \vee s_1 \vee \ldots \vee s_n}$$

NB: If p_i on the left hand side of one sentence is the same as q_j in the right hand side of the other sentence, it is okay to drop the two symbols, with the proviso that *only one* such pair may be dropped.

NB: If a constant is repeated on the same side of a single sentence, all but one of the occurrences can be deleted.

Examples

$$\frac{\begin{array}{ccc} p & \Rightarrow & q \\ & \Rightarrow & p \end{array}}{\begin{array}{ccc} & \Rightarrow & q \end{array}} \qquad \frac{\begin{array}{ccc} p & \Rightarrow & q \\ q & \Rightarrow & \end{array}}{\begin{array}{ccc} p & \Rightarrow & \end{array}} \qquad \frac{\begin{array}{ccc} p & \Rightarrow & q \\ q & \Rightarrow & r \end{array}}{\begin{array}{ccc} p & \Rightarrow & r \end{array}}$$

Logic Problem Revisited

If Mary loves Pat, then Mary loves Quincy. If it is Monday, then Mary loves Pat or Quincy. If it is Monday, does Mary love Quincy?

$$
\begin{array}{ccc}
p & \Rightarrow & q \\
m & \Rightarrow & p \vee q \\
\hline
m & \Rightarrow & q \vee q \\
m & \Rightarrow & q
\end{array}
$$

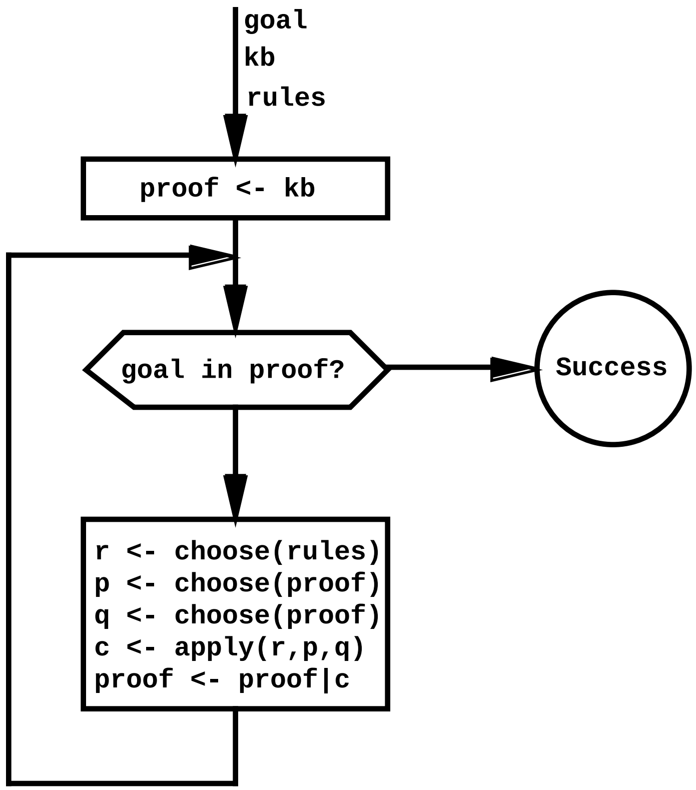
Logic Problem Concluded

Mary loves only one person at a time. If it is Monday, does Mary love Pat?

$$
\begin{array}{rcl}
m & \Rightarrow & q \\
p \wedge q & \Rightarrow & \\
\hline
m \wedge p & \Rightarrow &
\end{array}
$$

Computational Logic

Automated Reasoning



Comparison With Formal Logic

Formal Logic

Syntax, semantics, correctness and completeness

Emphasis on minimal sets of rules to simplify analysis

These rules are not always easy to implement or efficient

Computational Logic

Syntax, semantics, correctness, completeness

Also concerned with efficiency

Emphasis of different languages and different sets of rules

Attention to those that better suited to automation

Applications

Mathematics

Group Axioms

$$(x \times y) \times z = x \times (y \times z)$$

$$x \times e = x$$

$$e \times x = x$$

$$x \times x^{-1} = e$$

Theorem

$$x^{-1} \times x = e$$

Tasks:

Proof Checking

Proof Generation

Database Systems

Database in Tabular Form

parent

art	*bob*
art	*bea*
bea	*coe*

Database in Sentential Form

$$parent(art, bob)$$
$$parent(art, bea)$$
$$parent(bob, coe)$$

Constraints

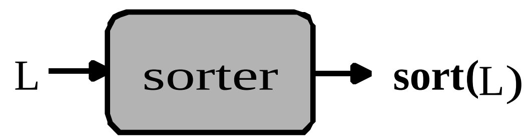
$$\neg parent(x, x)$$
$$parent(x, y) \Rightarrow \neg parent(y, x)$$

Definitions

$$parent(x, y) \wedge parent(y, z) \Rightarrow grandparent(x, z)$$

Program Verification

Program



Specification:

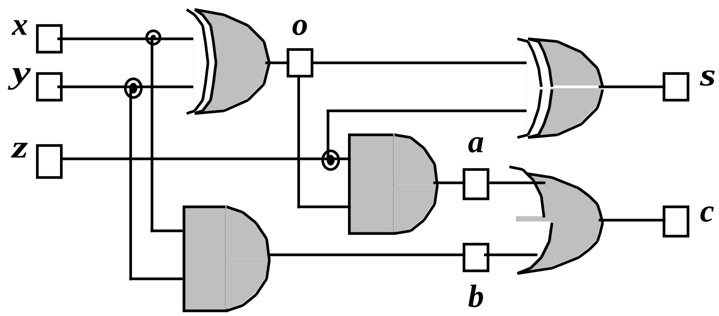
$$\forall i.\forall j.(i < j \Rightarrow sort(L)_i < sort(L)_j)$$

Tasks:

- Partial Evaluation
- Verification
- Proof of Termination
- Complexity Analysis

Hardware Engineering

Circuit:



Behavior:

$$o \Leftrightarrow (x \wedge \neg y) \vee (\neg x \wedge y)$$

$$a \Leftrightarrow z \wedge o$$

$$b \Leftrightarrow x \wedge y$$

$$s \Leftrightarrow (o \wedge \neg z) \vee (\neg o \wedge z)$$

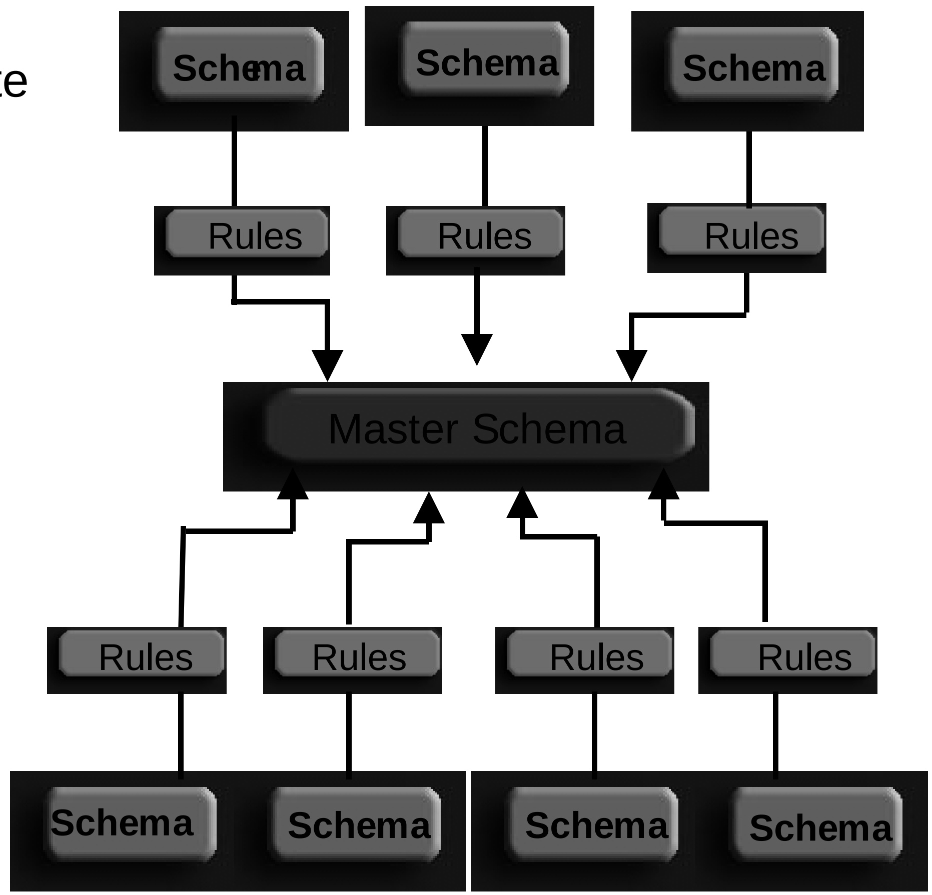
$$c \Leftrightarrow a \vee b$$

Applications:

- Simulation
- Configuration
- Diagnosis
- Test Generation

Information Integration

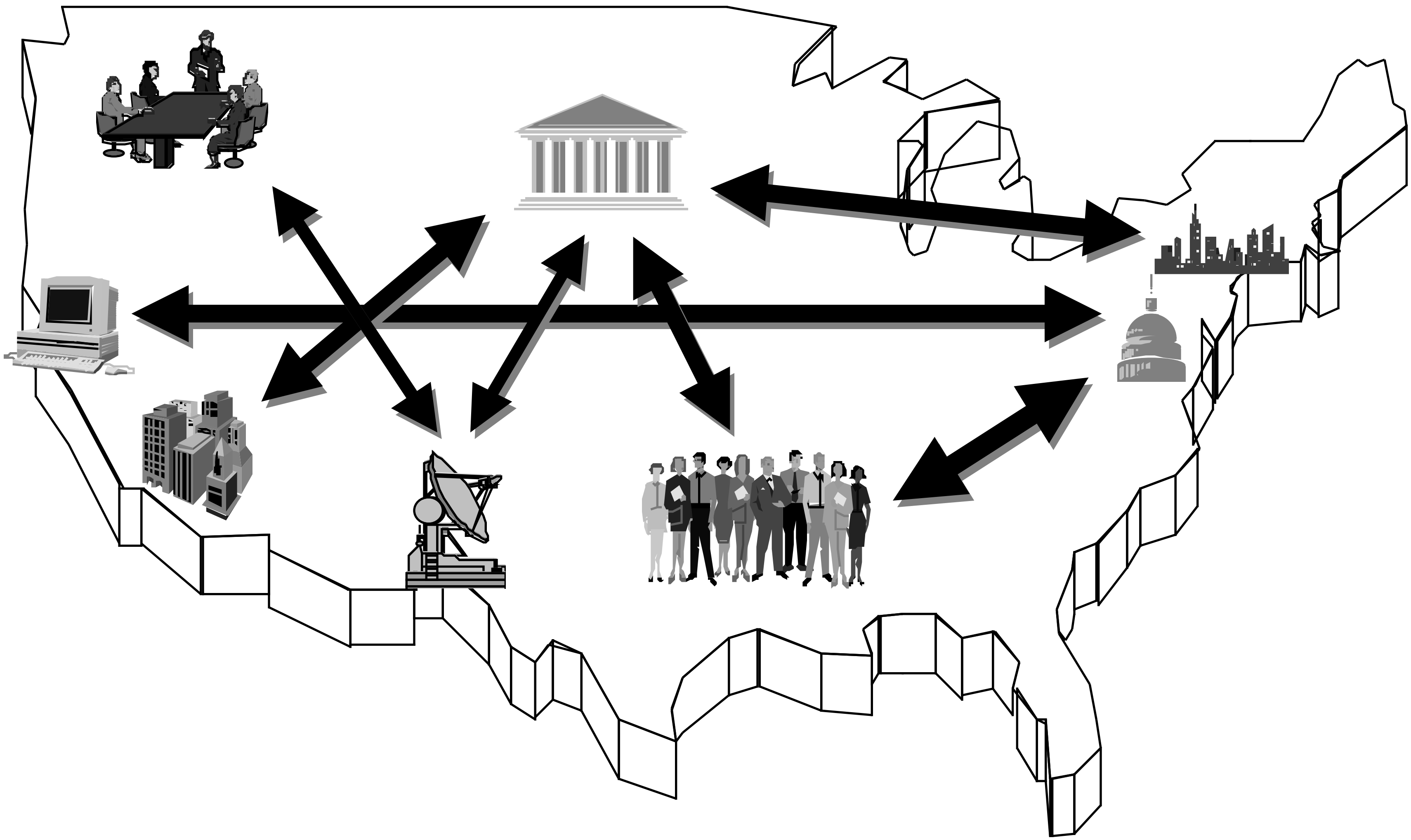
Consumers
Access and Update
Own Schema



Exchanges
Master Schema
Mapping Rules
Integration Data

Suppliers
Distributed Mgmt
Own Schema

Regulations and Business Rules



Logic Technology

Components
- Editors
- Automated Reasoning Systems
- Knowledge Bases

Some Popular Automated Reasoning Systems
- Boyer-Moore
- Otter
- PTTP
- Epilog

Knowledge Bases
- Definitions (e.g. *A bachelor is an unmarried adult male.*)
- Constraints (e.g. $PV=nRT$)
- Laws (e.g. *1040 necessary if earnings* > $10,000.)

Study Guide

Multiple Logics

Propositional Logic

If it is raining, the ground is wet.

Relational Logic

If x is a parent of y, then y is a child of x.

Metalevel Logic

John believes everything that Mary tells him.

Common Topics

Common Topics

- Syntax - expressions in the language
- Semantics - meaning of expressions
- Logical Entailment - premises and conclusions
- Proof Methods

Contrasts

- Expressiveness - operators, variables, expressions, ...
- Computational Hierarchy - linear, polynomial, decidable,...
- Tradeoffs - expressiveness versus computability

Meta

We will frequently write sentences *about* sentences.

Sentence: *When it rains, it pours.*
Metasentence: *That sentence contains a relative clause.*

We will frequently prove things about proofs.

Proofs: formal
Metaproofs: informal

Mike took it twice!

http://logic.stanford.edu/classes/cs157/cs157.html