

Computational Logic

Lecture 2

Propositional Logic

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Grammatical Complexity

The dog *chased* the cat.

The dog that *ate* the rat *chased* the cat.

The cherry *blossoms* in the spring ... *sank*.

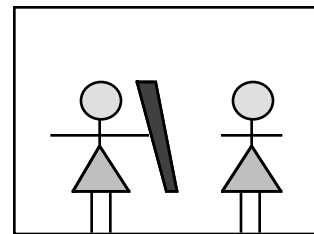
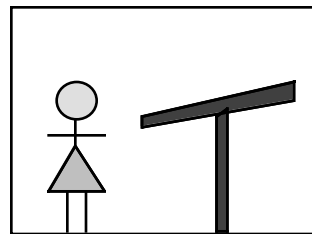
Ambiguity

Leland Stanford Junior University

Leland-Stanford Junior-University?

Leland-Stanford-Junior University?

There's a girl in the room with a telescope.



Ambiguity

Lettuce won't turn brown if you put your head
In a plastic bag before placing it in the
refrigerator.

The manager of a nudist colony complains that
a hole was cut in the wall surrounding the camp.
Police are looking into it.

from *Mistakes in Print*
edited by Kermit Shaefer

Propositional Constants

Examples:

raining

r32aining

rAiNiNg

rainingorsnowing

Non-Examples:

324567

raining.or.snowing

Compound Sentences

Negations:

$\neg \textit{raining}$

The argument of a negation is called the *target*.

Conjunctions:

$(\textit{raining} \wedge \textit{snowing})$

The arguments of a conjunction are called *conjuncts*.

Disjunctions:

$(\textit{raining} \vee \textit{snowing})$

The arguments of a disjunction are called *disjuncts*.

Compound Sentences (concluded)

Implications:

$$(raining \Rightarrow cloudy)$$

The left argument of an implication is the *antecedent*.

The right argument of an implication is the *consequent*.

Reductions:

$$(cloudy \Leftarrow raining)$$

The left argument of a reduction is the *consequent*.

The right argument of a reduction is the *antecedent*.

Equivalences:

$$(cloudy \Leftrightarrow raining)$$

Parenthesis Removal

Dropping Parentheses is good:

$$(p \wedge q) \rightarrow p \wedge q$$

But it can lead to ambiguities:

$$((p \vee q) \wedge r) \rightarrow p \wedge q \vee r$$

$$(p \vee (q \wedge r)) \rightarrow p \wedge q \vee r$$

Precedence

Parentheses can be dropped when the structure of an expression can be determined on the basis of precedence.

\neg
 \wedge
 \vee
 $\Rightarrow \Leftarrow \Leftrightarrow$

NB: An operand associates with operator of higher precedence.
If surrounded by operators of equal precedence, the operand associates with the operator to the right.

$$p \wedge q \vee r$$

$$p \vee q \wedge r$$

$$p \Rightarrow q \Rightarrow r$$

$$p \Rightarrow q \Leftarrow r$$

$$\neg p \wedge q$$

Propositional Logic Interpretation

A *propositional logic interpretation* is an association between the propositional constants in a propositional language and the truth values T or F.

$$p \xrightarrow{i} \text{T}$$

$$p^i = \text{T}$$

$$q \xrightarrow{i} \text{F}$$

$$q^i = \text{F}$$

$$r \xrightarrow{i} \text{T}$$

$$r^i = \text{T}$$

The notion of interpretation can be extended to all sentences by application of operator semantics.

Operator Semantics

Negation:

ϕ	$\neg\phi$
T	F
F	T

For example, if the interpretation of p is F, then the interpretation of $\neg p$ is T.

For example, if the interpretation of $\neg p$ is T, then the interpretation of $\neg\neg p$ is F.

Operator Semantics (continued)

Conjunction:

ϕ	ψ	$\phi \wedge \psi$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction:

ϕ	ψ	$\phi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	F

NB: The semantics of disjunction here is often called *inclusive or*, which says that a disjunction is true if and only if *at least* one of its disjuncts is true. This is in contrast with *exclusive or*, according to which a disjunction is true if and only if an odd number of its disjuncts is true. What is the truth table for exclusive or?

Operator Semantics (continued)

Implication:

ϕ	ψ	$\phi \Rightarrow \psi$
T	T	T
T	F	F
F	T	T
F	F	T

Reduction:

ϕ	ψ	$\phi \Leftarrow \psi$
T	T	T
T	F	T
F	T	F
F	F	T

NB: The semantics of implication here is called *material implication*. It has the peculiar characteristic that any implication is true if the antecedent is false, whether or not there is a connection to the consequent. For example, the following is a true sentence.

If George Washington is alive, I am a billionaire.

Operator Semantics (concluded)

Equivalence:

ϕ	ψ	$\phi \Leftrightarrow \psi$
T	T	T
T	F	F
F	T	F
F	F	T

Evaluation

Interpretation i :

$$p^i = \text{T}$$

$$q^i = \text{F}$$

$$r^i = \text{T}$$

Compound Sentence

$$(p \vee q) \wedge (\neg q \vee r)$$

Multiple Interpretations

Logic does not prescribe which interpretation is “correct”. In the absence of additional information, one interpretation is as good as another.

Interpretation i

$$p^i = T$$

$$q^i = F$$

$$r^i = T$$

Interpretation j

$$p^j = F$$

$$q^j = F$$

$$r^j = T$$

Examples:

Different days of the week

Different locations

Beliefs of different people

Truth Tables

A *truth table* is a table of all possible interpretations for the propositional constants in a language.

p	q	r
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

One column per constant.

One row per interpretation.

For a language with n constants, there are 2^n interpretations.

Evaluation and Disambiguation

Evaluation:

$$\begin{array}{l} p^i = \text{T} \\ q^i = \text{F} \end{array} \longrightarrow \begin{array}{l} (p \vee q)^i = \text{T} \\ (\neg q)^i = \text{T} \end{array}$$

Disambiguation:

$$\begin{array}{l} (p \vee q)^i = \text{T} \\ (\neg q)^i = \text{T} \end{array} \longrightarrow \begin{array}{l} p^i = \text{T} \\ q^i = \text{F} \end{array}$$

Disambiguation

By crossing out rows, it is possible to find interpretations implicit in a set of sentences.

<i>p</i>	<i>q</i>	<i>r</i>
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

Disambiguation

By crossing out rows, it is possible to find interpretations implicit in a set of sentences.

$$q \Rightarrow r$$

p	q	r	
1	1	1	
1	1	0	×
1	0	1	
1	0	0	
0	1	1	
0	1	0	×
0	0	1	
0	0	0	

Disambiguation

By crossing out rows, it is possible to find interpretations implicit in a set of sentences.

	p	q	r	
$q \Rightarrow r$	<u>1</u>	<u>1</u>	<u>1</u>	
	1	1	0	×
$p \Rightarrow q \wedge r$	1	0	1	×
	1	0	0	×
	0	1	1	
	0	1	0	×
	0	0	1	
	0	0	0	

Disambiguation

By crossing out rows, it is possible to find interpretations implicit in a set of sentences.

	p	q	r	
$q \Rightarrow r$	1	1	1	×
	1	1	0	×
$p \Rightarrow q \wedge r$	1	0	1	×
	1	0	0	×
$\neg r$	0	1	1	×
	0	1	0	×
	0	0	1	×
	0	0	0	

Properties of Sentences

Valid
Contingent
Unsatisfiable

A sentence is *valid* if and only if *every* interpretation satisfies it.

A sentence is *contingent* if and only if *some* interpretation satisfies it and *some* interpretation falsifies it.

A sentence is *unsatisfiable* if and only if *no* interpretation satisfies it.

Example of Validity

p	q	r	$(p \Rightarrow q)$	$(q \Rightarrow r)$	$(p \Rightarrow q) \vee (q \Rightarrow r)$
1	1	1			
1	1	0			
1	0	1			
1	0	0			
0	1	1			
0	1	0			
0	0	1			
0	0	0			

More Validities

Double Negation:

$$p \Leftrightarrow \neg\neg p$$

deMorgan's Laws:

$$\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$$

$$\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$$

Implication Introduction:

$$p \Rightarrow (q \Rightarrow p)$$

Implication Distribution

$$(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$$

The Big Game

Stanford people always tell the truth, and Berkeley people always lie. Unfortunately, by looking at a person, you cannot tell whether he is from Stanford or Berkeley.

You come to a fork in the road and want to get to the football stadium down one fork. However, you do not know which to take. There is a person standing there. What single question can you ask him to help you decide which fork to take?

Basic Idea

<i>left</i>	<i>su</i>	<i>Question</i>	<i>Response</i>
T	T		
T	F		
F	T		
F	F		

The Big Game Solved

Question: *The left road the way to the stadium if and only if you from Stanford. Is that correct?*

left \Leftrightarrow *su*