

CMSC 631, Spring 2006  
Homework 1  
Due Thursday, February 9, in class

1. Let  $S$  be a finite set, and let  $L$  be the lattice of subsets of  $S$ , with order  $\subseteq$ . Show that any function  $f(x)$  constructed from union, intersection, and constant sets is monotonic. Here, I mean that  $f(x) = e$  where  $e$  can be specified by the grammar

$$e ::= x \mid S' \mid e \cup e \mid e \cap e$$

where  $S'$  is any subset of  $S$ . Your proof should be by induction on the structure of  $e$ .

2. Suppose that we extend the grammar for  $e$  from problem 1 to include the complement operator  $!e$ , where  $!T = S - T$ . Is  $f$  still guaranteed to be monotonic? If it is, justify your answer. If it's not, explain why a transfer function defined by  $Out(stmt) = Gen(stmt) \cup (In(stmt) - Kill(stmt))$ , which seems to include negation, is monotonic.
3. Let  $A$  be a lattice, with order  $\leq$ . Define  $A \rightarrow A$  to be the set of all functions from  $A$  to  $A$ , and define  $f \leq' g$  if  $f(x) \leq g(x)$  for all  $x \in A$ .
- Show that  $A \rightarrow A$  with order  $\leq'$  is also a lattice. That is, show that for all  $f, g \in A \rightarrow A$ ,  $f \sqcup g$  and  $f \sqcap g$  always exist.
  - Suppose lattice  $A$  has height  $h$  and that  $A$  is finite with  $n$  elements. What is the height of the lattice  $(A \rightarrow A, \leq')$ ? (When counting height, count "edges" rather than "nodes," e.g., if  $A$  were the lattice  $\{a, b\}$  with  $a < b$ , then its height would be 1.)

4. (ASU, exercise 10.35) In class we talked about how an analysis is *conservative* if it models the behavior of the program in a way that is safe. As it turns out, "safe" is in the eye of the beholder.

When performing dataflow analysis to estimate the following properties, determine whether too-large or too-small estimates are conservative. Explain your answer in terms of the intended use of the information. (Hint: This is a bit of a trick question.)

- (a) Available expressions
- (b) Variables changed by a procedure
- (c) Variables not changed by a procedure
- (d) Copy statements reaching a given program point