1. Let $S$ be a finite set, and let $L$ be the lattice of subsets of $S$, with order $\subseteq$. Show that any function $f(x)$ constructed from union, intersection, and constant sets is monotonic. Here, I mean that $f(x) = e$ where $e$ can be specified by the grammar

$$e ::= x \mid S' \mid e \cup e \mid e \cap e$$

where $S'$ is any subset of $S$. Your proof should be by induction on the structure of $e$.

2. Suppose that we extend the grammar for $e$ from problem 1 to include the complement operator $!e$, where $!T = S - T$. Is $f$ still guaranteed to be monotonic? If it is, justify your answer. If it’s not, explain why a transfer function defined by $\text{Out}(\text{stmt}) = \text{Gen}(\text{stmt}) \cup (\text{In}(\text{stmt}) - \text{Kill}(\text{stmt}))$, which seems to include negation, is monotonic.

3. Let $A$ be a lattice, with order $\leq$. Define $A \rightarrow A$ to be the set of all functions from $A$ to $A$, and define $f \leq' g$ if $f(x) \leq g(x)$ for all $x \in A$.

- Show that $A \rightarrow A$ with order $\leq'$ is also a lattice. That is, show that for all $f, g \in A \rightarrow A$, $f \cup g$ and $f \cap g$ always exist.
- Suppose lattice $A$ has height $h$ and that $A$ is finite with $n$ elements. What is the height of the lattice $(A \rightarrow A, \leq')$? (When counting height, count “edges” rather than “nodes,” e.g., if $A$ were the lattice $\{a, b\}$ with $a < b$, then its height would be 1.)

4. (ASU, exercise 10.35) In class we talked about how an analysis is conservative if it models the behavior of the program in a way that is safe. As it turns out, “safe” is in the eye of the beholder.

When performing dataflow analysis to estimate the following properties, determine whether too-large or too-small estimates are conservative. Explain your answer in terms of the intended use of the information. (Hint: This is a bit of a trick question.)

(a) Available expressions
(b) Variables changed by a procedure
(c) Variables not changed by a procedure
(d) Copy statements reaching a given program point