CMSC 631, Spring 2006
Homework 5
Due Thursday, March 16, in class

1. Consider the following program, written in lambda calculus with tuples, integers, and strings:

   $let\ app2 = \lambda fxy.(f\ x, f\ y)\ in$
   $\quad app2\ (\lambda x.x)\ 1$ "foo"

   Write down the type for *app2* in simply-typed lambda calculus with Hindley-Milner style polymorphism. Does this program exhibit any run-time errors (i.e., will its evaluation ever be stuck)? Does the program type check using Hindley-Milner style polymorphism? Explain what goes wrong. Can you give a type for *app2* that is polymorphic but not Hindley-Milner such that this program would type check? (Note: You will not be able to construct a most-general type for *app2* without using intersection types, which we have not discussed, but your type should work for this particular use of *app2*.)

2. Consider the lambda calculus with Hindley-Milner style polymorphism. Here $e ::= x \mid \lambda x.e \mid e\ e \mid let\ x = e\ in\ e$ and types are given by $\sigma ::= \forall\alpha.\sigma \mid \tau$ and $\tau ::= \alpha \mid int \mid \tau \to \tau$, with the following type rules:

   $$\textsc{Var}\quad \frac{\Gamma(x) = \forall\vec{\alpha}.\tau \qquad \text{any } \vec{\tau}}{\Gamma \vdash x : \tau[\vec{\alpha} \mapsto \vec{\tau}]} \qquad\qquad \textsc{Lam}\quad \frac{\Gamma, x : \tau \vdash e : \tau'}{\Gamma \vdash \lambda x.e : \tau \to \tau'}$$

   $$\textsc{App}\quad \frac{\Gamma \vdash e_1 : \tau \to \tau' \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1\ e_2 : \tau'} \qquad\qquad \textsc{Let}\quad \frac{\Gamma \vdash e_1 : \tau_1 \qquad \vec{\alpha} = FV(\tau_1) - FV(\Gamma) \qquad \Gamma, x : \forall\vec{\alpha}.\tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash let\ x = e_1\ in\ e_2 : \tau_2}$$

   Here $FV(\cdot)$ stands for the free variables of a type or type environment. Prove the *polymorphic substitution lemma*, which is used in proving subject reduction: If $\Gamma, x : \forall\vec{\alpha}.\tau_1 \vdash e_2 : \tau_2$ where $\vec{\alpha} = FV(\tau_1) - FV(\Gamma)$ and $\Gamma \vdash e_1 : \tau_1$, then $\Gamma \vdash e_2[x \mapsto e_1] : \tau_2$.