Trees

- Trees are hierarchical data structures
  - One-to-many relationship between elements

- Tree node / element
  - Contains data
  - Referred to by only 1 (parent) node
  - Contains links to any number of (children) nodes
Trees

Terminology

- **Root** ⇒ node with no parent
- **Leaf** ⇒ all nodes with no children
- **Interior** ⇒ all nodes with children

![Tree diagram with labels for root, interior, and leaf nodes]
Trees

Terminology

- Sibling ⇒ node with same parent
- Descendant ⇒ children nodes & their descendents
- Subtree ⇒ portion of tree that is a tree by itself
  ⇒ a node and its descendents
Trees

Terminology

- **Level** \(\Rightarrow\) is a measure of a node’s distance from root
- **Definition of level**
  - If node is the root of the tree, its level is 1
  - Else, the node’s level is 1 + its parent’s level
- **Height (depth)** \(\Rightarrow\) max level of any node in tree

Height = 3
Binary Trees

- Binary tree
  - Tree with 0–2 children per node
  - Left & right child / subtree

Binary Tree
Tree Traversal

Often we want to
1. Find all nodes in tree
2. Determine their relationship

Can do this by
1. Walking through the tree in a prescribed order
2. Visiting the nodes as they are encountered

Process is called tree traversal
Tree Traversal

Goal

Visit every node in binary tree

Approaches

Depth first

Preorder ⇒ parent before children
Inorder ⇒ left child, parent, right child
Postorder ⇒ children before parent

Breadth first ⇒ closer nodes first
Tree Traversal Methods

- **Pre-order**
  1. Visit node // first
  2. Recursively visit left subtree
  3. Recursively visit right subtree

- **In-order**
  1. Recursively visit left subtree
  2. Visit node // second
  3. Recursively visit right subtree

- **Post-order**
  1. Recursively visit left subtree
  2. Recursively visit right subtree
  3. Visit node // last
Tree Traversal Methods

Breadth-first

BFS(Node n) {
    Queue Q = new Queue();
    Q.enqueue(n); // insert node into Q
    while ( !Q.empty()) {
        n = Q.dequeue(); // remove next node
        if ( !n.isEmpty()) {
            visit(n); // visit node
            Q.enqueue(n.Left()); // insert left subtree in Q
            Q.enqueue(n.Right()); // insert right subtree in Q
        }
    }
}
Tree Traversal Examples

- **Pre-order (prefix)**
  - $+ \times 2 \ 3 / \ 8 \ 4$

- **In-order (infix)**
  - $2 \times 3 + 8 / 4$

- **Post-order (postfix)**
  - $2 \ 3 \times 8 \ 4 / +$

- **Breadth-first**
  - $+ \times / \ 2 \ 3 \ 8 \ 4$

Expression tree
Tree Traversal Examples

- **Pre-order**
  - 44, 17, 32, 78, 50, 48, 62, 88

- **In-order**
  - 17, 32, 44, 48, 50, 62, 78, 88

- **Post-order**
  - 32, 17, 48, 62, 50, 88, 78, 44

- **Breadth-first**
  - 44, 17, 78, 32, 50, 88, 48, 62

Binary search tree
Types of Binary Trees

- **Degenerate**
  - Mostly 1 child / node
  - Height = $O(n)$
  - Similar to linear list

- **Balanced**
  - Mostly 2 child / node
  - Height = $O(\log(n))$
  - Useful for searches

Degenerate binary tree

Balanced binary tree
Binary Search Trees

Key property
- Value at node
  - Smaller values in left subtree
  - Larger values in right subtree
- Example
  - \( X > Y \)
  - \( X < Z \)
Binary Search Trees

Examples

Binary search trees

Non-binary search tree
Binary Tree Implementation

Class Node {
    Value data;
    Node left, right; // null if empty

    void insert ( Value data1 ) { ... }
    void delete ( Value data2 ) { ... }
    Node find ( Value data3 ) { ... }
    ...
}

Iterative Search of Binary Tree

Node Find( Node n, Value key) {
    while (n != null) {
        if (n.data == key)   // Found it
            return n;
        if (n.data > key)   // In left subtree
            n = n.left;
        else                // In right subtree
            n = n.right;
    }
    return null;
}

Find( root, keyValue );
Recursive Search of Binary Tree

Node Find( Node n, Value key) {
    if (n == null) // Not found
        return( n );
    else if (n.data == key) // Found it
        return( n );
    else if (n.data > key) // In left subtree
        return Find( n.left, key );
    else // In right subtree
        return Find( n.right, key );
}

Find( root, keyValue );
Example Binary Searches

Find (2)

10
→
5
→
2
2
→
10 > 2, left
5
→
30
→
25

5 > 2, left
2 = 2, found

45

2
→
5
→
10
→
30
→
25

5 > 2, left
2 = 2, found
Example Binary Searches

Find (25)

10 < 25, right
30 > 25, left
25 = 25, found

5 < 25, right
45 > 25, left
30 > 25, left
10 < 25, right
25 = 25, found
Binary Search Properties

- **Time of search**
  - Proportional to height of tree
  - Balanced binary tree
    - $O( \log(n) )$ time
  - Degenerate tree
    - $O( n )$ time
    - Like searching linked list / unsorted array

- **Requires**
  - Ability to compare key values
Binary Search Tree Construction

How to build & maintain binary trees?
- Insertion
- Deletion

Maintain key property (invariant)
- Smaller values in left subtree
- Larger values in right subtree
Binary Search Tree – Insertion

Algorithm
1. Perform search for value X
2. Search will end at node Y (if X not in tree)
3. If X < Y, insert new leaf X as new left subtree for Y
4. If X > Y, insert new leaf X as new right subtree for Y

Observations
- O( log(n) ) operation for balanced tree
- Insertions may unbalance tree
Example Insertion

Insert ( 20 )

- 10 < 20, right
- 30 > 20, left
- 25 > 20, left
- Insert 20 on left
Binary Search Tree – Deletion

Algorithm
1. Perform search for value X
2. If X is a leaf, delete X
3. Else  // must delete internal node
   a) Replace with largest value Y on left subtree
      OR smallest value Z on right subtree
   b) Delete replacement value (Y or Z) from subtree

Observation
- O( log(n) ) operation for balanced tree
- Deletions may unbalance tree
Example Deletion (Leaf)

Delete (25)

10 < 25, right
30 > 25, left
25 = 25, delete
Example Deletion (Internal Node)

Delete (10)

Replacing 10 with largest value in left subtree
Replacing 5 with largest value in left subtree
Deleting leaf
Example Deletion (Internal Node)

Delete (10)

Replacing 10 with smallest value in right subtree

Deleting leaf

Resulting tree
Building Maps w/ Search Trees

- Search trees often used to implement maps
  - Each non-empty node contains
    - Key
    - Value
    - Left and right child

- Need to be able to compare keys
  - Generic type <K extends Comparable<K>>
    - Denotes any type K that can be compared to K’s
Polymorphic Binary Search Trees

What do we mean by polymorphic?

Implement two subtypes of Tree

1. EmptyTree
2. NonEmptyTree

Use EmptyTree to represent the empty tree

- Rather than null

Invoke methods on tree nodes

- Without checking for null
- Get empty or nonempty functionality

- Selected by type of tree node
Polymorphic Binary Tree Implement.

Interface Tree {
    Tree insert ( Value data1 ) { … }
}

Class EmptyTree implements Tree {
    Tree insert ( Value data1 ) { … }
}

Class NonEmptyTree implements Tree {
    Value data;
    Tree left, right; // Either Empty or NonEmpty
    Tree insert ( Value data1 ) { … }
}
Example : Standard Binary Tree

Class Node {
    Node left, right;
}

Node X {
    left = Y;
    right = Z;
}

Node Y {
    left = null;
    right = null;
}

Node Z {
    left = null;
    right = W;
}

Node W {
    left = null;
    right = null;
}
Example: Polymorphic Binary Tree

Class EmptyTree {
}

Class NonEmptyTree {
    Tree left, right;
}

NonEmpty X {
    left = Y;
    right = Z;
}

NonEmpty Y {
    left = S;
    right = S;
}

NonEmpty Z {
    left = S;
    right = W;
}

NonEmpty W {
    left = S;
    right = S;
}

EmptyTree S {
}