Trees & Binary Search Trees

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Trees

- Trees are hierarchical data structures
  - One-to-many relationship between elements

- Tree node / element
  - Contains data
  - Referred to by only 1 (parent) node
  - Contains links to any number of (children) nodes
Trees

Terminology

- **Root** ⇒ node with no parent
- **Leaf** ⇒ all nodes with no children
- **Interior** ⇒ all nodes with children

Sibling

- **Sibling** ⇒ node with same parent

Descendent

- **Descendent** ⇒ children nodes & their descendents

Subtree

- **Subtree** ⇒ portion of tree that is a tree by itself
  - ⇒ a node and its descendents
Trees

Terminology
- Level ⇒ is a measure of a node’s distance from root
- Definition of level
  - If node is the root of the tree, its level is 1
  - Else, the node’s level is 1 + its parent’s level
- Height (depth) ⇒ max level of any node in tree

Binary Trees

Binary tree
- Tree with 0–2 children per node
  - Left & right child / subtree

Binary Tree
Tree Traversal

- Often we want to
  1. Find all nodes in tree
  2. Determine their relationship

- Can do this by
  1. Walking through the tree in a prescribed order
  2. Visiting the nodes as they are encountered

- Process is called tree traversal

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Tree Traversal

- Goal
  - Visit every node in binary tree

- Approaches
  - Depth first
    - Preorder ⇒ parent before children
    - Inorder ⇒ left child, parent, right child
    - Postorder ⇒ children before parent
  - Breadth first ⇒ closer nodes first
Tree Traversal Methods

- **Pre-order**
  1. Visit node // first
  2. Recursively visit left subtree
  3. Recursively visit right subtree

- **In-order**
  1. Recursively visit left subtree
  2. Visit node // second
  3. Recursively right subtree

- **Post-order**
  1. Recursively visit left subtree
  2. Recursively visit right subtree
  3. Visit node // last

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**Tree Traversal Methods**

- **Breadth-first**

```java
BFS(Node n) {
    Queue Q = new Queue();
    Q.enqueue(n); // insert node into Q
    while ( !Q.empty()) {
        n = Q.dequeue(); // remove next node
        if ( !n.isEmpty()) {
            visit(n); // visit node
            Q.enqueue(n.Left()); // insert left subtree in Q
            Q.enqueue(n.Right()); // insert right subtree in Q
        }
    }
}
```
Tree Traversal Examples

- **Pre-order (prefix)**
  - $+ \times 2 \ 3 \ / 8 \ 4$

- **In-order (infix)**
  - $2 \times 3 + 8 \ / 4$

- **Post-order (postfix)**
  - $2 \ 3 \times 8 \ 4 \ / +$

- **Breadth-first**
  - $+ \times / 2 \ 3 \ 8 \ 4$

Expression tree

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Tree Traversal Examples

- **Pre-order**
  - $44, 17, 32, 78, 50, 48, 62, 88$

- **In-order**
  - $17, 32, 44, 48, 50, 62, 78, 88$

- **Post-order**
  - $32, 17, 48, 62, 50, 88, 78, 44$

- **Breadth-first**
  - $44, 17, 78, 32, 50, 88, 48, 62$

Sorted order!

Binary search tree
**Types of Binary Trees**

- **Degenerate**
  - Mostly 1 child / node
  - Height = $O(n)$
  - Similar to linear list

- **Balanced**
  - Mostly 2 child / node
  - Height = $O(\log(n))$
  - Useful for searches

![Degenerate binary tree](image1)

![Balanced binary tree](image2)

**Binary Search Trees**

- **Key property**
  - Value at node
    - Smaller values in left subtree
    - Larger values in right subtree

- **Example**
  - $X > Y$
  - $X < Z$
Binary Search Trees

Examples

Binary search trees

Non-binary search tree

Binary Tree Implementation

Class Node {
    Value data;
    Node left, right;  // null if empty

    void insert ( Value data1 ) { ... }
    void delete ( Value data2 ) { ... }
    Node find ( Value data3 ) { ... }

    ... }
Iterative Search of Binary Tree

Node Find( Node n, Value key) {
    while (n != null) {
        if (n.data == key) // Found it
            return n;
        if (n.data > key) // In left subtree
            n = n.left;
        else // In right subtree
            n = n.right;
    }
    return null;
}
Find( root, keyValue );

Recursive Search of Binary Tree

Node Find( Node n, Value key) {
    if (n == null) // Not found
        return( n );
    else if (n.data == key) // Found it
        return( n );
    else if (n.data > key) // In left subtree
        return Find( n.left, key );
    else // In right subtree
        return Find( n.right, key );
}
Find( root, keyValue );
Example Binary Searches

Find (2)

10 > 2, left
5 > 2, left
2 = 2, found

Find (25)

10 < 25, right
30 > 25, left
25 = 25, found

5 < 25, right
45 > 25, left
30 > 25, left
10 < 25, right
25 = 25, found
Binary Search Properties

- Time of search
  - Proportional to height of tree
  - Balanced binary tree
    - O( \log(n) ) time
  - Degenerate tree
    - O( n ) time
    - Like searching linked list / unsorted array

- Requires
  - Ability to compare key values

Binary Search Tree Construction

- How to build & maintain binary trees?
  - Insertion
  - Deletion

- Maintain key property (invariant)
  - Smaller values in left subtree
  - Larger values in right subtree
Binary Search Tree – Insertion

Algorithm
1. Perform search for value X
2. Search will end at node Y (if X not in tree)
3. If X < Y, insert new leaf X as new left subtree for Y
4. If X > Y, insert new leaf X as new right subtree for Y

Observations
- $O(\log(n))$ operation for balanced tree
- Insertions may unbalance tree

Example Insertion

Insert (20)

10 < 20, right
30 > 20, left
25 > 20, left
Insert 20 on left
**Binary Search Tree – Deletion**

**Algorithm**
1. Perform search for value X
2. If X is a leaf, delete X
3. Else // must delete internal node
   a) Replace with largest value Y on left subtree OR smallest value Z on right subtree
   b) Delete replacement value (Y or Z) from subtree

**Observation**
- $O(\log(n))$ operation for balanced tree
- Deletions may unbalance tree

**Example Deletion (Leaf)**

Delete (25)

```
  10
  /   \
 5 30   \
/ \
2 25 45
```

$10 < 25$, right

```
  10
  /   \
 5 30   \
/ \
2 45
```

$30 > 25$, left

```
  10
  /   \
 5 30   \
/ 
2
```

$25 = 25$, delete

```
  10
  /   \
 5 45
```


Example Deletion (Internal Node)

Delete (10)

Replacing 10 with largest value in left subtree
Replacing 5 with largest value in left subtree
Deleting leaf

Replacing 10 with smallest value in right subtree
Deleting leaf
Resulting tree
Building Maps w/ Search Trees

- Search trees often used to implement maps
  - Each non-empty node contains
    - Key
    - Value
    - Left and right child

- Need to be able to compare keys
  - Generic type <K extends Comparable<K>>
    - Denotes any type K that can be compared to K’s

Polymorphic Binary Search Trees

- What do we mean by polymorphic?
- Implement two subtypes of Tree
  1. EmptyTree
  2. NonEmptyTree
- Use EmptyTree to represent the empty tree
  - Rather than null
- Invoke methods on tree nodes
  - Without checking for null
  - Get empty or nonempty functionality
    - Selected by type of tree node
Polymorphic Binary Tree Implement.

Interface Tree {
    Tree insert ( Value data1 ) { … }
}

Class EmptyTree implements Tree {
    Tree insert ( Value data1 ) { … }
}

Class NonEmptyTree implements Tree {
    Value data;
    Tree left, right; // Either Empty or NonEmpty
    Tree insert ( Value data1 ) { … }
}

Example: Standard Binary Tree

Class Node {
    Node left, right;
}

Node X {
    left = Y;
    right = Z;
}

Node Y {
    left = null;
    right = null;
}

Node Z {
    left = null;
    right = W;
}

Node W {
    left = null;
    right = null;
}
Example: Polymorphic Binary Tree

Class EmptyTree {
}
Class NonEmptyTree {
    Tree left, right;
}

NonEmpty X {
    left = Y;
    right = Z;
}

NonEmpty Y {
    left = S;
    right = S;
}

NonEmpty Z {
    left = S;
    right = W;
}

NonEmpty W {
    left = S;
    right = S;
}

EmptyTree S {
}