# CMSC 132: Object-Oriented Programming II

## Recursive Algorithms

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# Recursion

- Recursion is a strategy for solving problems
  - A procedure that calls itself
- Approach

If ( problem instance is simple / trivial )

Solve it directly

Else

1. Simplify problem instance into smaller instance(s) of the original problem
2. Solve smaller instance using same algorithm
3. Combine solution(s) to solve original problem

# Recursive Algorithm Format

1. Base case
   - Solve small problem directly
2. Recursive step
   - Simplify problem into smaller subproblem(s)
   - Recursively apply algorithm to subproblem(s)
   - Calculate overall solution

# Example – Find

- To find an element in an array
  - Base case
    - If array is empty, return false
  - Recursive step
    - If $1^{st}$ element of array is given value, return true
    - Skip $1^{st}$ element and recur on remainder of array

# Example – Count

- To count # of elements in an array
  - Base case
    - If array is empty, return 0
  - Recursive step
    - Skip 1st element and recur on remainder of array
    - Add 1 to result

# Example – Factorial

- **Factorial definition**
  - **$n! = n \times n\text{-}1 \times n\text{-}2 \times n\text{-}3 \times \ldots \times 3 \times 2 \times 1$**
  - **$0! = 1$**
- **To calculate factorial of n**
  - **Base case**
    - **If n = 0, return 1**
  - **Recursive step**
    - **Calculate the factorial of n-1**
    - **Return n × (the factorial of n-1)**

# Example – Factorial

- Code

```
int fact ( int n ) {
    if ( n == 0 ) return 1;          // base case
    return n * fact(n-1);            // recursive step
}
```

# Properties

- Recursion relies on the call stack
  - State of current procedure is saved when procedure is recursively invoked
  - Every procedure invocation gets own stack space

- Any problem solvable with recursion may be solved with iteration (and vice versa)
  - Use iteration with explicit stack to store state
  - Algorithm may be simpler for one approach

# Recursion vs. Iteration

- Recursive algorithm

```
int fact ( int n ) {
    if ( n == 0 ) return 1;
    return n * fact(n-1);
}
```

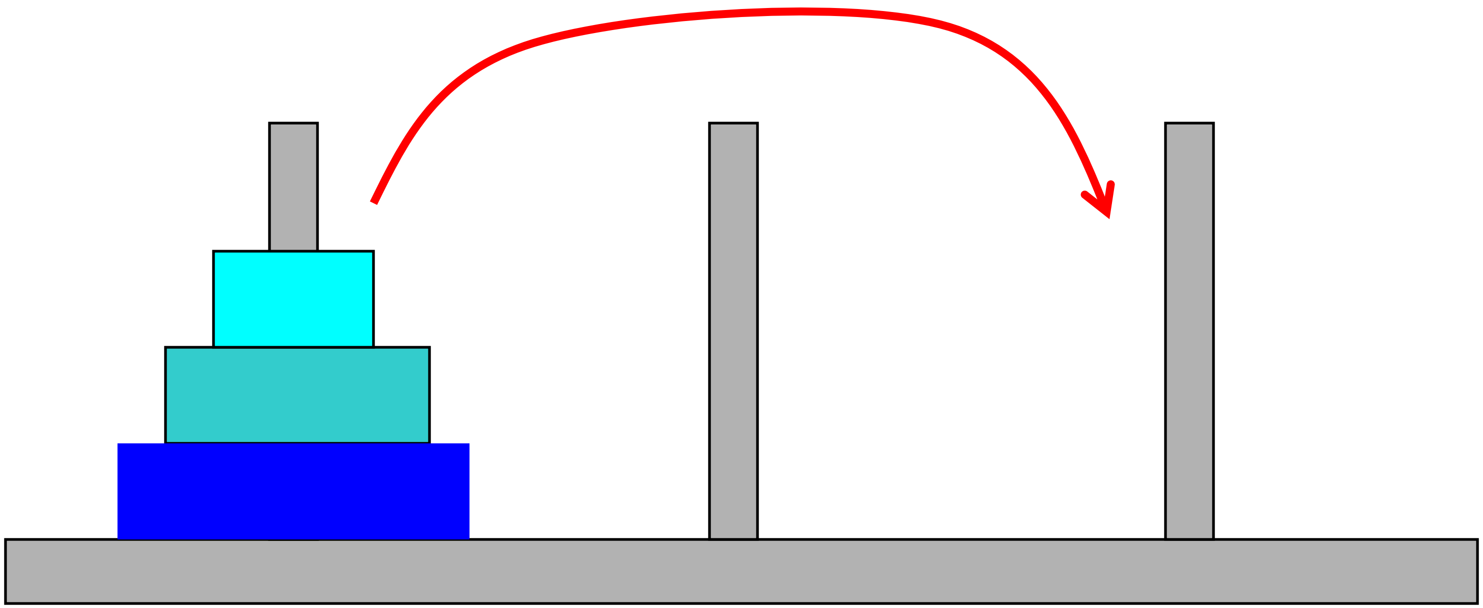
- Iterative algorithm

```
int fact ( int n ) {
    int i, res;
    res = 1;
    for (i=n; i>0; i--) {
        res = res * i;
    }
    return res;
}
```

Recursive algorithm is closer to factorial definition

# Example – Towers of Hanoi

- **Problem**
  - **Move stack of disks between pegs**
  - **Can only move top disk in stack**
  - **Only allowed to place disk on top of larger disk**



# Example – Towers of Hanoi

- To move a stack of n disks from peg X to Y
  - Base case
    - If n = 1, move disk from X to Y
  - Recursive step
    1. Move top n-1 disks from X to 3rd peg
    2. Move bottom disk from X to Y
    3. Move top n-1 disks from 3rd peg to Y

Iterative algorithm would take much longer to describe!

# Recursion vs. Iteration

- **Iterative algorithms**
  - **May be more efficient**
    - **No additional function calls**
    - **Run faster, use less memory**

# Recursion vs. Iteration

- Recursive algorithms
  - Higher overhead
    - Time to perform function call
    - Memory for call stack
  - May be simpler algorithm
    - Easier to understand, debug, maintain
  - Natural for backtracking searches
  - Suited for recursive data structures
    - Trees, graphs…

# Making Recursion Work

- Designing a correct recursive algorithm
- Verify
  1. Base case is
     - Recognized correctly
     - Solved correctly
  2. Recursive case
     - Solves 1 or more simpler subproblems
     - Can calculate solution from solution(s) to subproblems
- Uses principle of proof by induction

# Requirements

- Must have
  - Small version of problem solvable without recursion
  - Strategy to simplify problem into 1 or more smaller subproblems
  - Ability to calculate overall solution from solution(s) to subproblem(s)

# Proof By Induction

- Mathematical technique
- A theorem is true for all $n \geq 0$ if
  1. Base case
     - Prove theorem is true for $n = 0$, and
  2. Inductive step
     - Assume theorem is true for $n$
       (inductive hypothesis)
     - Prove theorem must be true for $n+1$

# Types of Recursion

- **Tail recursion**
  - **Single recursive call at end of function**
  - **Example**

```
int tail( int n ) {
    …
    return function( tail(n-1) );
}
```

  - **Can easily transform to iteration (loop)**

# Types of Recursion

- **Non-tail recursion**
  - Recursive call(s) not at end of function
  - Example

```
int nontail( int n ) {
    …
    x =  nontail(n-1) ;
    y =  nontail(n-2) ;
    z = x + y;
    return z;
}
```

  - Can transform to iteration using explicit stack

# Possible Problems – Infinite Loop

- **Infinite recursion**
  - **If recursion not applied to simpler problem**

```
int bad ( int n ) {
    if ( n == 0 ) return 1;
    return bad(n);
}
```

  - **Will infinite loop**
  - **Eventually halt when runs out of (stack) memory**
    - **Stack overflow**

# Possible Problems – Efficiency

- May perform excessive computation
  - If recomputing solutions for subproblems
- Example
  - Fibonacci numbers
    - fibonacci(0) = 1
    - fibonacci(1) = 1
    - fibonacci(n) = fibonacci(n-1) + fibonacci(n-2)

# Possible Problems – Efficiency

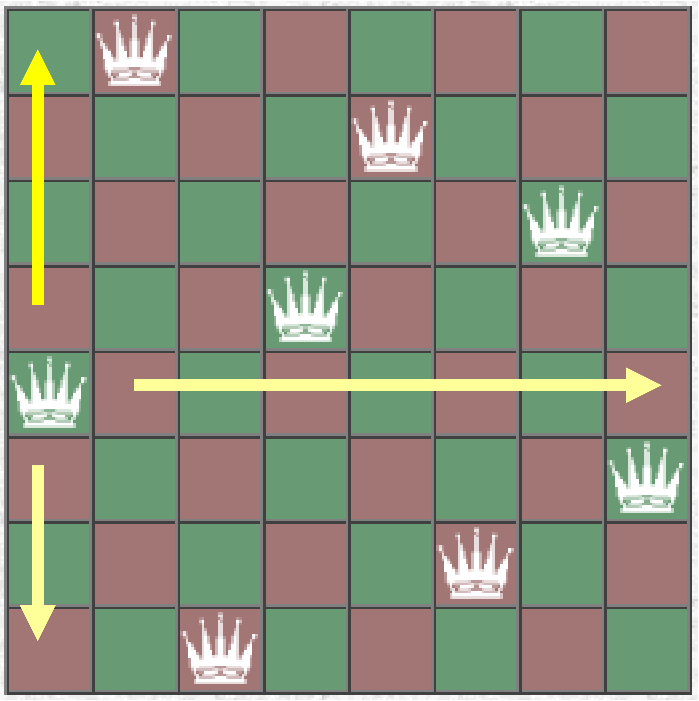
- Recursive algorithm to calculate fibonacci(n)
  - If n is 0 or 1, return 1
  - Else compute fibonacci(n-1) and fibonacci(n-2)
  - Return their sum
- Simple algorithm ⇒ exponential time O($2^n$)
  - Computes fibonacci(1) $2^n$ times
- Can solve efficiently using
  - Iteration
  - Dynamic programming
  - Will examine different algorithm strategies later…

# Examples of Recursive Algorithms

- Binary search
- Quicksort
- N-queens
- Fractals

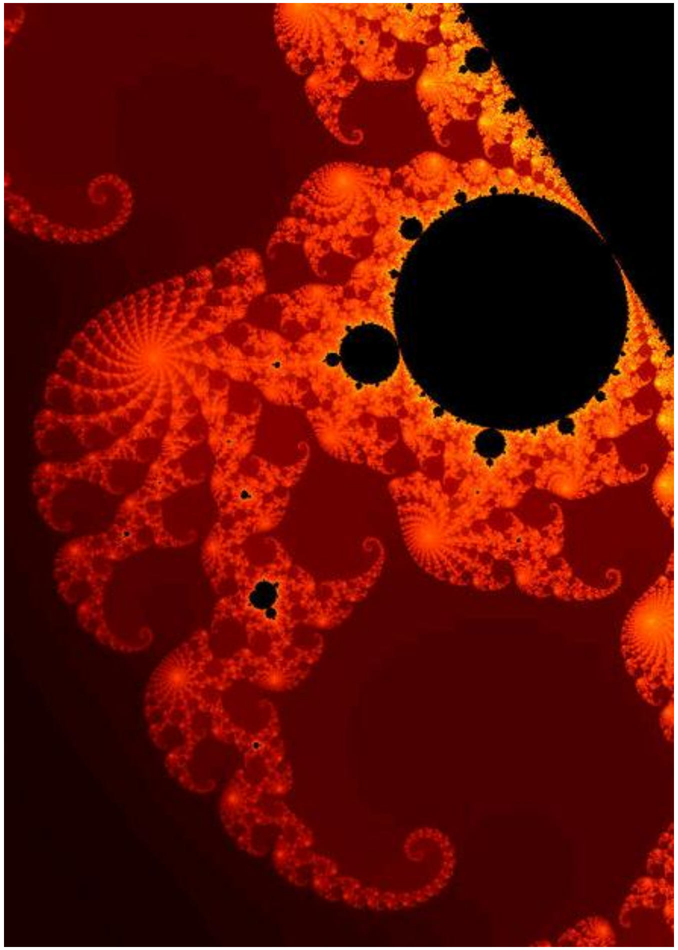
# N-Queens

- **Goal**
  - **Place queens on a board such that every row and column contains one queen, but no queen can attack another queen**
- **Recursive approach**
  - **To place queens on NxN board**
  - **Assume you've already placed K queens**



# Fractals

- Goal
  - Construct shapes using a simple recursive definition with a natural appearance
- Properties
  - Appears similar at all scales of magnification
    - Therefore “infinitely complex”
  - Not easily described in Euclidean geometry



Mandelbrot Set