CMSC 132:
Object-Oriented Programming II

Trees & Binary Search Trees

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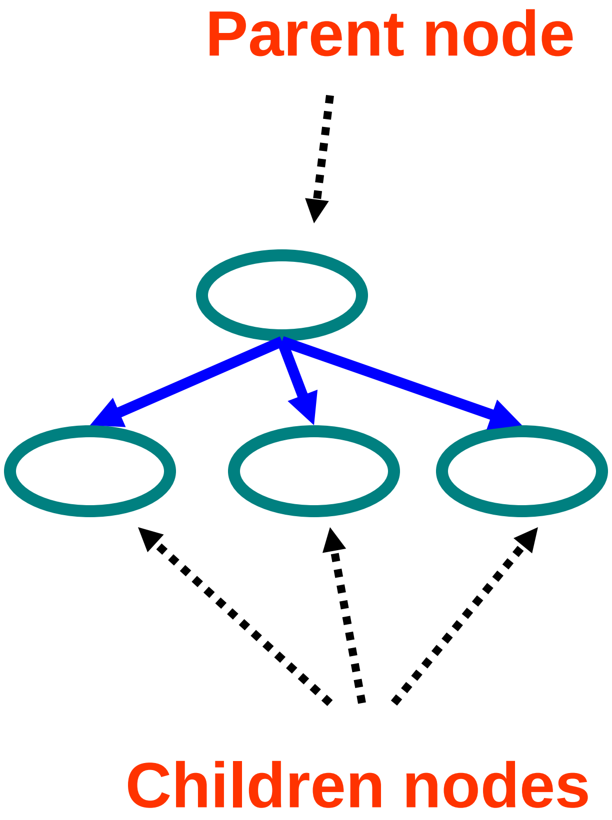
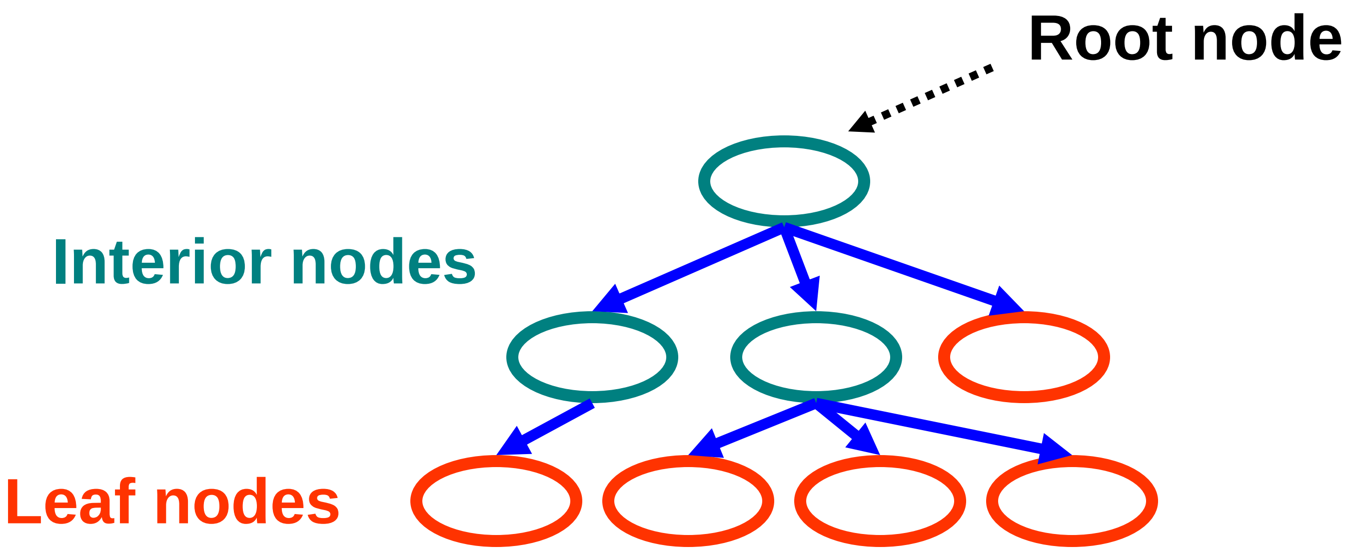
Trees

- Trees are hierarchical data structures
  - One-to-many relationship between elements
- Tree node / element
  - Contains data
  - Referred to by only 1 (parent) node
  - Contains links to any number of (children) nodes



Trees

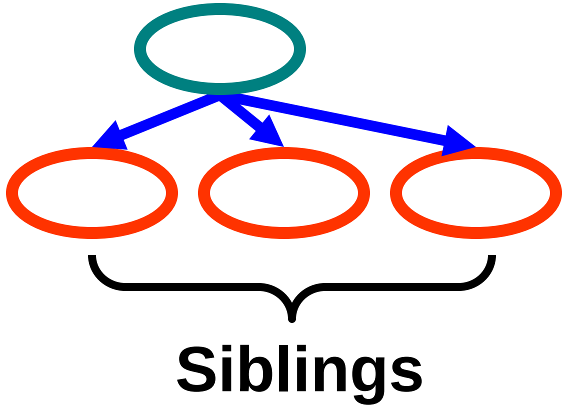
- **Terminology**
  - **Root ⇒ node with no parent**
  - **Leaf ⇒ all nodes with no children**
  - **Interior ⇒ all nodes with children**

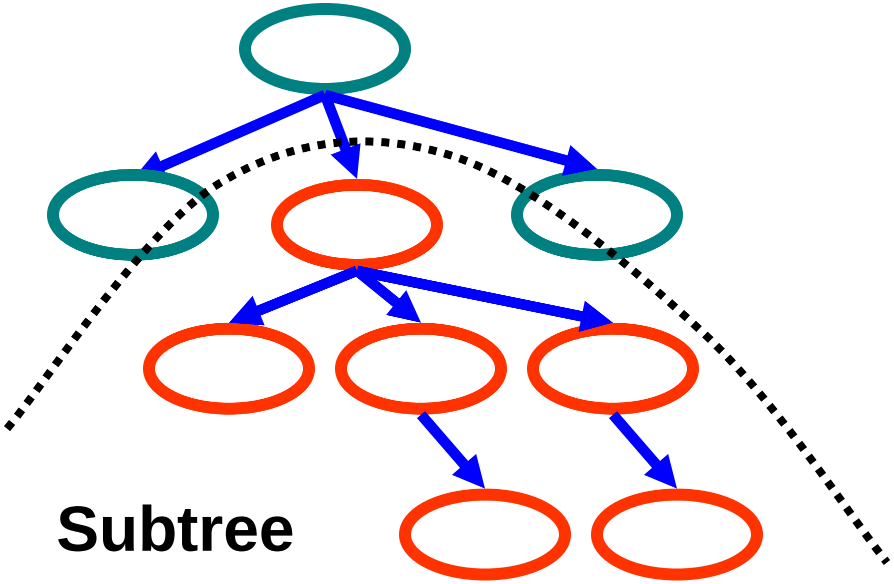


Trees

- Terminology
  - Sibling ⇒ node with same parent
  - Descendent ⇒ children nodes & their descendents
  - Subtree ⇒ portion of tree that is a tree by itself

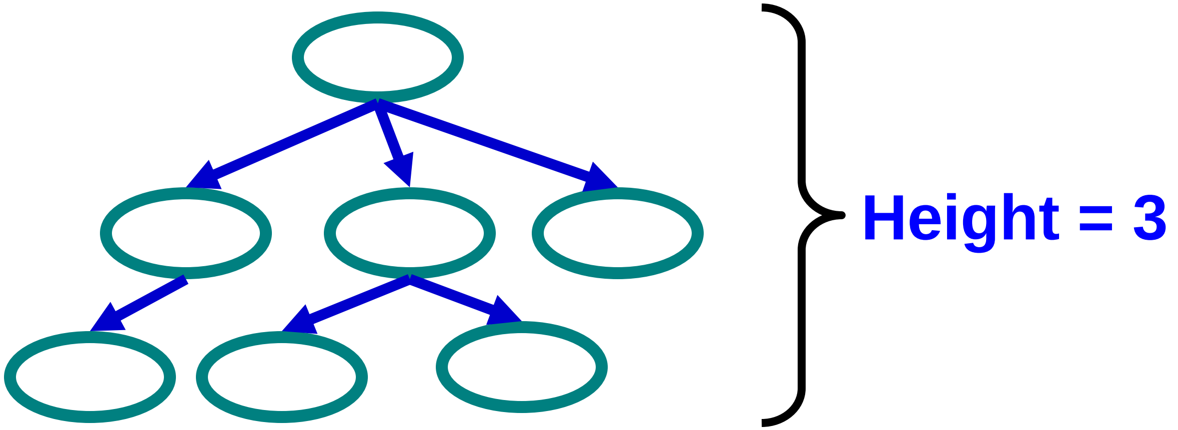
    ⇒ a node and its descendents





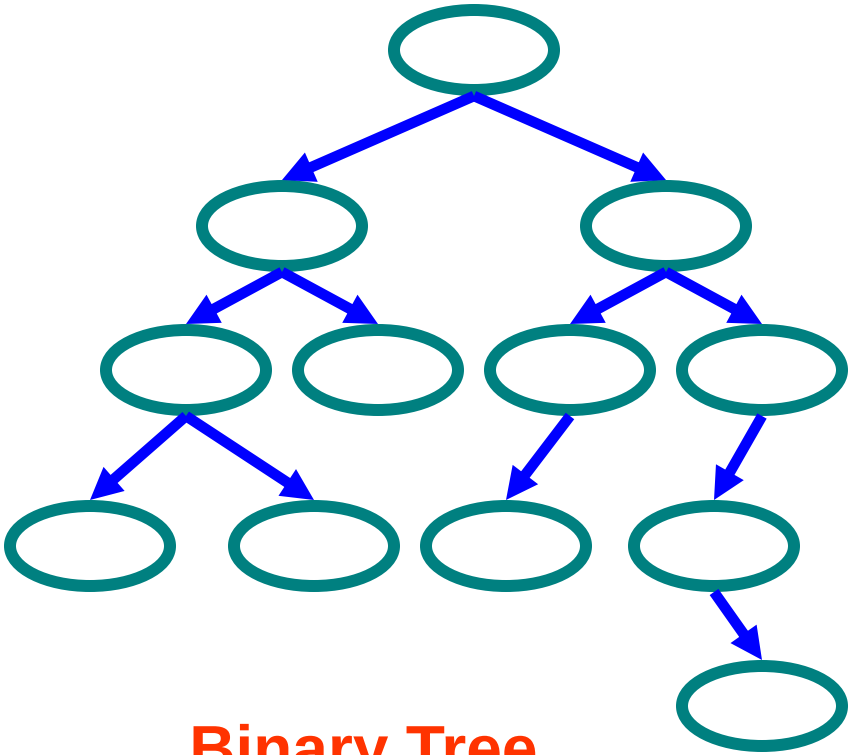
Trees

- Terminology
  - Level ⇒ is a measure of a node's distance from root
  - Definition of level
    - If node is the root of the tree, its level is 1
    - Else, the node's level is 1 + its parent's level
  - Height (depth) ⇒ max level of any node in tree

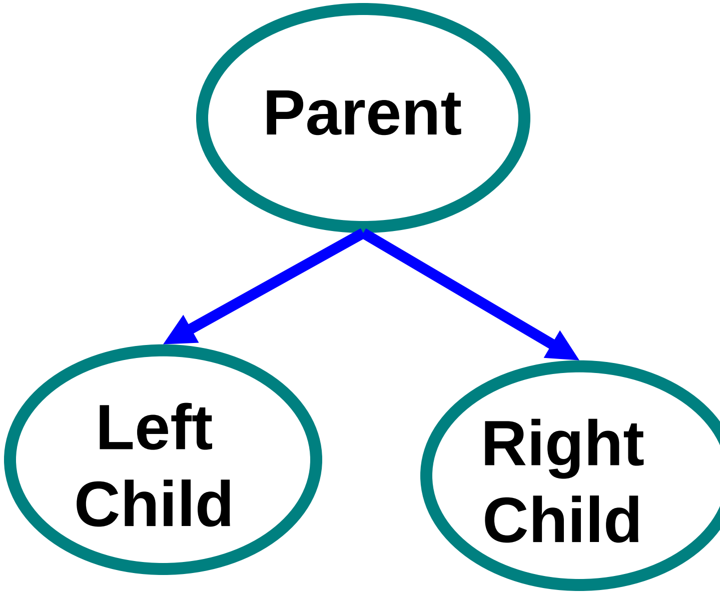


Binary Trees

- Binary tree
  - Tree with 0–2 children per node
    - Left & right child / subtree



Binary Tree



Tree Traversal

- **Often we want to**
  1. **Find all nodes in tree**
  2. **Determine their relationship**

- **Can do this by**
  1. **Walking through the tree in a prescribed order**
  2. **Visiting the nodes as they are encountered**

- **Process is called tree traversal**

Tree Traversal

- **Goal**
  - **Visit every node in binary tree**

- **Approaches**
  - **Depth first**
    - **Preorder ⇒ parent before children**
    - **Inorder ⇒ left child, parent, right child**
    - **Postorder ⇒ children before parent**
  - **Breadth first ⇒ closer nodes first**

Tree Traversal Methods

- **Pre-order**
  1. Visit node   // first
  2. Recursively visit left subtree
  3. Recursively visit right subtree
- **In-order**
  1. Recursively visit left subtree
  2. Visit node   // second
  3. Recursively right subtree
- **Post-order**
  1. Recursively visit left subtree
  2. Recursively visit right subtree
  3. Visit node   // last

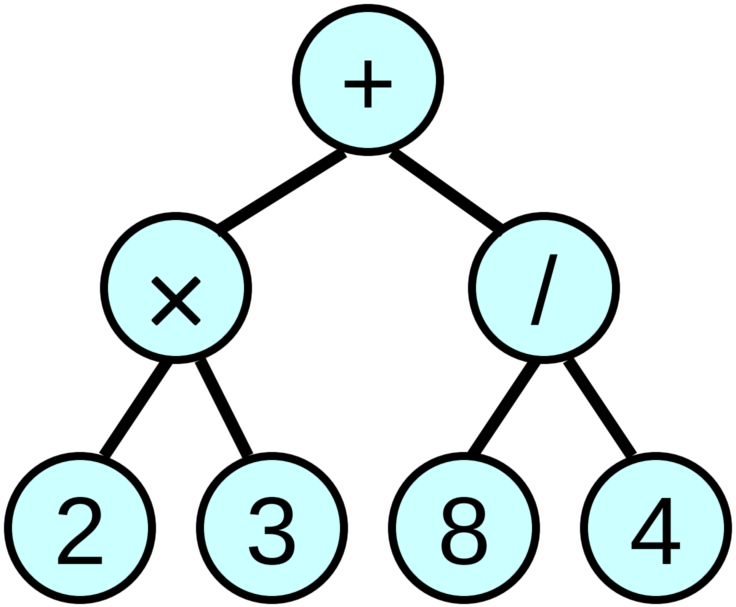
Tree Traversal Methods

- **Breadth-first**

```
BFS(Node n) {
    Queue Q = new Queue();
    Q.enqueue(n);                 // insert node into Q
    while ( !Q.empty()) {
        n = Q.dequeue();          // remove next node
        if ( !n.isEmpty()) {
            visit(n);             // visit node
            Q.enqueue(n.Left());  // insert left subtree in Q
            Q.enqueue(n.Right());// insert right subtree in Q
 } }
```

Tree Traversal Examples

- **Pre-order (prefix)**
  - **+ × 2 3 / 8 4**
- **In-order (infix)**
  - **2 × 3 + 8 / 4**
- **Post-order (postfix)**
  - **2 3 × 8 4 / +**
- **Breadth-first**
  - **+ × / 2 3 8 4**

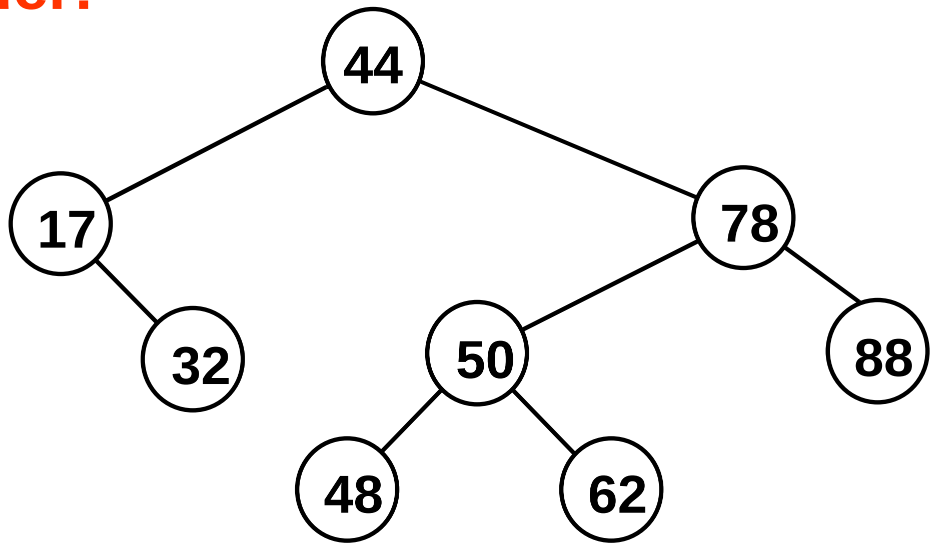


Expression tree

Tree Traversal Examples

- **Pre-order**
  - **44, 17, 32, 78, 50, 48, 62, 88**
- **In-order**
  - **17, 32, 44, 48, 50, 62, 78, 88**
- **Post-order**
  - **32, 17, 48, 62, 50, 88, 78, 44**
- **Breadth-first**
  - **44, 17, 78, 32, 50, 88, 48, 62**

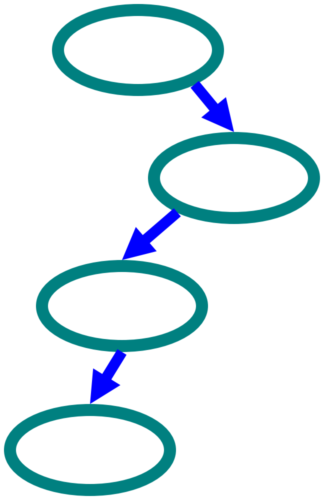
Sorted order!



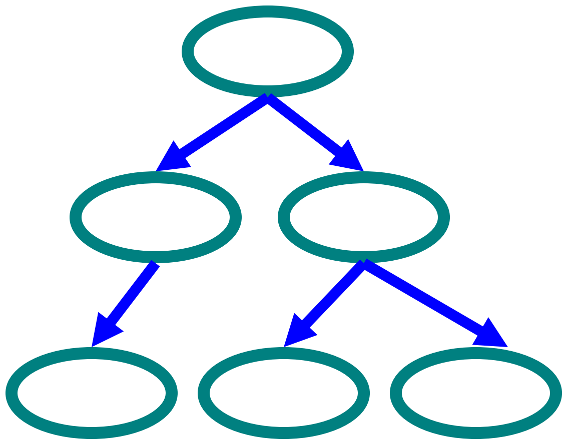
Binary search tree

Types of Binary Trees

- **Degenerate**
  - Mostly 1 child / node
  - Height = O(n)
  - Similar to linear list
- **Balanced**
  - Mostly 2 child / node
  - Height = O(log(n))
  - Useful for searches



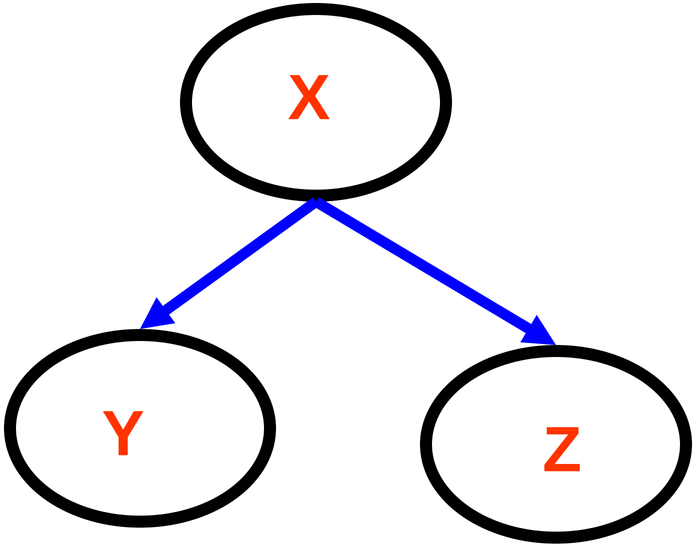
Degenerate binary tree



Balanced binary tree

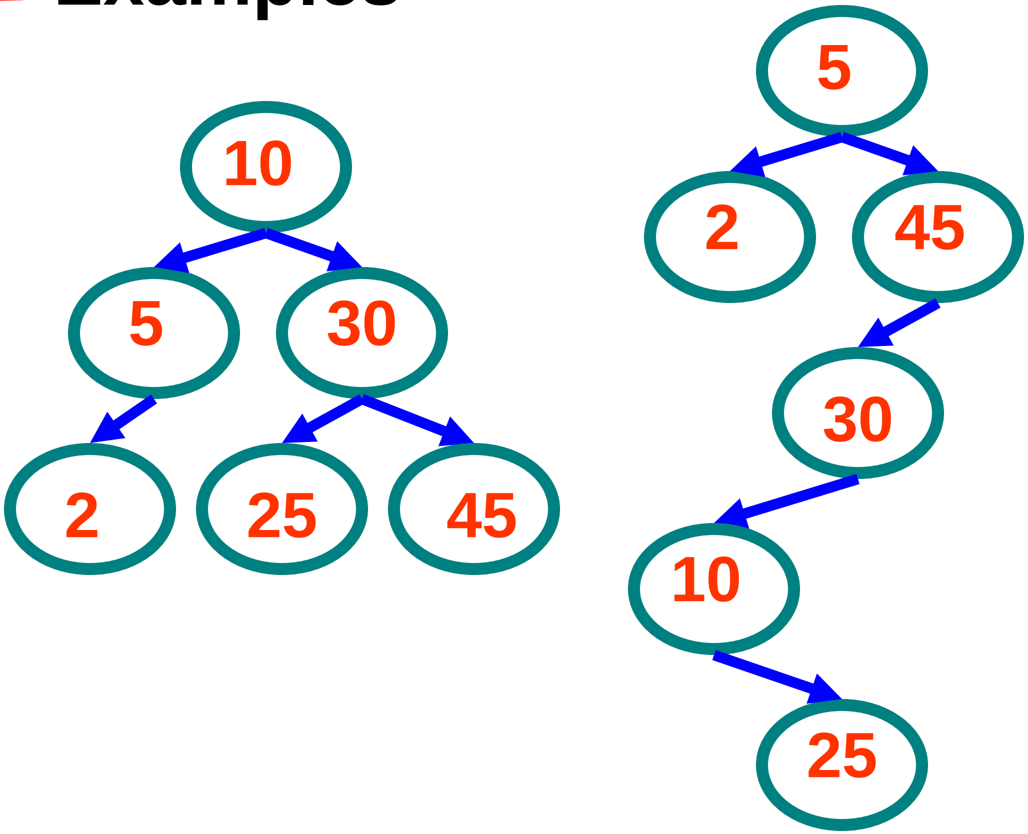
Binary Search Trees

- Key property
  - Value at node
    - Smaller values in left subtree
    - Larger values in right subtree
  - Example
    - X > Y
    - X < Z

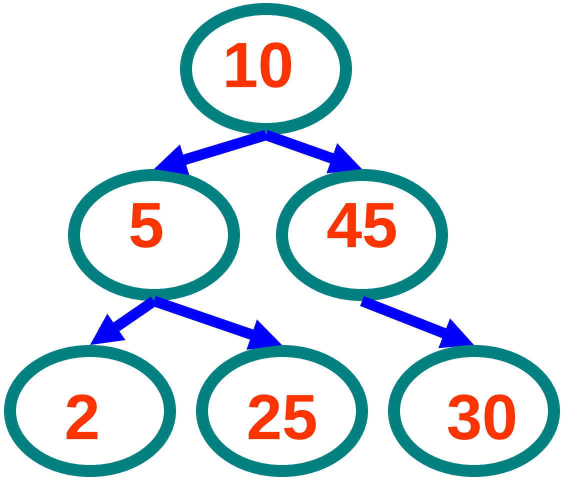


Binary Search Trees

- **Examples**



Binary
search trees



Non-binary
search tree

Binary Tree Implementation

```
Class Node {
    Value data;
    Node left, right;     // null if empty

    void insert ( Value data1 ) { … }
    void delete ( Value data2 ) { … }
    Node find ( Value data3 ) { … }
        …
}
```

Iterative Search of Binary Tree

```
Node Find( Node n, Value key) {
    while (n != null) {
        if (n.data == key)          // Found it
            return n;
        if (n.data > key)           // In left subtree
            n = n.left;
        else                        // In right subtree
            n = n.right;
    }
    return null;
}
Find( root, keyValue );
```

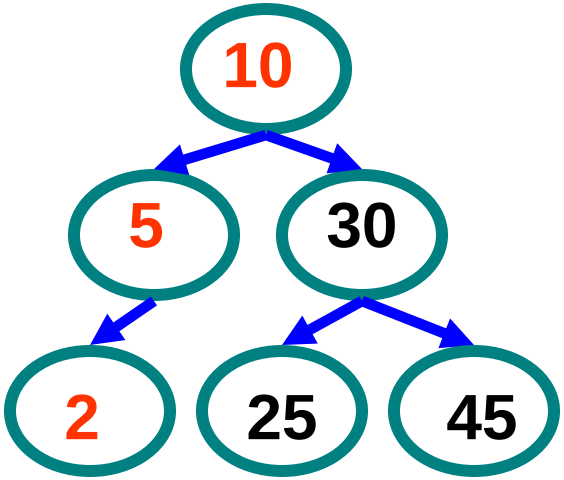
Recursive Search of Binary Tree

```
Node Find( Node n, Value key) {
    if (n == null)                  // Not found
        return( n );
    else if (n.data == key)         // Found it
        return( n );
    else if (n.data > key)          // In left subtree
        return Find( n.left, key );
    else                            // In right subtree
        return Find( n.right, key );
}

Find( root, keyValue );
```

Example Binary Searches

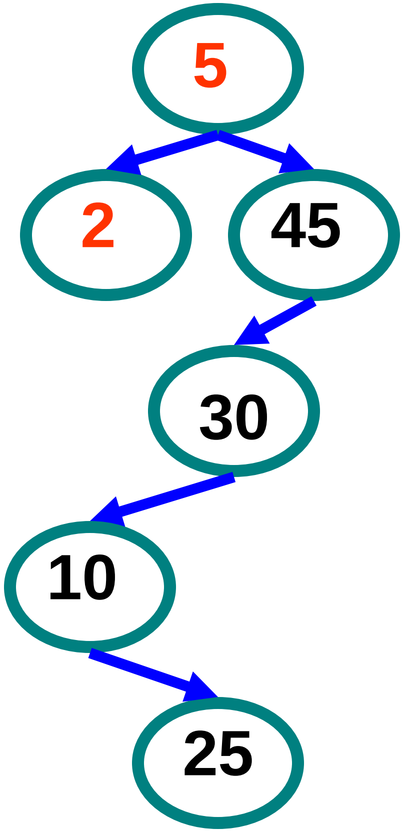
- **Find (2)**



10 > 2, left

5 > 2, left

2 = 2, found

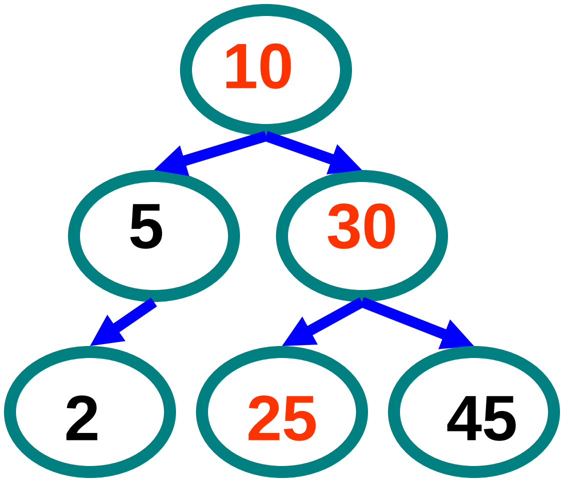


5 > 2, left

2 = 2, found

Example Binary Searches

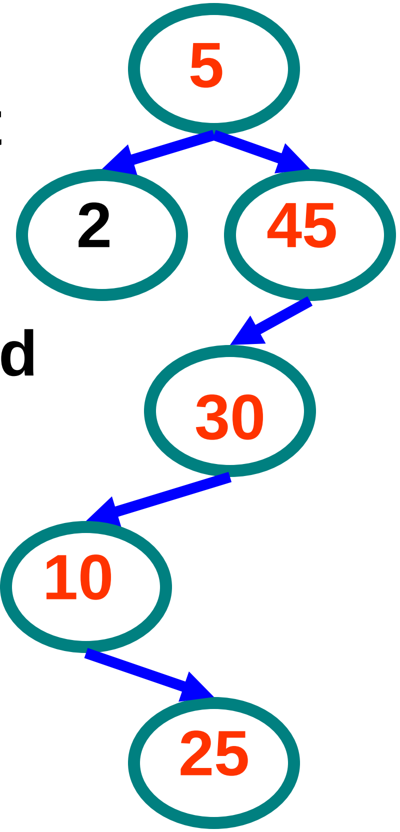
- **Find (25)**



10 < 25, right

30 > 25, left

25 = 25, found



5 < 25, right

45 > 25, left

30 > 25, left

10 < 25, right

25 = 25, found

Binary Search Properties

- Time of search
  - Proportional to height of tree
  - Balanced binary tree
    - O(log(n)) time
  - Degenerate tree
    - O(n) time
    - Like searching linked list / unsorted array
- Requires
  - Ability to compare key values

Binary Search Tree Construction

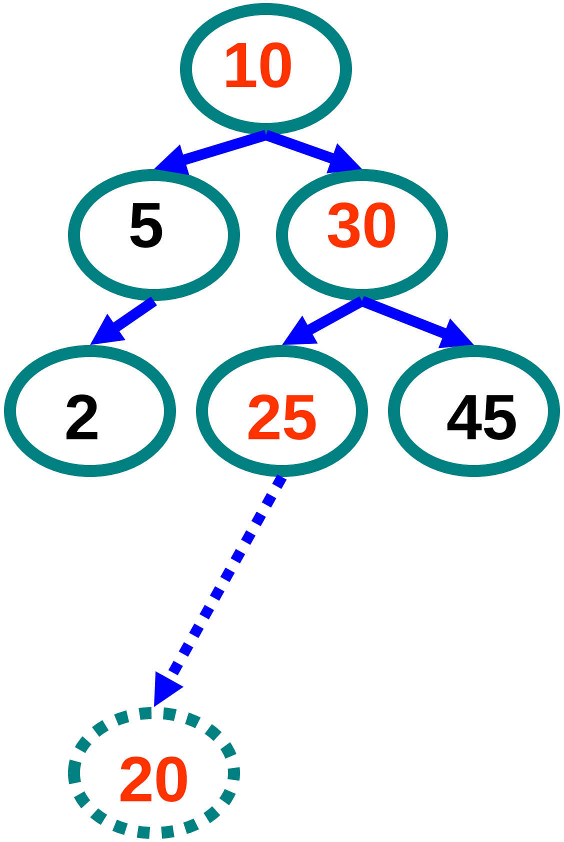
- **How to build & maintain binary trees?**
  - **Insertion**
  - **Deletion**
- **Maintain key property (invariant)**
  - **Smaller values in left subtree**
  - **Larger values in right subtree**

Binary Search Tree – Insertion

- **Algorithm**
  1. **Perform search for value X**
  2. **Search will end at node Y (if X not in tree)**
  3. **If X < Y, insert new leaf X as new left subtree for Y**
  4. **If X > Y, insert new leaf X as new right subtree for Y**
- **Observations**
  - **O(log(n)) operation for balanced tree**
  - **Insertions may unbalance tree**

Example Insertion

- Insert (20)



10 < 20, right

30 > 20, left

25 > 20, left

Insert 20 on left

Binary Search Tree – Deletion

- **Algorithm**
  1. **Perform search for value X**
  2. **If X is a leaf, delete X**
  3. **Else // must delete internal node**

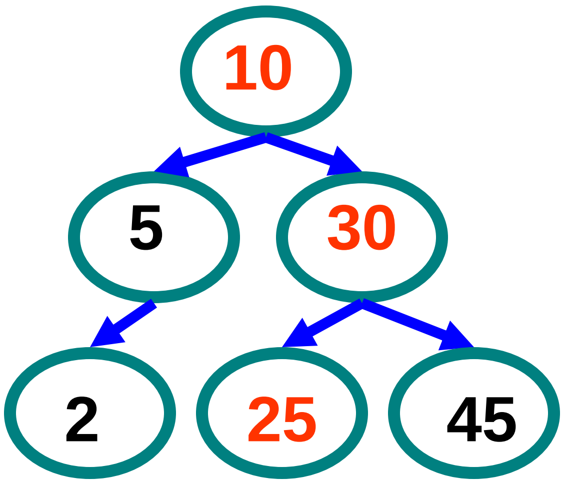
     a) Replace with largest value Y on left subtree

     OR smallest value Z on right subtree

     b) Delete replacement value (Y or Z) from subtree
- **Observation**
  - **O(log(n)) operation for balanced tree**
  - **Deletions may unbalance tree**

Example Deletion (Leaf)

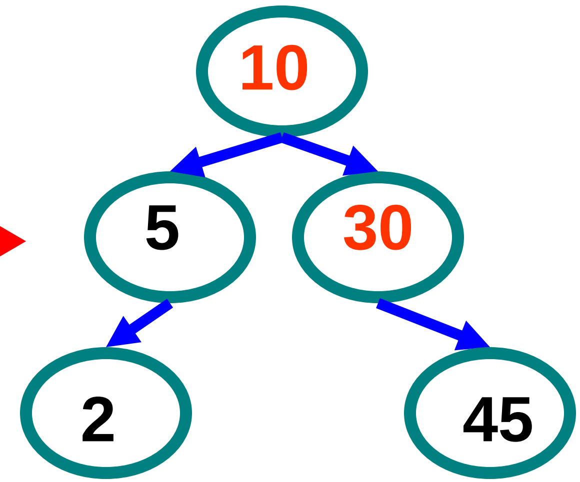
- **Delete (25)**



10 < 25, right

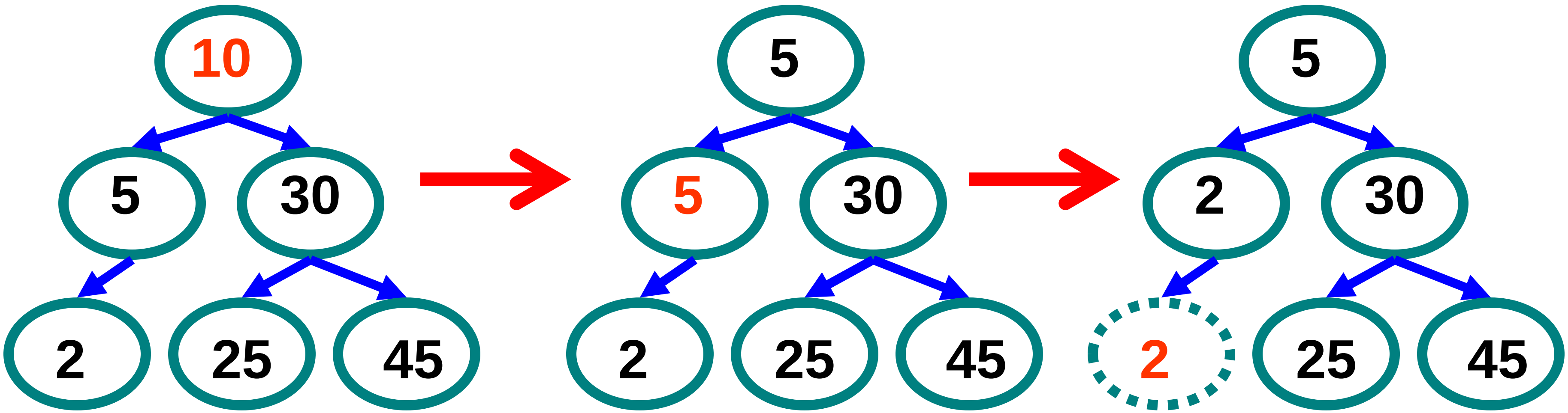
30 > 25, left

25 = 25, delete



Example Deletion (Internal Node)

- **Delete (10)**



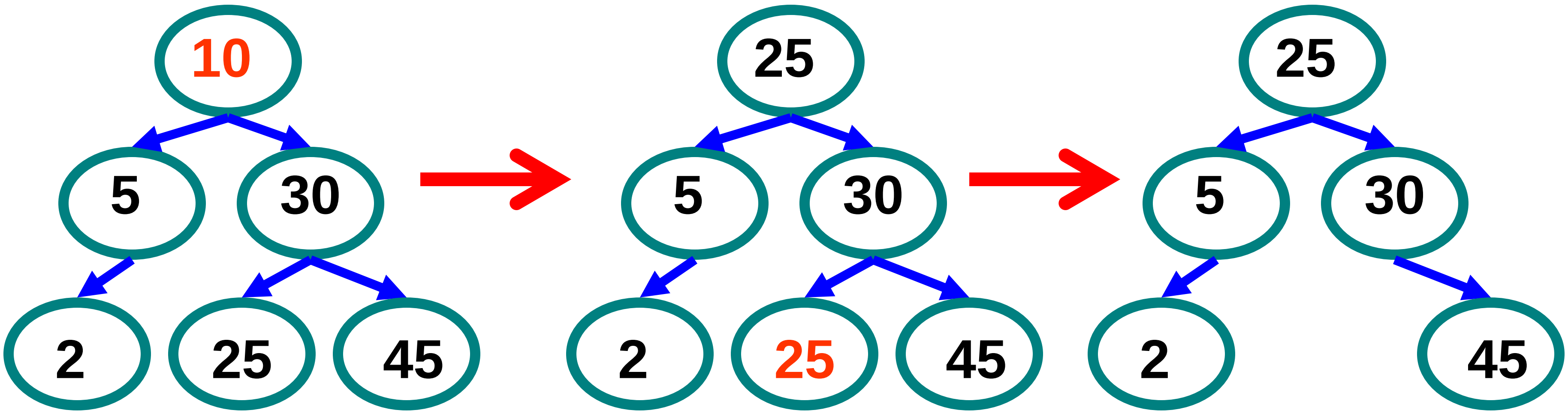
Replacing 10 with largest value in left subtree

Replacing 5 with largest value in left subtree

Deleting leaf

Example Deletion (Internal Node)

- **Delete (10)**



Replacing 10 with smallest value in right subtree

Deleting leaf

Resulting tree

Building Maps w/ Search Trees

- Search trees often used to implement maps
  - Each non-empty node contains
    - Key
    - Value
    - Left and right child
- Need to be able to compare keys
  - Generic type <K extends Comparable<K>>
    - Denotes any type K that can be compared to K's

Polymorphic Binary Search Trees

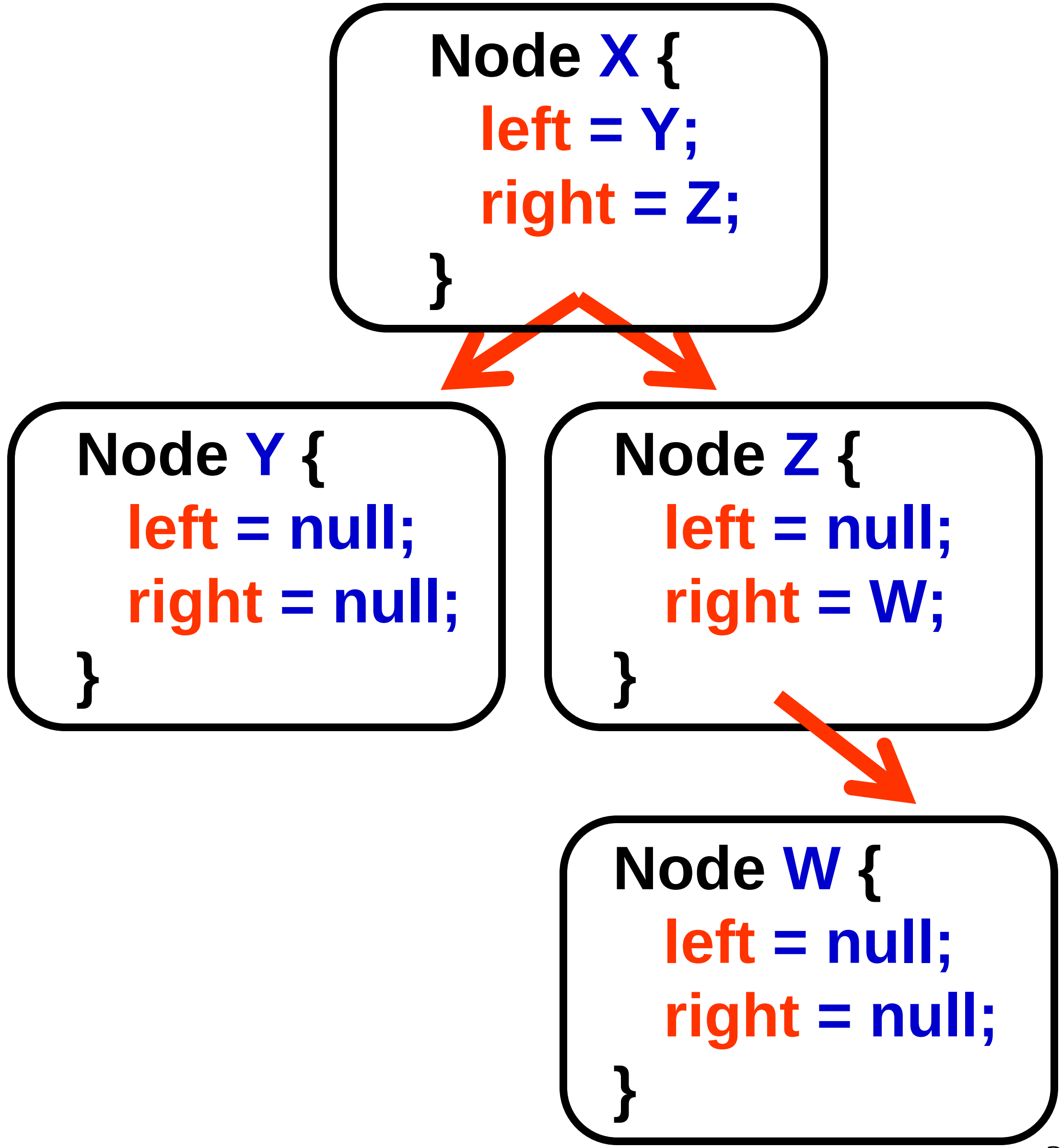
- What do we mean by polymorphic?
- Implement two subtypes of Tree
  1. EmptyTree
  2. NonEmptyTree
- Use EmptyTree to represent the empty tree
  - Rather than null
- Invoke methods on tree nodes
  - Without checking for null
  - Get empty or nonempty functionality
    - Selected by type of tree node

Polymorphic Binary Tree Implement.

```
Interface Tree {
    Tree insert ( Value data1 ) { … }
}
Class EmptyTree implements Tree {
    Tree insert ( Value data1 ) { … }
}
Class NonEmptyTree implements Tree {
    Value data;
    Tree left, right; // Either Empty or NonEmpty
    Tree insert ( Value data1 ) { … }
}
```

Example : Standard Binary Tree

```
Class Node {
    Node left, right;
}
```



Example : Polymorphic Binary Tree

```
Class EmptyTree {
}
Class NonEmptyTree {
    Tree left, right;
}
```