Algorithm Strategies

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General Concepts

- Algorithm strategy
  - Approach to solving a problem
  - May combine several approaches

- Algorithm structure
  - Iterative $\Rightarrow$ execute action in loop
  - Recursive $\Rightarrow$ reapply action to subproblem(s)

- Problem type
Problem Type

- **Satisfying**
  - Find any satisfactory solution
  - Example → Find path from A to F

- **Optimization**
  - Find best solution (vs. cost metric)
  - Example → Find shortest path from A to E
Some Algorithm Strategies

- Recursive algorithms
- Backtracking algorithms
- Divide and conquer algorithms
- Dynamic programming algorithms
- Greedy algorithms
- Brute force algorithms
- Branch and bound algorithms
- Heuristic algorithms
Recursive Algorithm

Based on reapplying algorithm to subproblem

Approach

1. Solves base case(s) directly
2. Recurs with a simpler subproblem
3. May need to convert solution(s) to subproblems
Backtracking Algorithm

Based on depth-first recursive search

Approach

1. Tests whether solution has been found
2. If found solution, return it
3. Else for each choice that can be made
   a) Make that choice
   b) Recur
   c) If recursion returns a solution, return it
4. If no choices remain, return failure

Tree of possible solutions → search tree
Backtracking Algorithm – Reachability

Find path in graph from A to F

1. Start with currentNode = A
2. If currentNode has edge to F, return path
3. Else select neighbor node X for currentNode
   - Recursively find path from X to F
     - If path found, return path
     - Else repeat for different X
   - Return false if no path from any neighbor X
Backtracking Algorithm – Path Finding

Search tree

Internal nodes → partial solutions
Leaves → complete solutions
Backtracking Algorithm – Map Coloring

- Color a map with no more than four colors
  - If all countries have been colored return success
  - Else for each color c of four colors and country n
    - If country n is not adjacent to a country that has been colored c
      - Color country n with color c
      - Recursively color country n+1
      - If successful, return success
  - Return failure
Divide and Conquer

Based on dividing problem into subproblems

Approach

1. Divide problem into smaller subproblems
   - Subproblems must be of same type
   - Subproblems do not need to overlap
2. Solve each subproblem recursively
3. Combine solutions to solve original problem

Usually contains two or more recursive calls
Example

1. Divide into subproblem for each neighboring node
2. Solve subproblems (recursively)
3. Combine by comparing costs
Divide and Conquer – Sorting

- **Quicksort**
  - Partition array into **two parts** around pivot
  - Recursively quicksort each part of array
  - Concatenate solutions

- **Mergesort**
  - Partition array into **two parts**
  - Recursively mergesort each half
  - Merge two sorted arrays into single sorted array
Dynamic Programming Algorithm

Based on remembering past results

Approach

1. Divide problem into smaller subproblems
   - Subproblems must be of same type
   - Subproblems must overlap
2. Solve each subproblem recursively
   - May simply look up solution (if previously solved)
3. Combine solutions into to solve original problem
4. Store solution to problem

Generally applied to optimization problems
Fibonacci Algorithm

Fibonacci numbers

- \( \text{fibonacci}(0) = 1 \)
- \( \text{fibonacci}(1) = 1 \)
- \( \text{fibonacci}(n) = \text{fibonacci}(n-1) + \text{fibonacci}(n-2) \)

Recursive algorithm to calculate \( \text{fibonacci}(n) \)

- If \( n \) is 0 or 1, return 1
- Else compute \( \text{fibonacci}(n-1) \) and \( \text{fibonacci}(n-2) \)
- Return their sum

Simple algorithm \( \Rightarrow \) exponential time \( O(2^n) \)
Dynamic Programming – Fibonacci

Dynamic programming version of fibonacci(n)

- If n is 0 or 1, return 1
- Else solve fibonacci(n-1) and fibonacci(n-2)
  - Look up value if previously computed
  - Else recursively compute
- Find their sum and store
- Return result

Dynamic programming algorithm ⇒ O(n) time
- Since solving fibonacci(n-2) is just looking up value
Dynamic Programming – Shortest Path

Dijkstra’s Shortest Path Algorithm

S = ∅

C[X] = 0

C[Y] = ∞ for all other nodes

while ( not all nodes in S )

find node K not in S with smallest C[K]

add K to S

for each node M not in S adjacent to K

\[ C[M] = \min ( C[M] , C[K] + \text{cost of (K,M)} ) \]

Stores results of smaller subproblems
**Greedy Algorithm**

- Based on trying best current (local) choice
- Approach
  - At each step of algorithm
  - Choose best local solution
- Avoid backtracking, exponential time $O(2^n)$
- Hope local optimum lead to **global** optimum
Greedy Algorithm – Shortest Path

Example

Choose lowest-cost neighbor

Does not yield overall (global) shortest path
Greedy Algorithm – MST

Kruskal’s Minimal Spanning Tree Algorithm

sort edges by weight (from least to most)
tree = ∅
for each edge (X, Y) in order
    if it does not create a cycle
        add (X, Y) to tree

stop when tree has N–1 edges

Picks best local solution at each step
Brute Force Algorithm

- Based on trying all possible solutions

- Approach

  - Generate and evaluate possible solutions until
    - Satisfactory solution is found
    - Best solution is found (if can be determined)
    - All possible solutions found
      - Return best solution
      - Return failure if no satisfactory solution

- Generally most expensive approach
Brute Force Algorithm – Shortest Path

Example

Examines all paths in graph
Brute Force Algorithm – TSP

Traveling Salesman Problem (TSP)
- Given weighted undirected graph (map of cities)
- Find lowest cost path visiting all nodes (cities) once
- No known polynomial-time general solution

Brute force approach
- Find all possible paths using recursive backtracking
- Calculate cost of each path
- Return lowest cost path

Requires exponential time $O(2^n)$
Branch and Bound Algorithm

- Based on limiting search using current solution

Approach

- Track best current solution found
- Eliminate (prune) partial solutions that can not improve upon best current solution

- Reduces amount of backtracking

- Not guaranteed to avoid exponential time $O(2^n)$
Branch & Bound Alg. – Shortest Path

Example

- **A → B**
  - **A → B → C**
  - **A → B → E**
  - Cost: 6

- **A → C**
  - **A → C → E**
  - Cost: 5

- **A → D**
  - Pruned paths beginning with **A → B → C** & **A → D**

Graph:

- A, B, C, D, E, F
- Edges and weights: 1, 2, 3, 4, 5, 6, 7, 8
- Path costs: 6, 5
Branch and Bound – TSP

- Branch and bound algorithm for TSP
  - Find possible paths using recursive backtracking
  - Track cost of best current solution found
  - Stop searching path if cost > best current solution
  - Return lowest cost path
- If good solution found early, can reduce search
- May still require exponential time $O(2^n)$
Heuristic Algorithm

- Based on trying to guide search for solution
- Heuristic $\Rightarrow$ “rule of thumb”

Approach

- Generate and evaluate possible solutions
  - Using “rule of thumb”
  - Stop if satisfactory solution is found

- Can reduce complexity
- Not guaranteed to yield best solution
Heuristic – Shortest Path

Example

Try only edges with cost < 5

Worked...in this case
Heuristic Algorithm – TSP

- Heuristic algorithm for TSP
  - Find possible paths using recursive backtracking
    - Search 2 lowest cost edges at each node first
  - Calculate cost of each path
  - Return lowest cost path from first 100 solutions

- Not guaranteed to find best solution

- Heuristics used frequently in real applications
Summary

- Wide range of strategies

Choice depends on

- Properties of problem
- Expected problem size
- Available resources