

CMSC 330: Organization of Programming Languages

Practice Problem 2 Solutions

Problem 1 –  $(a | b)^*$

a) Construct NFA

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Problem 1 –  $(a | b)^*$

a) Construct NFA



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Problem 1 –  $(a | b)^*$

a) Construct NFA

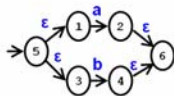


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Problem 1 –  $(a | b)^*$

a) Construct NFA

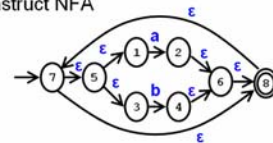


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Problem 1 –  $(a | b)^*$

a) Construct NFA



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## Problem 1 – (a | b)\*

### b) Reduce NFA to DFA

- Start =  $\epsilon$ -closure(7) = {7,5,1,3,8}
- $R = \{\{7,5,1,3,8\}\}$
- $r = R = \{7,5,1,3,8\}$  // mark DFA state
- move ({7,5,1,3,8}, a) = {2}
  - $e = \epsilon$ -closure(2) = {2,6,8,7,5,1,3} // new DFA state
  - $R = R \cup \{2,6,8,7,5,1,3\}$  // add to R
  - $\delta = \delta \cup \{(7,5,1,3,8), a, (2,6,8,7,5,1,3)\}$
- move ({7,5,1,3,8}, b) = {4}
  - $e = \epsilon$ -closure(4) = {4,6,8,7,5,1,3} // new DFA state
  - $R = R \cup \{4,6,8,7,5,1,3\}$  // add to R
  - $\delta = \delta \cup \{(7,5,1,3,8), b, (4,6,8,7,5,1,3)\}$

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## Problem 1 – (a | b)\*

- $R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
- $r = R = \{2,6,8,7,5,1,3\}$  // mark DFA state
- move ({2,6,8,7,5,1,3}, a) = {2}
  - $e = \epsilon$ -closure(2) = {2,6,8,7,5,1,3} // existing DFA state
  - $\delta = \delta \cup \{(2,6,8,7,5,1,3), a, (2,6,8,7,5,1,3)\}$
- move ({2,6,8,7,5,1,3}, b) = {4}
  - $e = \epsilon$ -closure(4) = {4,6,8,7,5,1,3} // existing DFA state
  - $\delta = \delta \cup \{(2,6,8,7,5,1,3), b, (4,6,8,7,5,1,3)\}$

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## Problem 1 – (a | b)\*

- $R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
- $r = R = \{4,6,8,7,5,1,3\}$  // mark DFA state
- move ({4,6,8,7,5,1,3}, a) = {2}
  - $e = \epsilon$ -closure(2) = {2,6,8,7,5,1,3} // existing DFA state
  - $\delta = \delta \cup \{(4,6,8,7,5,1,3), a, (2,6,8,7,5,1,3)\}$
- move ({4,6,8,7,5,1,3}, b) = {4}
  - $e = \epsilon$ -closure(4) = {4,6,8,7,5,1,3} // existing DFA state
  - $\delta = \delta \cup \{(4,6,8,7,5,1,3), b, (4,6,8,7,5,1,3)\}$
- $R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$ 
  - No more unmarked states to process
- $F_d = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$ 
  - Since  $8 \in F_n$

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## Problem 1 – (a | b)\*

- Resulting DFA
  - $\Sigma = \{a, b\}$
  - $R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
  - $r_0 = \{7,5,1,3,8\}$
  - $F_d = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
  - $\delta = \{ \{ \{7,5,1,3,8\}, a, \{2,6,8,7,5,1,3\} \}, \{ \{7,5,1,3,8\}, b, \{4,6,8,7,5,1,3\} \}, \{ \{2,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\} \}, \{ \{2,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\} \}, \{ \{4,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\} \}, \{ \{4,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\} \} \}$

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## Problem 1 – (a | b)\*

- NFA to DFA reduction pictorial

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## Problem 1 – (a | b)\*

- NFA to DFA reduction pictorial

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### Problem 1 – (a | b)\*

– NFA to DFA reduction pictorial

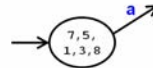


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### Problem 1 – (a | b)\*

– NFA to DFA reduction pictorial

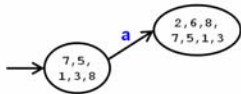


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### Problem 1 – (a | b)\*

– NFA to DFA reduction pictorial

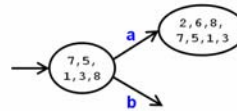


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– NFA to DFA reduction pictorial

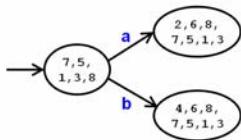


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### Problem 1 – (a | b)\*

– NFA to DFA reduction pictorial

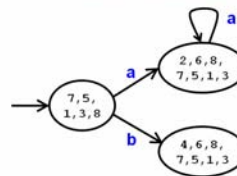


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### Problem 1 – (a | b)\*

– NFA to DFA reduction pictorial

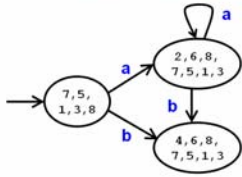


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### Problem 1 – $(a | b)^*$

– NFA to DFA reduction pictorial

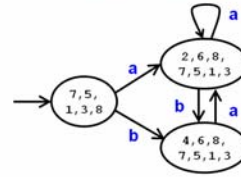


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### Problem 1 – $(a | b)^*$

– NFA to DFA reduction pictorial

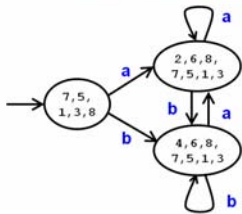


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### Problem 1 – $(a | b)^*$

– NFA to DFA reduction pictorial

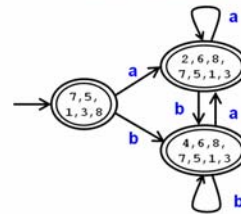


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### Problem 1 – $(a | b)^*$

– NFA to DFA reduction pictorial

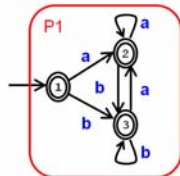


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### Problem 1 – $(a | b)^*$

c) Minimize DFA



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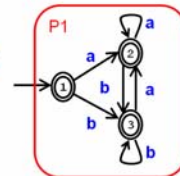
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### Problem 1 – $(a | b)^*$

c) Minimize DFA

– Initial partitions

- Accept  $\rightarrow \{1,2,3\} \rightarrow P1$
- Reject  $\rightarrow \emptyset$



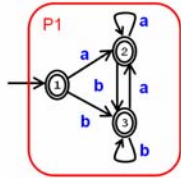
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### Problem 1 – $(a | b)^*$

#### c) Minimize DFA

- Initial partitions
  - Accept  $\rightarrow \{1,2,3\} \rightarrow P1$
  - Reject  $\rightarrow \emptyset$
- Split partition?
  - move(1,a)  $\rightarrow P1$
  - move(2,a)  $\rightarrow P1$
  - move(3,a)  $\rightarrow P1$
  - move(1,b)  $\rightarrow P1$
  - move(2,b)  $\rightarrow P1$
  - move(3,b)  $\rightarrow P1$



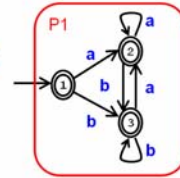
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### Problem 1 – $(a | b)^*$

#### c) Minimize DFA

- Initial partitions
  - Accept  $\rightarrow \{1,2,3\} \rightarrow P1$
  - Reject  $\rightarrow \emptyset$
- Split partition?
  - move(1,a)  $\rightarrow P1$
  - move(2,a)  $\rightarrow P1$
  - move(3,a)  $\rightarrow P1$
  - move(1,b)  $\rightarrow P1$
  - move(2,b)  $\rightarrow P1$
  - move(3,b)  $\rightarrow P1$



Not required, minimization done

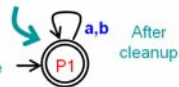
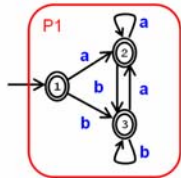
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### Problem 1 – $(a | b)^*$

#### c) Minimize DFA

- Initial partitions
  - Accept  $\rightarrow \{1,2,3\} \rightarrow P1$
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  - move(3,b)  $\rightarrow P1$



Not required, minimization done

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### Problem 1 – $(a^* | b^*)^*$

#### a) Construct NFA

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### Problem 1 – $(a^* | b^*)^*$

#### a) Construct NFA

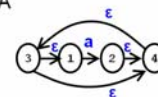


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### Problem 1 – $(a^* | b^*)^*$

#### a) Construct NFA

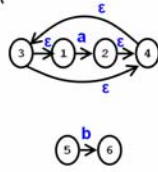


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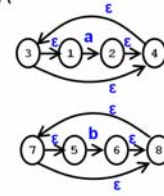
Problem 1 –  $(a^* | b^*)^*$

a) Construct NFA



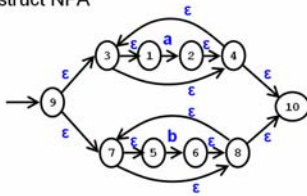
Problem 1 –  $(a^* | b^*)^*$

a) Construct NFA



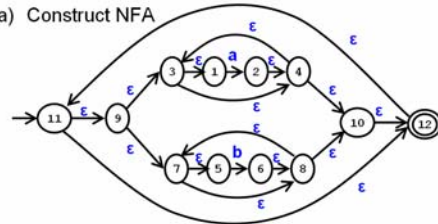
Problem 1 –  $(a^* | b^*)^*$

a) Construct NFA



Problem 1 –  $(a^* | b^*)^*$

a) Construct NFA



Problem 1 –  $(a^* | b^*)^*$

b) Reduce NFA to DFA

- Start =  $\epsilon$ -closure(1)
- = {11,9,3,1,4,7,5,8,10,12}
- $R = \{\{11,9\dots12\}\}$
- $r \in R = \{\{11,9\dots12\}\}$
- move ( $\{11,9\dots12\}$ , a) = 2
  - $\epsilon$ -closure(2) = {2,11,9...12}
- move ( $\{11,9\dots12\}$ , b) = 6
  - $\epsilon$ -closure(6) = {6,11,9...12}
- ...
- NFA to DFA reduction pictorial

Problem 1 –  $(a^* | b^*)^*$

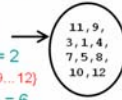
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- move ( $\{11,9\dots12\}$ , a) = 2
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- NFA to DFA reduction pictorial

### Problem 1 – $(a^* | b^*)^*$

#### b) Reduce NFA to DFA

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- NFA to DFA reduction pictorial



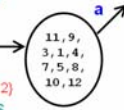
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### Problem 1 – $(a^* | b^*)^*$

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- NFA to DFA reduction pictorial



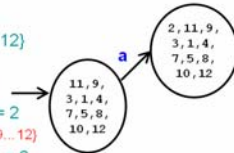
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### Problem 1 – $(a^* | b^*)^*$

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- NFA to DFA reduction pictorial



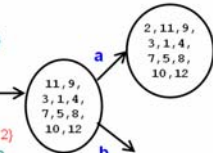
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### Problem 1 – $(a^* | b^*)^*$

#### b) Reduce NFA to DFA

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- NFA to DFA reduction pictorial



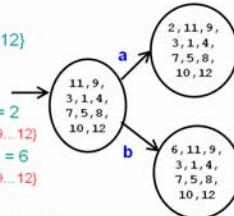
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### Problem 1 – $(a^* | b^*)^*$

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- NFA to DFA reduction pictorial



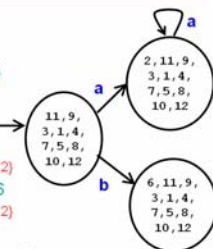
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### Problem 1 – $(a^* | b^*)^*$

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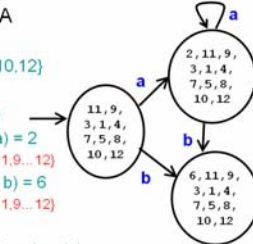
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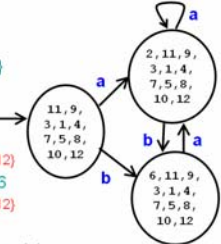
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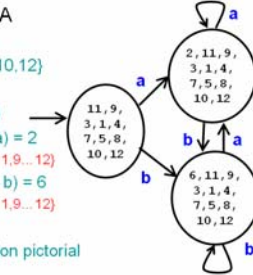
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### Problem 1 – $(a^* | b^*)^*$

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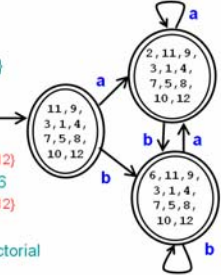
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### Problem 1 – $(a^* | b^*)^*$

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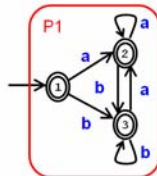


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### Problem 1 – $(a^* | b^*)^*$

#### c) Minimize DFA



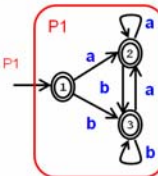
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### Problem 1 – $(a^* | b^*)^*$

#### c) Minimize DFA

- Initial partitions
- Accept  $\rightarrow \{1,2,3\} \rightarrow P1$
- Reject  $\rightarrow \emptyset$



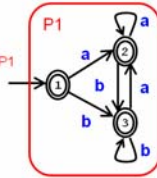
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Problem 1 –  $(a^* | b^*)^*$

c) Minimize DFA

- Initial partitions
  - Accept  $\rightarrow \{1,2,3\} \rightarrow P1$
  - Reject  $\rightarrow \emptyset$
- Split partition?
  - move(1,a)  $\rightarrow P1$
  - ...



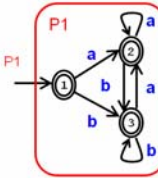
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- Not required,  
minimization done



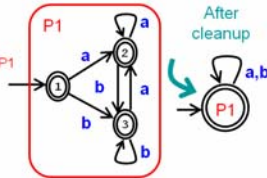
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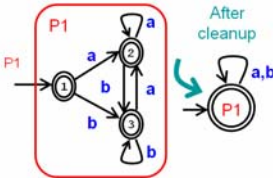
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- Not required,  
minimization done



d) Compare 2 minimized DFAs

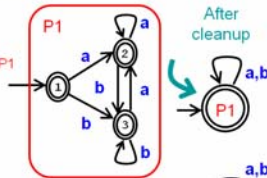
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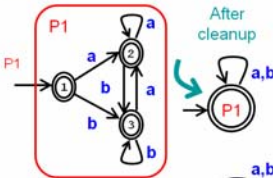
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Problem 1 –  $(a^* | b^*)^*$

c) Minimize DFA

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    - ...
- Not required,  
minimization done



d) Compare 2 minimized DFAs

- Identical up to state names!



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### Problem 2a

Write a grammar for the following languages

- a) All strings of 0's and 1's that have the **same** number of 0's and 1's

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$S \rightarrow ASB \mid BSA \mid \epsilon$  // same # of 0's & 1's

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 $A \rightarrow 1S \mid S1$  // one more 1 than 0

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 $B \rightarrow 0S \mid S0$  // one more 0 than 1

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### Problem 2a

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or  
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Write a grammar for the following languages

- a) All strings of 0's and 1's that have the **same** number of 0's and 1's

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 $A \rightarrow 1S \mid S1$  // one more 1 than 0  
 $B \rightarrow 0S \mid S0$  // one more 0 than 1  
or  
 $S \rightarrow 1S0S \mid 0S1S \mid \epsilon$  // same # of 0's & 1's  
And many other possibilities!

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### Problem 2b

Write a grammar for the following languages  
b) All strings of 0's and 1's that have **more** 0's than 1's

```
S → A S B | B S A | ε      // same # of 0's & 1's
A → 1 S | S 1              // one more 1 than 0
B → 0 S | S 0              // one more 0 than 1
```

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### Problem 2b

Write a grammar for the following languages  
b) All strings of 0's and 1's that have **more** 0's than 1's

```
S' → B S' | B S           // add extra 0
S → A S B | B S A | ε     // same # of 0's & 1's
A → 1 S | S 1             // one more 1 than 0
B → 0 S | S 0             // one more 0 than 1
Where S' is the new start symbol
```

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### Problem 2c

Write a grammar for the following languages  
c) All balanced pairs of left and right parentheses  
(e.g., "()", "()()")

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Write a grammar for the following languages  
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(e.g., "()", "()()")

```
S → ( S )      // left & right parentheses are balance
```

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### Problem 2c

Write a grammar for the following languages  
c) All balanced pairs of left and right parentheses  
(e.g., "()", "()()")

```
S → ( S )      // left & right parentheses are balance
    | S S       // both sides are balanced
```

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### Problem 2c

Write a grammar for the following languages  
c) All balanced pairs of left and right parentheses  
(e.g., "()", "()()")

```
S → ( S )      // left & right parentheses are balance
    | S S       // both sides are balanced
    | ε         // empty string
```

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## Definitions

- Left-most derivation
  - Left-most nonterminal is replaced in each step

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  - Left-most nonterminal is replaced in each step
- Right-most derivation
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- Right-most derivation
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- Example
  - Grammar
    - $S \rightarrow AB, A \rightarrow a, B \rightarrow b$

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  - Grammar
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    - $S \rightarrow AB \rightarrow aB \rightarrow ab$
  - Rightmost derivation for "ab"
    - $S \rightarrow AB \rightarrow Ab \rightarrow ab$

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## Problem 3

Consider the string "(a,a)" and the grammar

$S \rightarrow (L) | a$   
 $L \rightarrow L, S | S$

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Consider the string "(a,a)" and the grammar

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- a) Find all left-most derivations

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$S \rightarrow (L) \rightarrow (L,S) \rightarrow (S,S) \rightarrow (a,S) \rightarrow (a,a)$

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- b) Find all right-most derivations

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$S \rightarrow (L) \rightarrow (L,S) \rightarrow (L,a) \rightarrow (S,a) \rightarrow (a,a)$

- c) Find the parse tree for each derivation

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$S \rightarrow (L) \rightarrow (L,S) \rightarrow (L,a) \rightarrow (S,a) \rightarrow (a,a)$

- c) Find the parse tree for each derivation

$S$   
/ $\backslash$   
(  $L$  )  
/ $\backslash$   
 $L, S$   
 $S$   $a$   
 $a$

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### Problem 3

Consider the string "(a,a)" and the grammar

$S \rightarrow (L) | a$   
 $L \rightarrow L, S | S$

- Find all left-most derivations
- Find all right-most derivations
- Find the parse tree for each derivation
- Can you distinguish between the parse trees?

S  
/|\  
( L )  
/|\  
L , S  
| |  
S a  
|  
a

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### Problem 3

Consider the string "(a,a)" and the grammar

$S \rightarrow (L) | a$   
 $L \rightarrow L, S | S$

- Find all left-most derivations
- Find all right-most derivations
- Find the parse tree for each derivation
- Can you distinguish between the parse trees? **No**

S  
/|\  
( L )  
/|\  
L , S  
| |  
S a  
|  
a

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Consider the string "(a,(a,a))" and grammar

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 $L \rightarrow L, S | S$

- Find all left-most derivations
- $S \rightarrow (L) \rightarrow (L,S) \rightarrow (S,S) \rightarrow (a,S) \rightarrow (a,(L)) \rightarrow (a,(L,S)) \rightarrow (a,(S,S)) \rightarrow (a,(a,S)) \rightarrow (a,(a,a))$

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  - Find all right-most derivations
- $S \rightarrow (L) \rightarrow (L,S) \rightarrow (S,S) \rightarrow (a,S) \rightarrow (a,(L)) \rightarrow (a,(L,S)) \rightarrow (a,(S,S)) \rightarrow (a,(a,S)) \rightarrow (a,(a,a))$

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### Problem 3

Consider the string " $(a,(a,a))$ " and grammar

$S \rightarrow (L) \mid a$   
 $L \rightarrow L, S \mid S$

a) Find all left-most derivations

$S \rightarrow (L) \rightarrow (L,S) \rightarrow (S,S) \rightarrow (a,S) \rightarrow (a,(L)) \rightarrow$   
 $(a,(L,S)) \rightarrow (a,(S,S)) \rightarrow (a,(a,S)) \rightarrow (a,(a,a))$

b) Find all right-most derivations

$S \rightarrow (L) \rightarrow (L,S) \rightarrow (L,(L)) \rightarrow (L,(L,S)) \rightarrow (L,(L,a)) \rightarrow$   
 $(L,(S,a)) \rightarrow (L,(a,a)) \rightarrow (S,(a,a)) \rightarrow (a,(a,a))$

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 $(L,(S,a)) \rightarrow (L,(a,a)) \rightarrow (S,(a,a)) \rightarrow (a,(a,a))$

c) Find the parse tree for each derivation

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 $/ \backslash$   
 $(L)$   
 $/ \backslash$   
 $L, S$   
 $| / \backslash$   
 $S (L)$   
 $| / \backslash$   
 $a L, S$   
 $| / \backslash$   
 $S a$   
 $|$   
 $a$

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 $(L,(S,a)) \rightarrow (L,(a,a)) \rightarrow (S,(a,a)) \rightarrow (a,(a,a))$

c) Find the parse tree for each derivation

d) Can you distinguish between the parse trees?

$S$   
 $/ \backslash$   
 $(L)$   
 $/ \backslash$   
 $L, S$   
 $| / \backslash$   
 $S (L)$   
 $| / \backslash$   
 $a L, S$   
 $| / \backslash$   
 $S a$   
 $|$   
 $a$

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Consider the string " $(a,(a,a))$ " and grammar

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 $(L,(S,a)) \rightarrow (L,(a,a)) \rightarrow (S,(a,a)) \rightarrow (a,(a,a))$

c) Find the parse tree for each derivation

d) Can you distinguish between the parse trees? **No**

$S$   
 $/ \backslash$   
 $(L)$   
 $/ \backslash$   
 $L, S$   
 $| / \backslash$   
 $S (L)$   
 $| / \backslash$   
 $a L, S$   
 $| / \backslash$   
 $S a$   
 $|$   
 $a$

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### Problem 4

Consider the string " $abab$ " and the grammar

$S \rightarrow aSbS \mid bSaS \mid \epsilon$

a) Find all left-most derivations

#### Problem 4

Consider the string "abab" and the grammar

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a) Find all left-most derivations

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Consider the string "abab" and the grammar

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a) Find all left-most derivations

$S \rightarrow aSbS \rightarrow abS \rightarrow abaSbS \rightarrow ababS \rightarrow abab$

$S \rightarrow aSbS \rightarrow abSaSbS \rightarrow abaSbS \rightarrow ababS \rightarrow abab$

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b) Find all right-most derivations

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$S \rightarrow aSbS \rightarrow abSaSbS \rightarrow abaSbS \rightarrow ababS \rightarrow abab$

b) Find all right-most derivations

$S \rightarrow aSbS \rightarrow aSbaSbS \rightarrow aSbaSb \rightarrow aSbab \rightarrow abab$

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b) Find all right-most derivations

$S \rightarrow aSbS \rightarrow aSbaSbS \rightarrow aSbaSb \rightarrow aSbab \rightarrow abab$

$S \rightarrow aSbS \rightarrow aSb \rightarrow abSaSb \rightarrow abSab \rightarrow abab$

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#### Problem 4

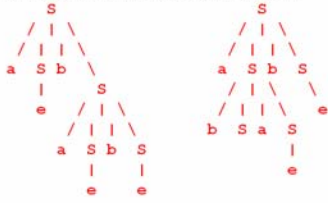
c) Find the parse tree for each derivation

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### Problem 4

c) Find the parse tree for each derivation

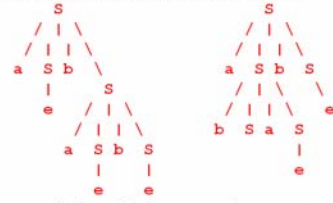


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### Problem 4

c) Find the parse tree for each derivation



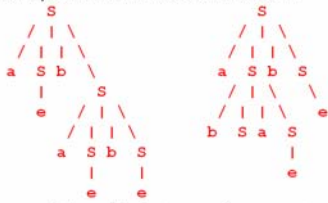
d) Can you distinguish between the parse trees?

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### Problem 4

c) Find the parse tree for each derivation



d) Can you distinguish between the parse trees?

Yes

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