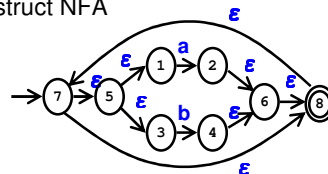


CMSC 330: Organization of Programming Languages

Practice Problem 2 Solutions

Problem 1 – (a | b)*

a) Construct NFA



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Problem 1 – (a | b)*

b) Reduce NFA to DFA

- Start = ϵ -closure(7) = {7,5,1,3,8}
- R = {{7,5,1,3,8}}
- $r \in R = \{7,5,1,3,8\}$ // mark DFA state
- move ({7,5,1,3,8}, a) = {2}
 - e = ϵ -closure({2}) = {2,6,8,7,5,1,3} // new DFA state
 - R = R \cup {2,6,8,7,5,1,3} // add to R
 - $\delta = \delta \cup \{(7,5,1,3,8), a, \{2,6,8,7,5,1,3\}\}$
- move ({7,5,1,3,8}, b) = {4}
 - e = ϵ -closure({4}) = {4,6,8,7,5,1,3} // new DFA state
 - R = R \cup {4,6,8,7,5,1,3} // add to R
 - $\delta = \delta \cup \{(7,5,1,3,8), b, \{4,6,8,7,5,1,3\}\}$

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Problem 1 – (a | b)*

- R = {{7,5,1,3,8}, {2,6,8,7,5,1,3}, {4,6,8,7,5,1,3}}
- $r \in R = \{2,6,8,7,5,1,3\}$ // mark DFA state
- move ({2,6,8,7,5,1,3}, a) = {2}
 - e = ϵ -closure({2}) = {2,6,8,7,5,1,3} // existing DFA state
 - $\delta = \delta \cup \{(2,6,8,7,5,1,3), a, \{2,6,8,7,5,1,3\}\}$
- move ({2,6,8,7,5,1,3}, b) = {4}
 - e = ϵ -closure({4}) = {4,6,8,7,5,1,3} // existing DFA state
 - $\delta = \delta \cup \{(2,6,8,7,5,1,3), b, \{4,6,8,7,5,1,3\}\}$

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Problem 1 – (a | b)*

- R = {{7,5,1,3,8}, {2,6,8,7,5,1,3}, {4,6,8,7,5,1,3}}
- $r \in R = \{4,6,8,7,5,1,3\}$ // mark DFA state
- move ({4,6,8,7,5,1,3}, a) = {2}
 - e = ϵ -closure({2}) = {2,6,8,7,5,1,3} // existing DFA state
 - $\delta = \delta \cup \{(4,6,8,7,5,1,3), a, \{2,6,8,7,5,1,3\}\}$
- move ({4,6,8,7,5,1,3}, b) = {4}
 - e = ϵ -closure({4}) = {4,6,8,7,5,1,3} // existing DFA state
 - $\delta = \delta \cup \{(4,6,8,7,5,1,3), b, \{4,6,8,7,5,1,3\}\}$
- R = {{7,5,1,3,8}, {2,6,8,7,5,1,3}, {4,6,8,7,5,1,3}}
- No more unmarked states to process
- $F_d = \{(7,5,1,3,8), \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
 - Since $8 \in F_n$

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Problem 1 – (a | b)*

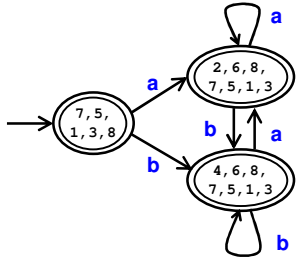
- Resulting DFA
 - $\Sigma = \{a,b\}$
 - R = {{7,5,1,3,8}, {2,6,8,7,5,1,3}, {4,6,8,7,5,1,3}}
 - $r_0 = \{7,5,1,3,8\}$
 - $F_d = \{(7,5,1,3,8), \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
 - $\delta = \{$
 - $((7,5,1,3,8), a, \{2,6,8,7,5,1,3\}),$
 - $((7,5,1,3,8), b, \{4,6,8,7,5,1,3\}),$
 - $((2,6,8,7,5,1,3), a, \{2,6,8,7,5,1,3\}),$
 - $((2,6,8,7,5,1,3), b, \{4,6,8,7,5,1,3\}),$
 - $((4,6,8,7,5,1,3), a, \{2,6,8,7,5,1,3\}),$
 - $((4,6,8,7,5,1,3), b, \{4,6,8,7,5,1,3\})$

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Problem 1 – $(a | b)^*$

- NFA to DFA reduction pictorial



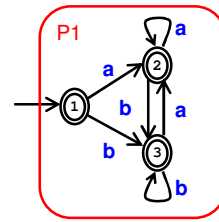
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Problem 1 – $(a | b)^*$

- c) Minimize DFA

- Initial partitions
 - Accept $\rightarrow \{1,2,3\} \rightarrow P1$
 - Reject $\rightarrow \emptyset$
- Split partition?
 - move(1,a) $\rightarrow P1$
 - move(2,a) $\rightarrow P1$
 - move(3,a) $\rightarrow P1$
 - move(1,b) $\rightarrow P1$
 - move(2,b) $\rightarrow P1$
 - move(3,b) $\rightarrow P1$



Not required, minimization done

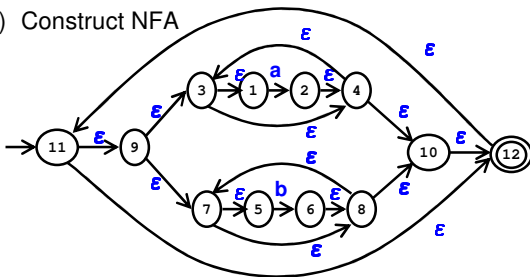


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Problem 1 – $(a^* | b^*)^*$

- a) Construct NFA



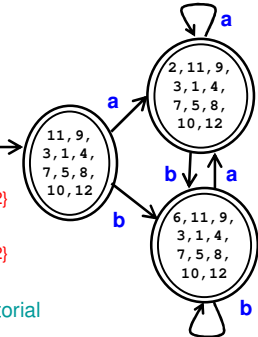
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Problem 1 – $(a^* | b^*)^*$

- b) Reduce NFA to DFA

- Start = ϵ -closure(1) = $\{11,9,3,1,4,7,5,8,10,12\}$
- $R = \{\{11,9\dots12\}\}$
- $r \in R = \{11,9\dots12\}$
 - move($\{11,9\dots12\}, a$) = 2
 - ϵ -closure($\{2\}$) = $\{2,11,9\dots12\}$
 - move($\{11,9\dots12\}, b$) = 6
 - ϵ -closure($\{6\}$) = $\{6,11,9\dots12\}$
- ...
- NFA to DFA reduction pictorial



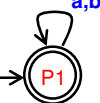
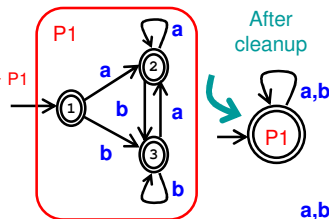
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Problem 1 – $(a^* | b^*)^*$

- c) Minimize DFA

- Initial partitions
 - Accept $\rightarrow \{1,2,3\} \rightarrow P1$
 - Reject $\rightarrow \emptyset$
 - Split partition?
 - move(1,a) $\rightarrow P1$
 - ...
- Not required, minimization done



- d) Compare 2 minimized DFAs

- Identical up to state names!

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Problem 2a

Write a grammar for the following languages

- a) All strings of 0's and 1's that have the **same** number of 0's and 1's

$S \rightarrow A S B | B S A | \epsilon$ // same # of 0's & 1's
 $A \rightarrow 1 S | S 1$ // one more 1 than 0
 $B \rightarrow 0 S | S 0$ // one more 0 than 1

or
 $S \rightarrow 1 S 0 S | 0 S 1 S | \epsilon$ // same # of 0's & 1's

And many other possibilities!

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Problem 2b

Write a grammar for the following languages
 b) All strings of 0's and 1's that have **more** 0's than 1's

$S' \rightarrow B S' \mid B S$ // add extra 0
 $S \rightarrow A S B \mid B S A \mid \epsilon$ // same # of 0's & 1's
 $A \rightarrow 1 S \mid S 1$ // one more 1 than 0
 $B \rightarrow 0 S \mid S 0$ // one more 0 than 1
 Where S' is the new start symbol

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Problem 2c

Write a grammar for the following languages
 c) All balanced pairs of left and right parentheses (e.g., "()", "()(())")

$S \rightarrow (S)$ // left & right parentheses are balance
 $\mid S S$ // both sides are balanced
 $\mid \epsilon$ // empty string

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Definitions

- Left-most derivation
 - Left-most nonterminal is replaced in each step
- Right-most derivation
 - Right-most nonterminal is replaced in each step
- Example
 - Grammar
 - $S \rightarrow AB, A \rightarrow a, B \rightarrow b$
 - Leftmost derivation for "ab"
 - $S \rightarrow AB \rightarrow aB \rightarrow ab$
 - Rightmost derivation for "ab"
 - $S \rightarrow AB \rightarrow Ab \rightarrow ab$

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Problem 3

Consider the string "(a,a)" and the grammar

$S \rightarrow (L) \mid a$
 $L \rightarrow L , S \mid S$

- Find all left-most derivations
- Find all right-most derivations
- Find the parse tree for each derivation
- Can you distinguish between the parse trees? **No**

S
 $/ \backslash$
 (L)
 $/ \backslash$
 L , S
 $| \quad |$
 $S \quad a$
 $|$
 a

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Problem 3

Consider the string "(a,(a,a))" and grammar

$S \rightarrow (L) \mid a$
 $L \rightarrow L , S \mid S$

- Find all left-most derivations
- Find all right-most derivations
- Find the parse tree for each derivation
- Can you distinguish between the parse trees? **No**

S
 $/ \backslash$
 (L)
 $/ \backslash$
 L , S
 $| \quad / \backslash$
 $S \quad (L)$
 $| \quad / \backslash$
 $a \quad L , S$
 $| \quad |$
 $S \quad a$
 $|$
 a

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Problem 4

Consider the string "abab" and the grammar

$S \rightarrow aSbS \mid bSaS \mid \epsilon$

- Find all left-most derivations
- Find all right-most derivations

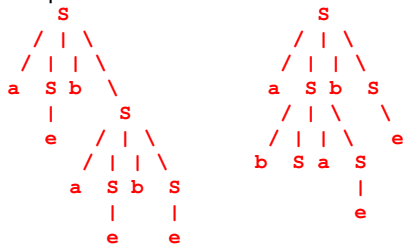
$S \rightarrow aSbS \rightarrow abS \rightarrow abaSbS \rightarrow ababS \rightarrow abab$
 $S \rightarrow aSbS \rightarrow abSaSbS \rightarrow abaSbS \rightarrow ababS \rightarrow abab$
 $S \rightarrow aSbS \rightarrow aSbaSbS \rightarrow aSbaSb \rightarrow aSbab \rightarrow abab$
 $S \rightarrow aSbS \rightarrow aSb \rightarrow abSaSb \rightarrow abSab \rightarrow abab$

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Problem 4

c) Find the parse tree for each derivation



d) Can you distinguish between the parse trees?

Yes