B-Trees

CMSC 420: Lecture 9
Another way to achieve “balance”

• Height of a perfect binary tree of n nodes is $O(\log n)$.

• Idea: Force the tree to be perfect.
  
  - Problem: can’t have an arbitrary # of nodes.
  
  - Perfect binary trees only have $2^h - 1$ nodes

• So: relax the condition that the search tree be binary.

• As we’ll see, this lets you have any number of nodes while keeping the leaves all at the same depth.

*Global balance instead of the local balance of AVL trees.*
2,3 Trees

- All leaves are at the same level.
- Each internal node has either 2 or 3 children.
- If it has:
  - 2 children => it has 1 key
  - 3 children => it has 2 keys
2,3 Tree Find (multiway searching)

Find 19

Standard BST-type walk down the tree.
At each node have to examine each key stored there.
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2,3 Tree Insertion
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![2,3 Tree Diagram]
2,3 Tree Insertion
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22  68
11  17  25  69  70
73
76  80
98  102
94
2,3 Tree Insertion
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Overflow!
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Try **Key Rotation**: Look for left or right sibling with some space, move a parent key into it, and a child key into the parent.
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2,3 Tree Insertion – When key rotation fails
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Overflow!
If both siblings are filled, you have to split the node.
Overflow!
If both siblings are filled, you have to split the node.
2,3 Tree Insertion – When key rotation fails

Overflow!

May have to recursively split nodes, working back to the root.
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2,3 Tree Insertion – When key rotation fails

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2,3 Tree Insertion – Another Splitting Example
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2,3 Tree Insertion – Splitting at the root
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From 2,3-Trees to $a,b$-trees

- An $a,b$-tree is a generalization of a 2,3-tree, where each node (except the root) has between $a$ and $b$ children.
- Root can have between 2 and $b$ children.
- We require that:
  - $a \geq 2$ (can’t allow internal nodes to have 1 child)
  - $b \geq 2a - 1$ (need enough children to make split work)
a,b Insertions & Deletions

key rotation

split

merge
Deletion Details

- Try to borrow a key from a sibling if you have one that has an extra ($\geq 1$ more than the minimum)
- Otherwise, you have a sibling with exactly the minimum number $a-1$ of keys.
- Since you are underflowing, you must have one less than the minimum number of keys = $a-2$.
- Therefore, merging with your sibling produces a node with $a-1 + a-2 = 2a-3$ keys.
- This is one less than the maximum ($2a-2$ keys), so we have room to bring down the key that split us from our sibling.
B-trees

• A B-tree of order $b$ is an $a,b$-tree with $b = 2a - 1$
  
  – In other words, we choose the largest allowed $a$.

• Want to have large $b$ if bringing a node into memory is slow (say reading a disc block), but scanning the node once in memory is fast.

• $b$ is usually chosen to match characteristics of the device.

• Ex. B-tree of order 1023 has $a = 512$.
  
  – If this B-tree stores $n = 10$ million records, its height no more than $O(\log_a n) \approx 2.58$. So only around 3 blocks need to be read from disk.
What if $b$ is very large?

- Need to be able to find which subtree to traverse.

- Could linearly search through keys – technically constant time if $b$ is a constant, but may be time consuming.

- Solution: Store a balanced tree (AVL or splay) at each node so that you can search for keys efficiently.