Trees

- Trees are hierarchical data structures
  - One-to-many relationship between elements

- Tree node / element
  - Contains data
  - Referred to by only 1 (parent) node
  - Contains links to any number of (children) nodes

Parent node

Children nodes
Trees

**Terminology**

- **Root** ⇒ node with no parent
- **Leaf** ⇒ all nodes with no children
- **Interior** ⇒ all nodes with children
Trees

**Terminology**

- **Sibling** ⇒ node with same parent
- **Descendent** ⇒ children nodes & their descendents
- **Subtree** ⇒ portion of tree that is a tree by itself
  ⇒ a node and its descendents
Trees

**Terminology**

- **Level** ⇒ is a measure of a node’s distance from root.

**Definition of level**

- If node is the root of the tree, its level is 1.
- Else, the node’s level is 1 + its parent’s level.

- **Height (depth)** ⇒ max level of any node in tree.

```
  ┌───┐
  │   │
  │   │
  │   │
  │   │
  │   │
  └───┘
     
Height = 3
```
Binary Trees

- Binary tree
  - Tree with 0–2 children per node
  - Left & right child / subtree

Binary Tree

Parent

Left Child

Right Child
Tree Traversal

Often we want to
1. Find all nodes in tree
2. Determine their relationship

Can do this by
1. Walking through the tree in a prescribed order
2. Visiting the nodes as they are encountered

Process is called tree traversal
Tree Traversal

Goal

Visit every node in binary tree

Approaches

Depth first

- Preorder ⇒ parent before children
- Inorder ⇒ left child, parent, right child
- Postorder ⇒ children before parent

Breadth first ⇒ closer nodes first
Tree Traversal Methods

- **Pre-order**
  1. Visit node // first
  2. Recursively visit left subtree
  3. Recursively visit right subtree

- **In-order**
  1. Recursively visit left subtree
  2. Visit node // second
  3. Recursively visit right subtree

- **Post-order**
  1. Recursively visit left subtree
  2. Recursively visit right subtree
  3. Visit node // last
Tree Traversal Methods

Breadth-first

BFS(Node n) {

    Queue Q = new Queue();
    Q.enqueue(n);  // insert node into Q

    while ( !Q.empty()) {
        n = Q.dequeue();  // remove next node
        if ( !n.isEmpty()) {
            visit(n);  // visit node
            Q.enqueue(n.Left());  // insert left subtree in Q
            Q.enqueue(n.Right());  // insert right subtree in Q
        }
    }
}
Tree Traversal Examples

- **Pre-order (prefix)**
  - $+ \times 2 \ 3 \ / \ 8 \ 4$

- **In-order (infix)**
  - $2 \times 3 + 8 / 4$

- **Post-order (postfix)**
  - $2 \ 3 \ \times \ 8 \ 4 \ / \ +$

- **Breadth-first**
  - $+ \times / \ 2 \ 3 \ 8 \ 4$

Expression tree
Binary Tree Implementation

- Using a class to represent a Node

  ```java
  Class Node {
      KeyType key;
      Node left, right; // null if empty
  }
  
  Node root = null; // Empty Tree
  ```

- Using a Polymorphic Binary Tree
  - We will talk about this implementation later on
Types of Binary Trees

- **Degenerate**
  - Mostly 1 child / node
  - Height = $O(n)$
  - Similar to linear list

- **Balanced**
  - Mostly 2 child / node
  - Height = $O(\log(n))$
  - $2^{\text{Height}} - 1 = n$ (# of nodes)
  - Useful for searches

Degenerate binary tree

Balanced binary tree
Binary Search Trees

Key property

Value at node
- Smaller values in left subtree
- Larger values in right subtree

Example
- X > Y
- X < Z
Binary Search Trees

Examples

Binary search trees

Non-binary search tree
Tree Traversal Examples

- Pre-order
  - 44, 17, 32, 78, 50, 48, 62, 88

- In-order
  - 17, 32, 44, 48, 50, 62, 78, 88

- Post-order
  - 32, 17, 48, 62, 50, 88, 78, 44

- Breadth-first
  - 44, 17, 78, 32, 50, 88, 48, 62

Binary search tree

Sorted order!
Example Binary Searches

Find (2)

2 < 10, left
2 < 5, left
2 = 2, found

2 < 5, left
2 = 2, found
Example Binary Searches

Find (25)

- 25 > 10, right
- 25 < 30, left
- 25 = 25, found

- 25 > 5, right
- 25 < 45, left
- 25 < 30, left
- 25 > 10, right
- 25 = 25, found
Binary Search Properties

- **Time of search**
  - Proportional to height of tree
  - Balanced binary tree
    - $O(\log(n))$ time
  - Degenerate tree
    - $O(n)$ time
    - Like searching linked list / unsorted array

- Requires
  - Ability to compare key values
Binary Search Tree Construction

How to build & maintain binary trees?
- Insertion
- Deletion

Maintain key property (invariant)
- Smaller values in left subtree
- Larger values in right subtree
Binary Search Tree – Insertion

Algorithm
1. Perform search for value X
2. Search will end at node Y (if X not in tree)
3. If X < Y, insert new leaf X as new left subtree for Y
4. If X > Y, insert new leaf X as new right subtree for Y

Observations
- O( log(n) ) operation for balanced tree
- Insertions may unbalance tree
Example Insertion

Insert ( 20 )

20 > 10, right
20 < 30, left
20 < 25, left
Insert 20 on left
Binary Search Tree – Deletion

Algorithm

1. Perform search for value X
2. If X is a leaf, delete X
3. Else // must delete internal node
   a) Replace with largest value Y on left subtree
      OR smallest value Z on right subtree
   b) Delete replacement value (Y or Z) from subtree

Observation

- \( O(\log(n)) \) operation for balanced tree
- Deletions may unbalance tree
Delete (25)

25 > 10, right
25 < 30, left
25 = 25, delete
Delete ( 10 )

Replacing 10 with largest value in left subtree

Replacing 5 with largest value in left subtree

Deleting leaf
Example Deletion (Internal Node)

Delete (10)

Replacing 10 with the smallest value in the right subtree

Deleting leaf

Resulting tree
Building Maps w/ Search Trees

- Binary Search trees often used to implement maps
  - Each non-empty node contains
    - Key
    - Value
    - Left and right child

- Need to be able to compare keys
  - Generic type <K extends Comparable<K>>
    - Denotes any type K that can be compared to K’s
BST (Binary Search Tree) Implementation

- Implementing Tree using traditional approach
- Based on the BST definition below let’s see how to implement typical BST Operations (constructor, add, print, find, isEmpty, isFull, size, height, etc.)

```java
public class BinarySearchTree <K extends Comparable<K>, V> {
    private class Node {
        private K key;
        private V data;
        private Node left, right;
        public Node(K key, V data) {
            this.key = key;
            this.data = data;
        }
    }
    private Node root;
}
```

See code distribution BinaryTreeCode.zip
BST Testing

How can we test the correctness of BST Methods?

What is the best approach?
Polymorphic Binary Search Trees

Second approach to implement BST

What do we mean by polymorphic?

Implement two subtypes of Tree
1. EmptyTree
2. NonEmptyTree

Use EmptyTree to represent the empty tree

Rather than null

Invoke methods on tree nodes

Without checking for null (IMPORTANT!)
Polymorphic Binary Tree Implementation

Interface Tree {
    Tree insert ( Value data1 ) { … }
}

Class EmptyTree implements Tree {
    Tree insert ( Value data1 ) { … }
}

Class NonEmptyTree implements Tree {
    Value data;
    Tree left, right; // Either Empty or NonEmpty
    Tree insert ( Value data1 ) { … }
}
Standard vs. Polymorphic Binary Tree

Class Node {
    Node left, right;
}

Node X {
    left = Y;
    right = Z;
}

Node Y {
    left = null;
    right = null;
}

Node Z {
    left = null;
    right = W;
}

Node W {
    left = null;
    right = null;
}

Class EmptyTree {} 

Class NonEmptyTree {
    Tree left, right;
}

NonEmptyTree X {
    left = Y;
    right = Z;
}

NonEmptyTree Y {
    left = ET;
    right = ET;
}

NonEmptyTree Z {
    left = ET;
    right = W;
}

NonEmptyTree W {
    left = ET;
    right = ET;
}

EmptyTree {}
Singleton Design Pattern

Definition

- One instance of a class or value accessible globally

Where to use & benefits

- Ensure unique instance by defining class final
- Access to the instance only via methods provided

EmptyTree class will be a singleton class
public final class MySingleton {
  // declare the unique instance of the class
  private static MySingleton uniq = new MySingleton();
  // private constructor only accessed from this class
  private MySingleton() { … }
  // return reference to unique instance of class
  public static MySingleton getInstance() {
    return uniq;
  }
}
Using Singleton EmptyTree

Class Node {
    Node left, right;
}

Class EmptyTree {}
Class NonEmptyTree {
    Tree left, right;
}

Node X {
    left = Y;
    right = Z;
}

Node Y {
    left = null;
    right = null;
}

Node Z {
    left = null;
    right = W;
}

Node W {
    left = null;
    right = null;
}

NonEmptyTree X {
    left = Y;
    right = Z;
}

NonEmptyTree Y {
    left = ET;
    right = ET;
}

NonEmptyTree Z {
    left = ET;
    right = W;
}

EmptyTree ET {
}

NonEmptyTree W {
    left = ET;
    right = ET;
}

NonEmptyTree X {
    left = Y;
    right = Z;
}

NonEmptyTree Y {
    left = ET;
    right = ET;
}

NonEmptyTree Z {
    left = ET;
    right = W;
}

EmptyTree ET {
}
Polymorphic List Implementation

Let’s see a polymorphic list implementation

See code distribution PolymorphicListCode.zip