Feb 5

**Problem 1.** For each pair of expressions \( (A, B) \) below, indicate whether \( A \) is \( O, o, \Omega, \omega, \) or \( \Theta \) of \( B \). Note that zero, one or more of these relations may hold for a given pair; list all correct ones.

\[
\begin{array}{c|c|c}
A & B \\
\hline
(a) & n^{100} & 2^n \\
(b) & 10^n & 100^n \\
(c) & \log (n!) & n \log n \\
\end{array}
\]

**Problem 2.** Prove by induction

\[
\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}.
\]

**Problem 3.** Assume that there are \( n \) numbers (some possibly negative) in a circle, and we wish to find the maximum contiguous sum on the circle. Give an efficient algorithm for solving this problem. What is its worst case running time?

**Problem 4.** Solve the recurrence

\[
T(n) = \begin{cases} 
T(n/5) + 2 & \text{if } n > 1 \\
3 & \text{otherwise}
\end{cases}
\]

assuming \( n \) is a power of 5. Show your calculations.

**Problem 5.** A coin is tossed \( n \) times, each time with an independent probability \( p \) of coming up heads and \( 1 - p \) of coming up tails. Let \( H \) be the number of heads occurring. What is

(a) \( E[H] \), the expected number of heads?

(b) \( V[H] \), the variance of \( H \)?

(c) the standard deviation of \( H \)?

(d) the probability that \( H > 2 \)?

Show your calculations.