Problem 1. Draw the deterministic finite state machine for each of the following:

(a) \( w \in \{a, b, c\}^* : \) no letter occurs twice in a row.
(b) \( w \in \{a, b\}^* : \) aabb is not a substring.
(c) \( w \in \{a, b\}^* : \) neither aabb nor abab is a substring.
(d) \( w \in \{a, b\}^* : \) exactly one of aabb and abab is a substring.
(e) \( w \in \{a, b\}^* : \) at least one of aabb and abab is a substring.
(f) \( w \in \{a, b\}^* : \) both aabb and abab are substrings.
(g) \( w \in \{a, b\}^* : \) the number of a’s minus 1 is divisible by 4 and the number of b’s is even.

Give the full machine description for part (b) only.

Problem 2.

(a) Draw a nondeterministic finite state machine that accepts a string if the number in binary notation is even. (Your machine should be “simpler” than the one in part (b).)
(b) Draw a deterministic finite state machine that accepts a string if the number in binary notation is even.
(c) Draw a nondeterministic finite state machine that accepts a string if the number in binary notation is divisible by 3.
(d) Draw a nondeterministic finite state machine that accepts a string if the number in binary notation is even or is divisible by 3.

Give the full machine description for part (a) only.

Problem 3. Do Exercises 2.11(b) (page 59) and 2.11(c) (page 59) in Kimber and Smith. (Will put problem on board.)

Problem 4. Give regular expressions for the following languages.

(a) \( w \in \{a, b\}^* : \) exactly one a.
(b) \( w \in \{a, b\}^* : \) exactly one a or one b.
(c) \( w \in \{a, b\}^* : \) begins with aa and ends with bb.
(d) \( w \in \{a, b\}^* : \) begins with ab and ends with ba.
(e) \( w \in \{a, b\}^* : \) does not begin with ab.
(f) \( w \in \{a, b\}^* : \) both aabb and bbba are substrings.
(g) \( w \in \{a, b\}^* : \) either aabb or bbba is a substring (or both).
Problem 5. Consider a deterministic finite state machine that has one memory location, which can store an element of its own alphabet. At each step the machine knows what state it is in, what is the next symbol on the input string, and what symbol is stored in the memory location. It can change its state and change the symbol in the memory location.

(a) Formally define this machine.
(b) Write a program to tell if the first character of a string occurs at least three times (total) in the string, where the alphabet for words in the language is \{a, b, c, d, e\}.
(c) Is there some problem that can be solved on this version of a finite state machine that cannot be solve on a standard finite state machine. Justify.