

CMSC 631, Spring 2009
Homework 5 Sample Solution (partial)

1. (a) $(\lambda x.x(xy))(\lambda u.u) \rightarrow (\lambda u.u)((\lambda u.u)y) \rightarrow ((\lambda u.u)y) \rightarrow y$
(b) $(\lambda xyz.zyx)aa(\lambda pq.q) \rightarrow (\lambda pq.q)aa \rightarrow a$
(c) $(\lambda xyz.xz(yz))(\lambda xy.x)(\lambda xy.x) \rightarrow \lambda z.(\lambda xy.x)z((\lambda xy.x)z) \rightarrow \lambda z.z$
2. (a) $\lambda xy.y$
(b) $\lambda xyz.xzy$
(c) No term has the type $\alpha \rightarrow \beta$
(d) $\lambda xy.(\lambda fwz.((\lambda v.fw)(fz)))(\lambda u.u)xy$
3. No, the simply typed lambda calculus with integers does not have the subject expansion property. For example, the term 3 has a valid type, but the term $(\lambda xy.x) 3 (\lambda z.\Delta \Delta) \rightarrow^* 3$ yet it does not have a valid type. (Recall that $\Delta = \lambda x.xx$.) Note that we must put $\Delta \Delta$ under a λ because we're using call-by-value semantics. In particular, $(\lambda xy.x) 3 (\Delta \Delta)$ does *not* reduce to 3 under call-by-value semantics because the evaluation of $\Delta \Delta$ never terminates.