CMSC 132: Object-Oriented Programming II

Graph Implementation

Department of Computer Science
University of Maryland, College Park
Graph Implementation

How do we represent edges?

- **Adjacency matrix**
  - 2D array of neighbors
- **Adjacency list**
  - List of neighbors
- **Adjacency set / map**
  - Set / map of neighbors

Important for very large graphs

- Affects efficiency / storage
Adjacency Matrix

**Representation**

- **2D array**
- **Position** $j, k \Rightarrow$ edge between nodes $n_j, n_k$

**Example**

![Diagram of a graph and its adjacency matrix](image-url)
Adjacency Matrix

Representation (cont.)

- Single array for entire graph
- Undirected graph
  - Only upper / lower triangle matrix needed
  - Since $n_j, n_k$ implies $n_k, n_j$
- Unweighted graph
  - Matrix elements $\Rightarrow$ boolean
- Weighted graph
  - Matrix elements $\Rightarrow$ weight
Adjacency List/Set

**Representation**

- For each node, store
  - List/Set of neighbors / successors
    - Linked list
    - Array list
- For weighted graph
  - Also store weight for each edge
  - Using a Map is a good choice
- For undirected graph with edge \((a \leftrightarrow b)\)
  - Nodes a & b need to store each other as neighbor
- For directed graph with edge \((a \rightarrow b)\)
  - Node a needs to store node b as neighbor
Adjacency List

Example

Unweighted graph

node 1: {2, 3}
node 2: {1, 3, 4}
node 3: {1, 2, 4, 5}
node 4: {2, 3, 5}
node 5: {3, 4, 5}

Weighted graph

node 1: {2=3.7, 3=5}
node 2: {1=3.7, 3=1, 4=10.2}
node 3: {1=5, 2=1, 4=8, 5=3}
node 4: {2=10.2, 3=8, 5=1.5}
node 5: {3=3, 4=1.5, 5=6}
Adjacency Set / Map

**Representation**

- For each node, store
  - Set or map of neighbors / successors
- For unweighted graph
  - Use *set* of neighbors
- For weighted graph
  - Use *map* of neighbors, w/ value = weight of edge
- For undirected graph with edge \((a \leftrightarrow b)\)
  - Nodes a & b need to store each other as neighbor
- For directed graph with edge \((a \rightarrow b)\)
  - Node a needs to store node b as neighbor
Graph Space Requirements

- **Adjacency matrix**
  - $\frac{1}{2} N^2$ entries (for graph with $N$ nodes, $E$ edges)
  - Many empty entries for large, sparse graphs

- **Adjacency list**
  - $2 \times E$ entries

- **Adjacency set / map**
  - $2 \times E$ entries
  - Space overhead per entry
    - Higher than for adjacency list
Graph Time Requirements

**Adjacency matrix**
- Can find individual edge (a,b) quickly
  - Examine entry in array Edge[a,b]
    - Constant time operation

**Adjacency list / set / map**
- Can find all edges for node (a) quickly
  - Iterate through collection of edges for a
    - On average E / N edges per node
# Graph Time Requirements

**Average Complexity of operations**

**For graph with N nodes, E edges**

<table>
<thead>
<tr>
<th>Operation</th>
<th>Adj Matrix</th>
<th>Adj List</th>
<th>Adj Set/Map</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find edge</td>
<td>$O(1)$</td>
<td>$O(E/N)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Insert edge</td>
<td>$O(1)$</td>
<td>$O(E/N)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Delete edge</td>
<td>$O(1)$</td>
<td>$O(E/N)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Enumerate edges for node</td>
<td>$O(N)$</td>
<td>$O(E/N)$</td>
<td>$O(E/N)$</td>
</tr>
</tbody>
</table>
Choosing Graph Implementations

- **Graph density**
  - Ratio edges to nodes (dense vs. sparse)

- **Graph algorithm**
  - **Neighbor based**
    - For each node X in graph
      - For each neighbor Y of X // adj list faster if sparse
        - doWork( )
  - **Connection based**
    - For each node X in ...
      - For each node Y in ...
        - if (X,Y) is an edge // adj matrix faster if dense
        - doWork( )