CMSC 132:
Object-Oriented Programming II

Graphs & Graph Traversal

Department of Computer Science

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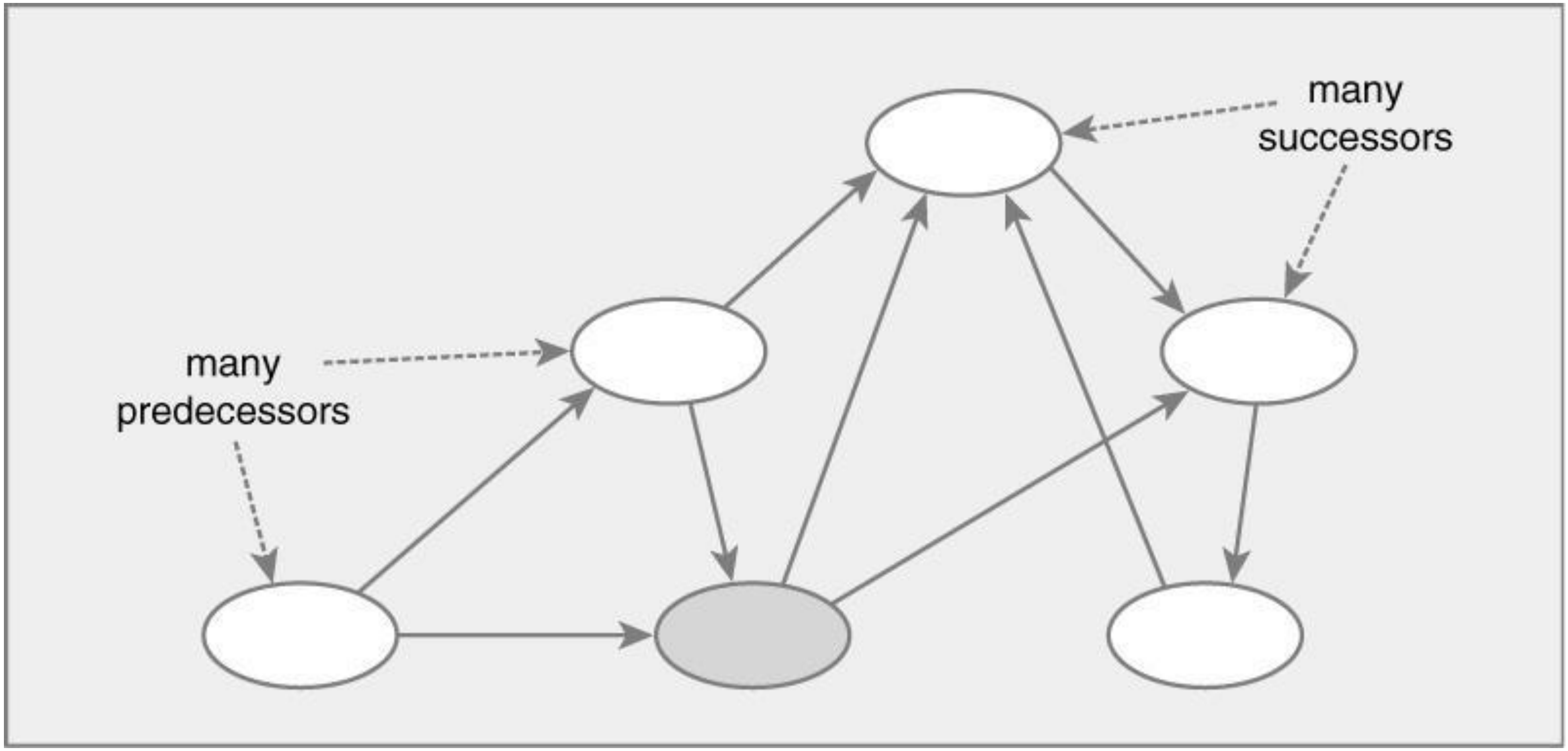
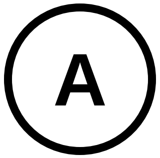
Graph Data Structures

- **Many-to-many relationship between elements**
  - **Each element has multiple predecessors**
  - **Each element has multiple successors**

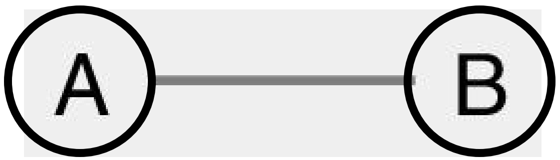


Graph Definitions

- **Node**
  - **Element of graph**
  - **State**
    - **List of adjacent/neighbor/successor nodes**

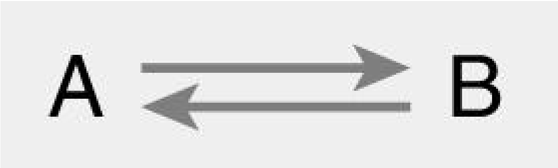


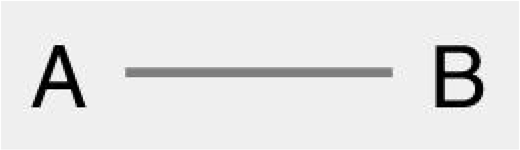
- **Edge**
  - **Connection between two nodes**
  - **State**
    - **Endpoints of edge**

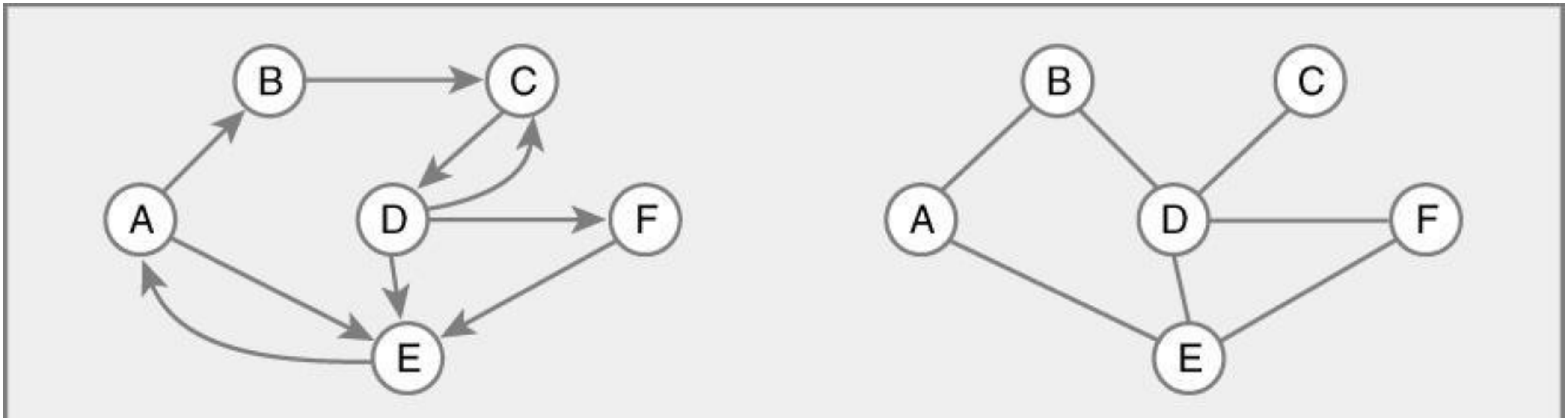


Graph Definitions

- **Directed graph**
  - **Directed edges**
- **Undirected graph**
  - **Undirected edges**





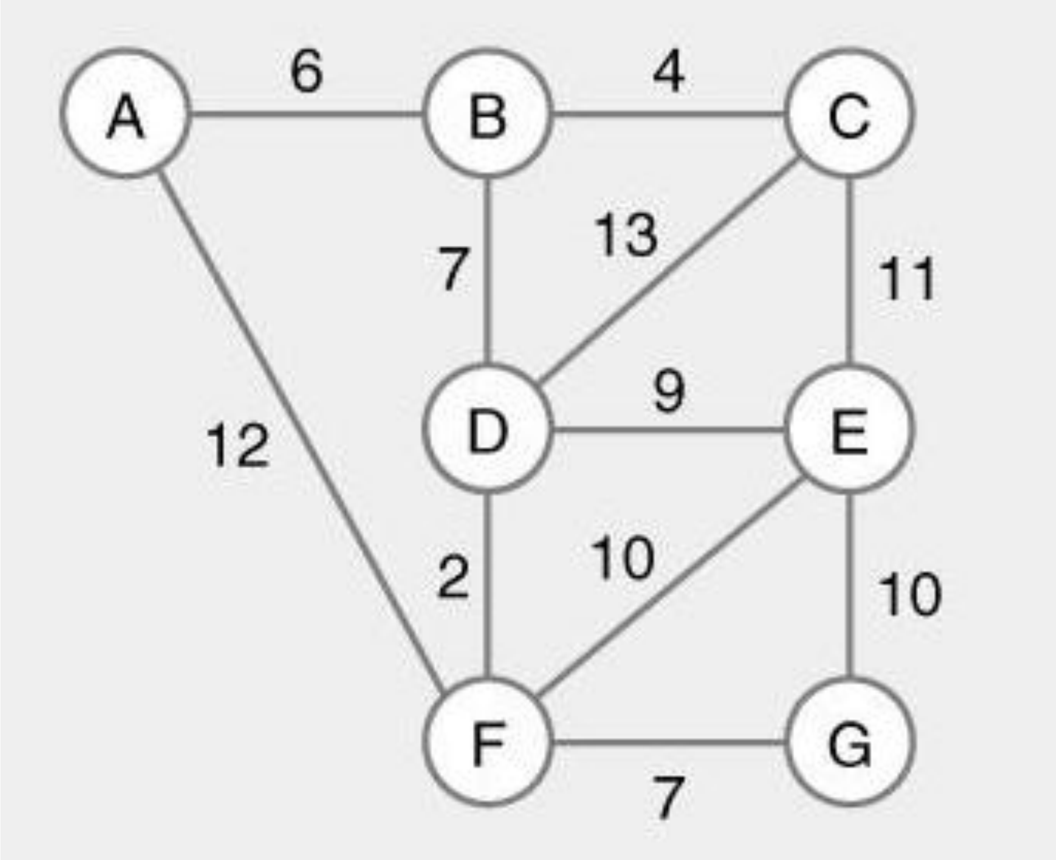


(a) Directed graph

(b) Undirected graph

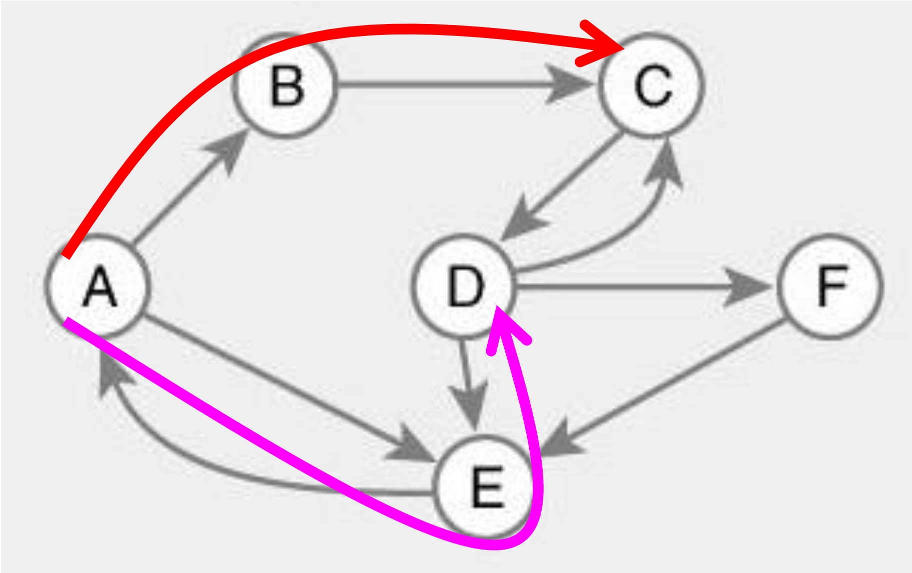
Graph Definitions

- **Weighted graph**
  - **Weight (cost) associated with each edge**



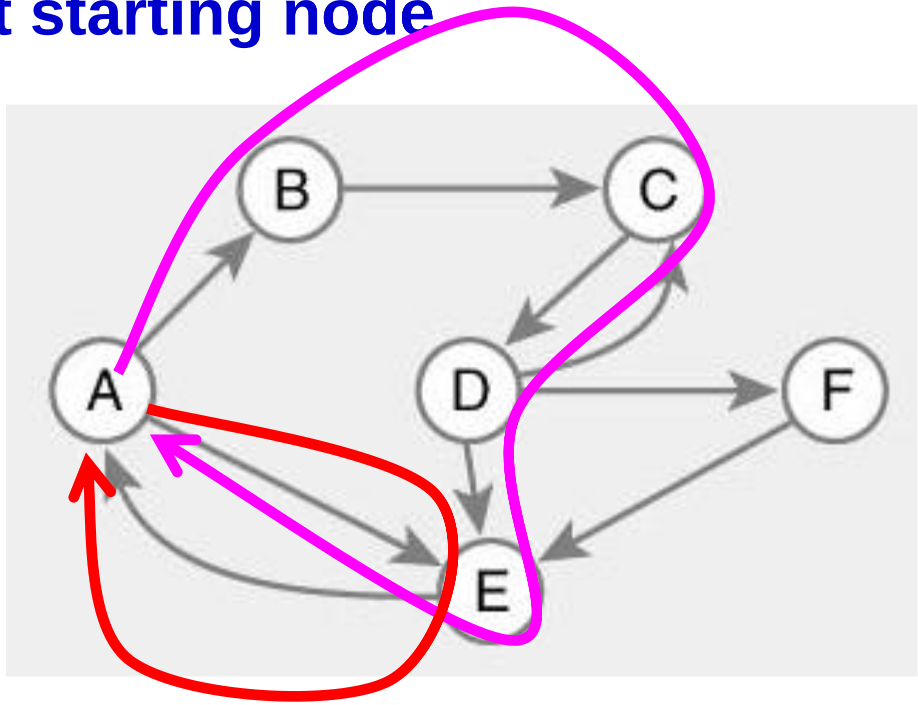
Graph Definitions

- **Path**
  - **Sequence of nodes n_1, n_2, … n_k**
  - **Edge exists between each pair of nodes n_i , n_{i+1}**
  - **Example**
    - **A, B, C is a path**
    - **A, E, D is not a path**



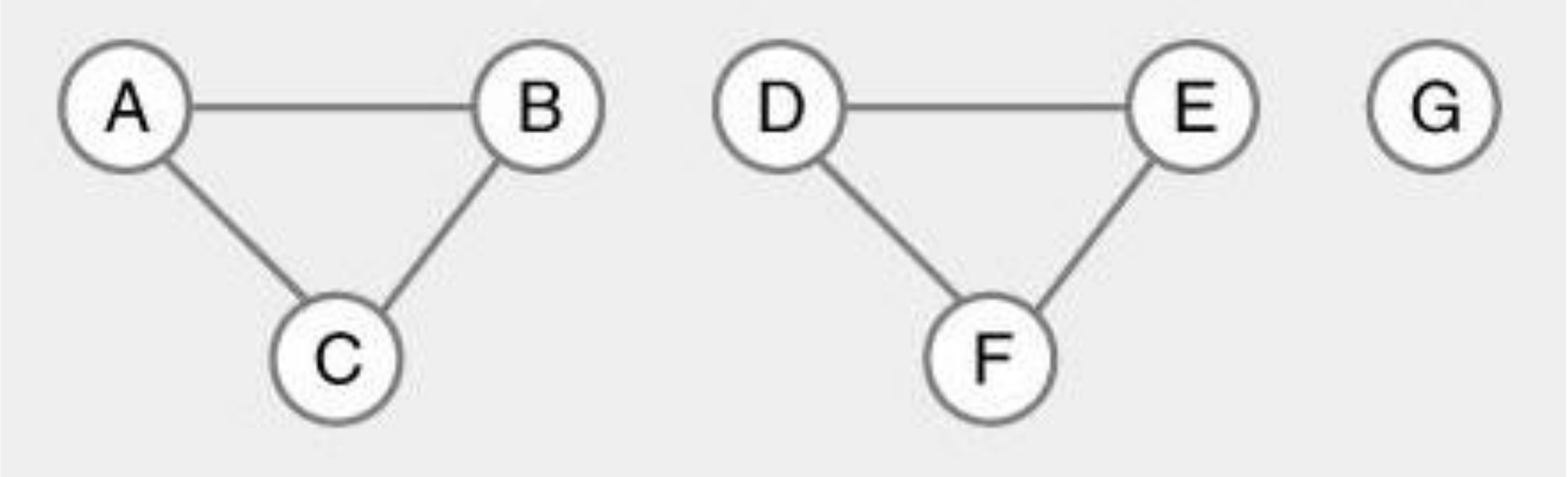
Graph Definitions

- **Cycle**
  - **Path that ends back at starting node**
  - **Example**
    - **A, E, A**
    - **A, B, C, D, E, A**
- **Simple path**
  - **No cycles in path**
- **Acyclic graph**
  - **No cycles in graph**



Graph Definitions

- **Connected Graph**
  - **Every node in the graph is reachable from every other node in the graph**
- **Unconnected graph**
  - **Graph that has several disjoint components**



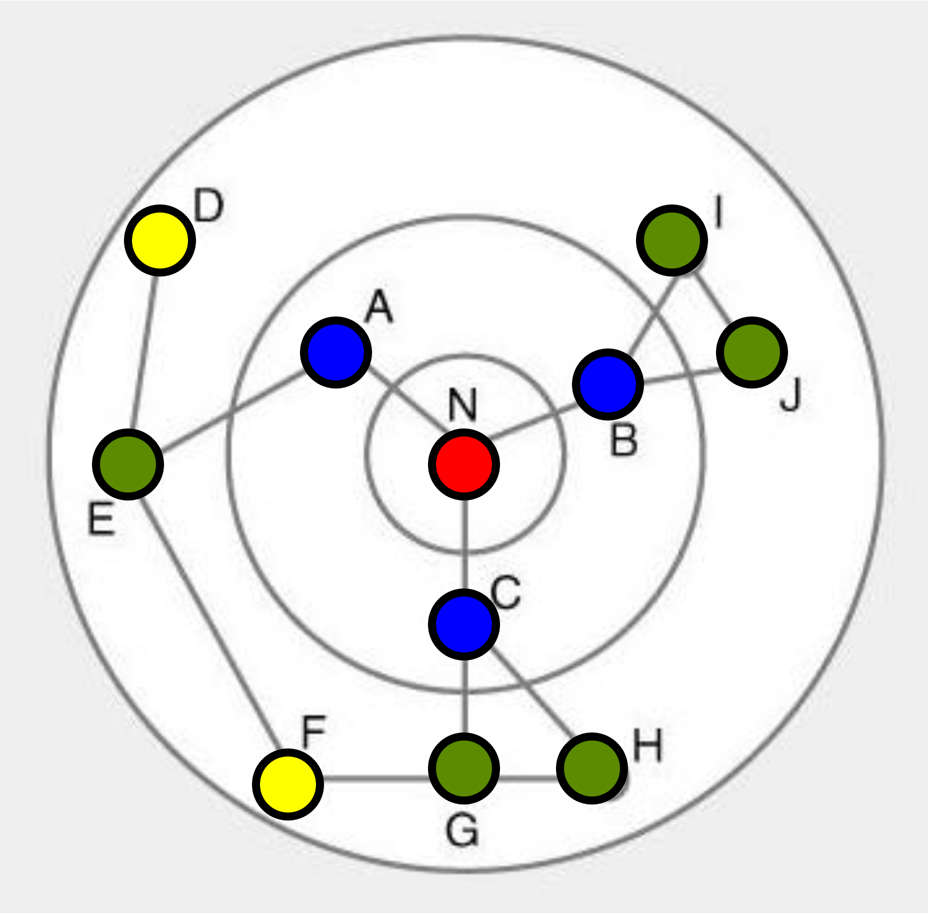
Unconnected graph

Graph Operations

- **Traversal (search)**
  - **Visit each node in graph exactly once**
  - **Usually perform computation at each node**
  - **Two approaches**
    - **Breadth first search (BFS)**
    - **Depth first search (DFS)**

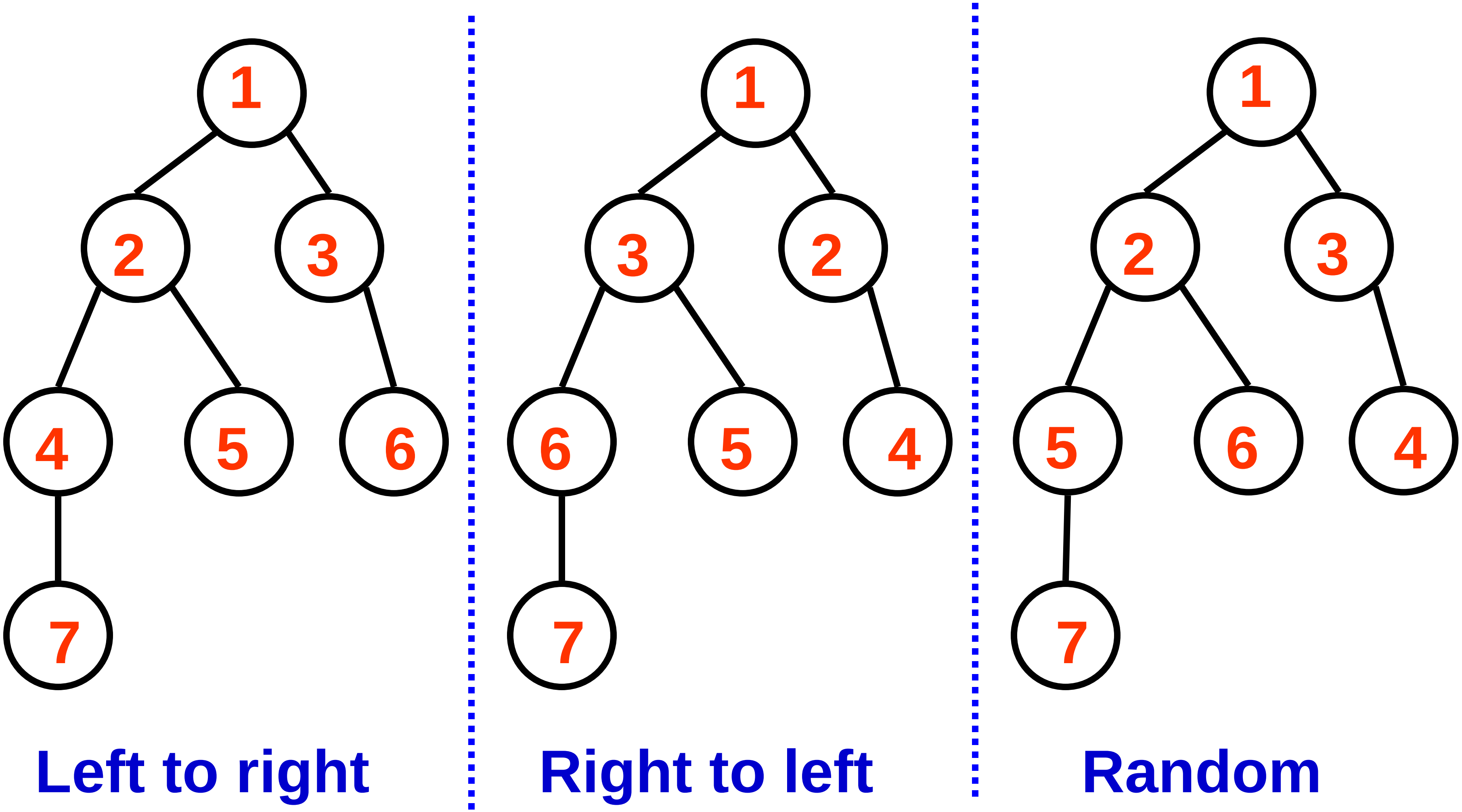
Breadth-first Search (BFS)

- **Approach**
  - **Visit all neighbors of node first**
  - **View as series of expanding circles**
  - **Keep list of nodes to visit in queue**
- **Example traversal**
  1. **n**
  2. **a, c, b**
  3. **e, g, h, i, j**
  4. **d, f**



Breadth-first Tree Traversal

- **Example traversals starting from 1**



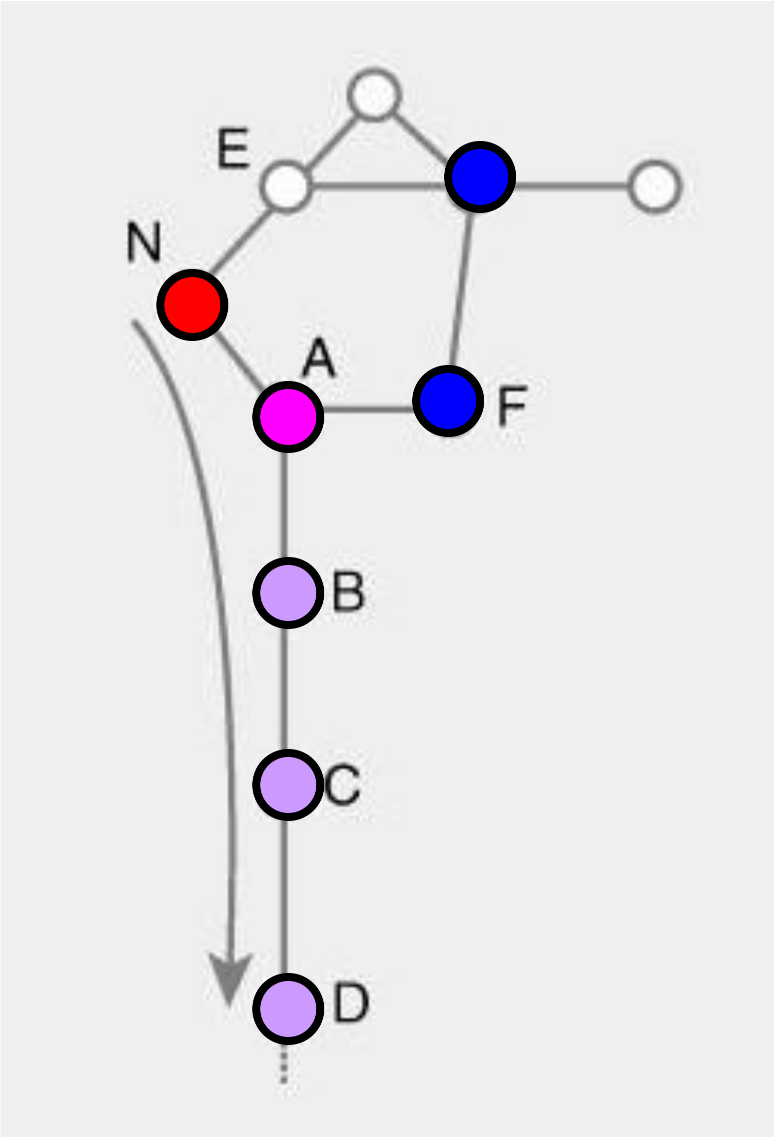
Left to right | **Right to left** | **Random**

Traversals Orders

- Order of successors
  - For tree
    - Can order children nodes from left to right
  - For graph
    - Left to right doesn’t make much sense
    - Each node just has a set of successors and predecessors; there is no order among edges
- For breadth first search
  - Visit all nodes at distance k from starting point
  - Before visiting any nodes at (minimum) distance k+1 from starting point

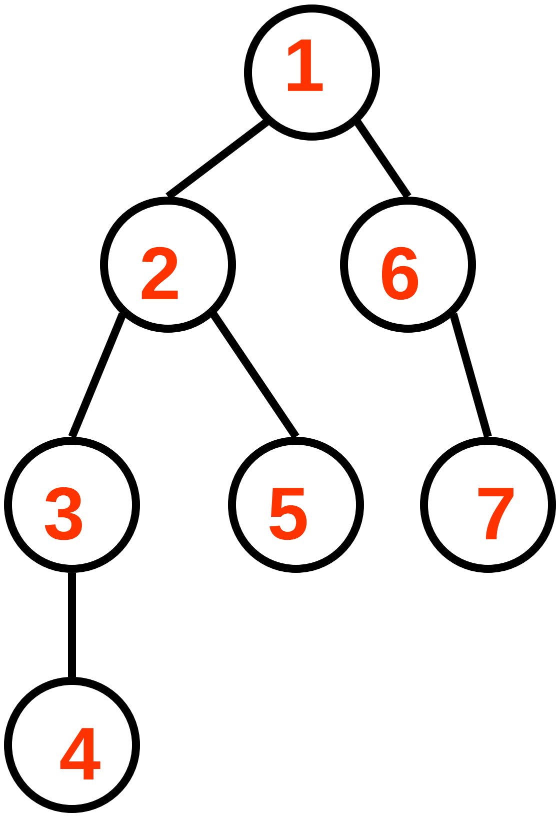
Depth-first Search (DFS)

- **Approach**
  - **Visit all nodes on path first**
  - **Backtrack when path ends**
  - **Keep list of nodes to visit in a stack**
- **Example traversal**
  1. **N**
  2. **A**
  3. **B, C, D, …**
  4. **F…**

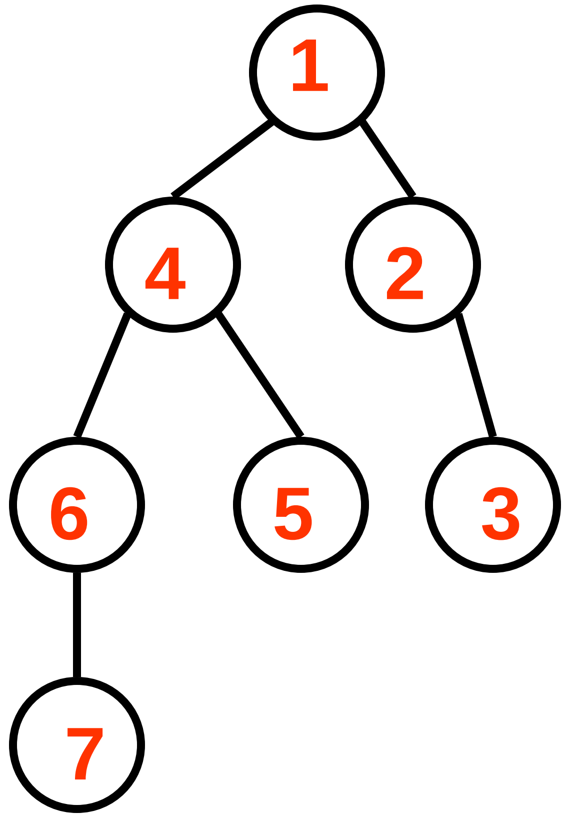


Depth-first Tree Traversal

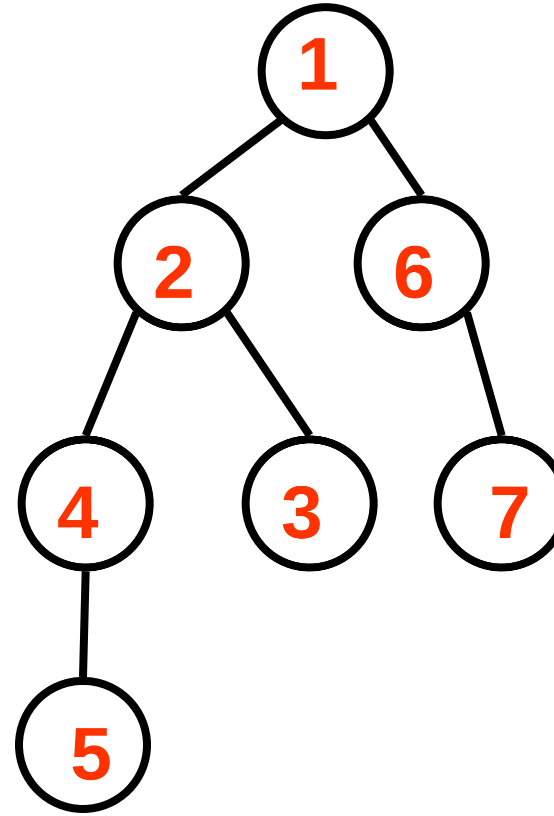
- **Example traversals from 1 (preorder)**



Left to right



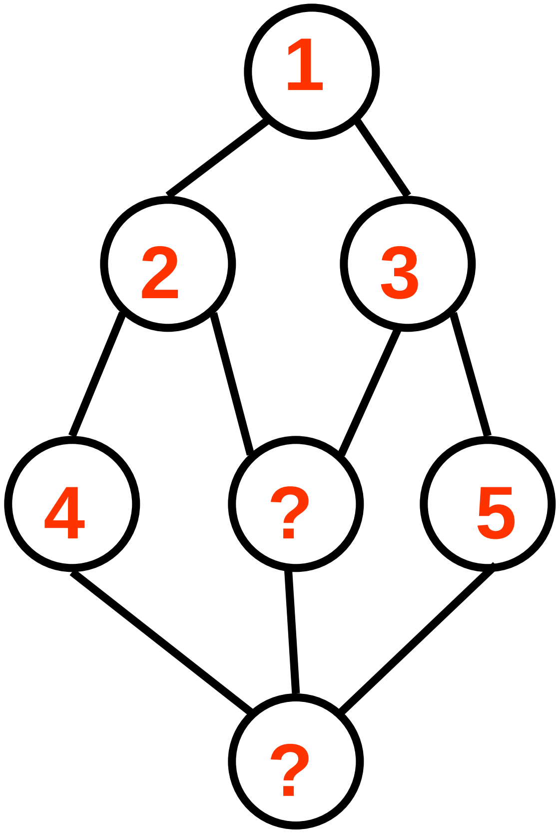
Right to left



Random

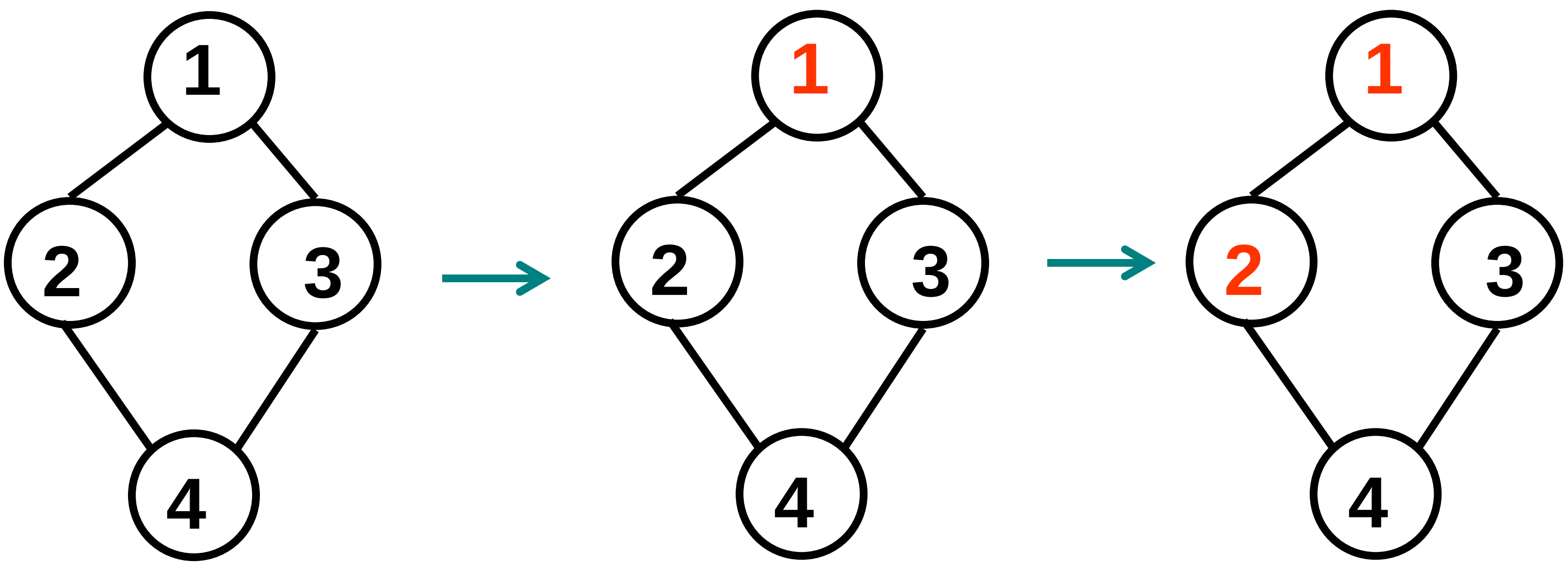
Traversal Algorithms

- **Issue**
  - **How to avoid revisiting nodes**
  - **Infinite loop if cycles present**
- **Approaches**
  - **Record set of visited nodes**
  - **Mark nodes as visited**



Traversal – Avoid Revisiting Nodes

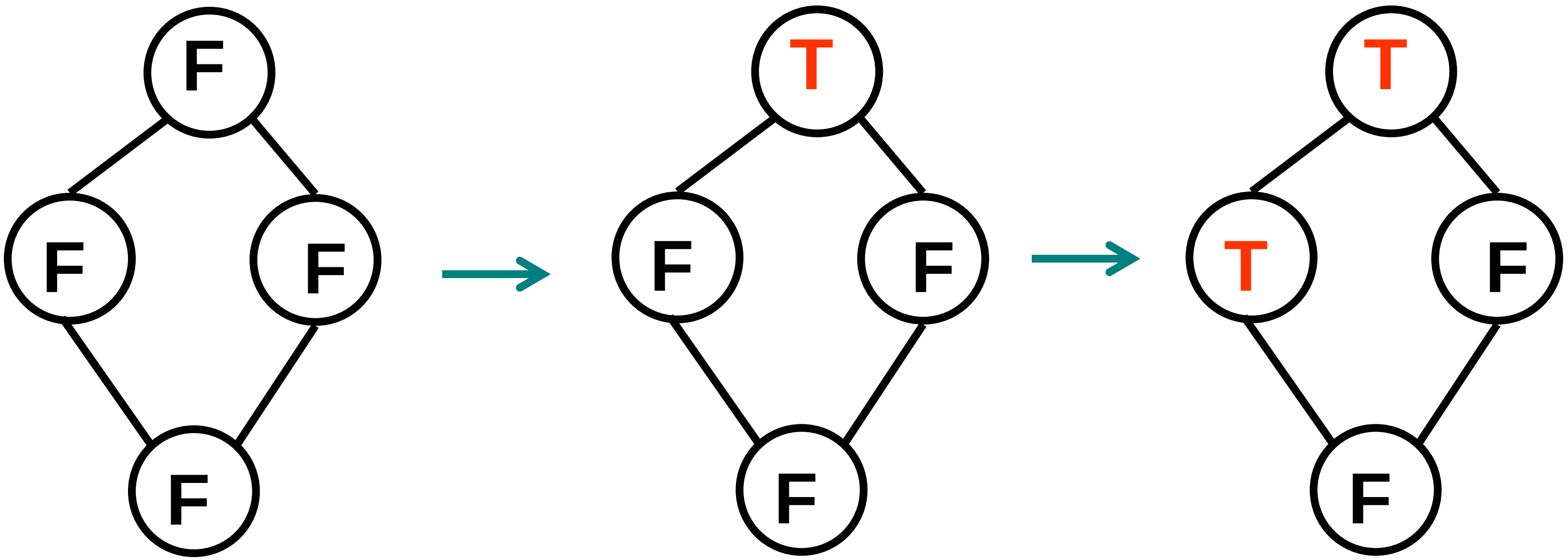
- **Record set of visited nodes**
  - **Initialize { Visited } to empty set**
  - **Add to { Visited } as nodes is visited**
  - **Skip nodes already in { Visited }**



V = ∅ **V = { 1 }** **V = { 1, 2 }**

Traversal – Avoid Revisiting Nodes

- **Mark nodes as visited**
  - **Initialize tag on all nodes (to False)**
  - **Set tag (to True) as node is visited**
  - **Skip nodes with tag = True**



Traversal Algorithm Using Sets

{ Visited } = ∅

{ Discovered } = { 1st node }

while ({ Discovered } ≠ ∅)

    take node X out of { Discovered }

    if X not in { Visited }

        add X to { Visited }

        for each successor Y of X

            if (Y is not in { Visited })

                add Y to { Discovered }

Traversal Algorithm Using Tags

```
for all nodes X
     set X.tag = False
{ Discovered } = { 1st node }
while ( { Discovered } ≠ ∅ )
    take node X out of { Discovered }
    if (X.tag = False)
        set X.tag = True
        for each successor Y of X
            if (Y.tag = False)
                add Y to { Discovered }
```

BFS vs. DFS Traversal

- Order nodes taken out of { Discovered } key
- Implement { Discovered } as Queue
  - First in, first out
  - Traverse nodes breadth first
- Implement { Discovered } as Stack
  - First in, last out
  - Traverse nodes depth first

BFS Traversal Algorithm

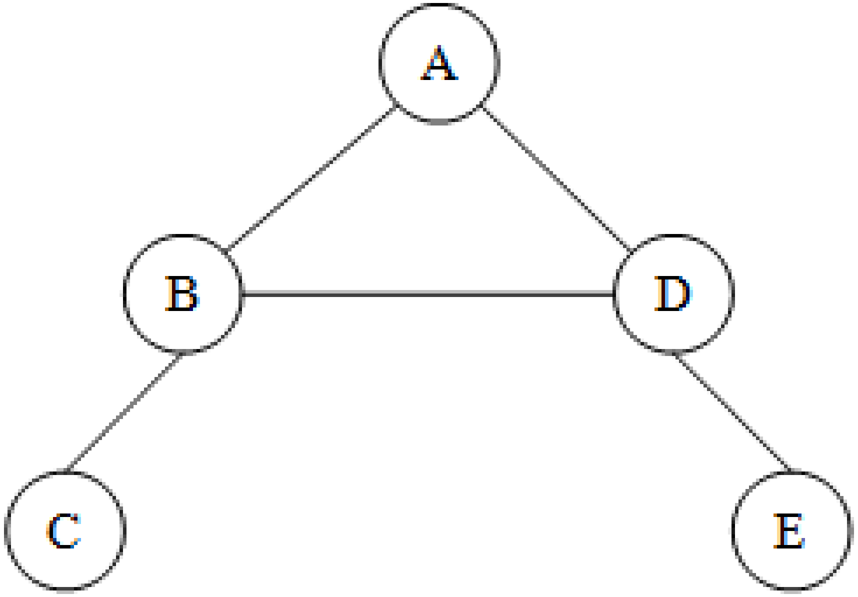
```
for all nodes X
    X.tag = False
put 1st node in Queue
while ( Queue not empty )
    take node X out of Queue
    if (X.tag = False)
        set X.tag = True
        for each successor Y of X
            if (Y.tag = False)
                put Y in Queue
```

DFS Traversal Algorithm

```
for all nodes X
     X.tag = False
put 1st node in Stack
while (Stack not empty )
    pop X off Stack
    if (X.tag = False)
        set X.tag = True
        for each successor Y of X
            if (Y.tag = False)
                push Y onto Stack
```

Example

- **Let's do a BFS/DFS using the following graph (start vertex A)**



Recursive Graph Traversal

- **Can traverse graph using recursive algorithm**
  - **Recursively visit successors**

- **Approach**

Visit (X)

  for each successor Y of X

    Visit (Y)

- **Implicit call stack & backtracking**
  - **Results in depth-first traversal**

Recursive DFS Algorithm

```
Traverse( )
    for all nodes X
        set X.tag = False
    Visit ( 1st node )
Visit ( X )
    set X.tag = True
    for each successor Y of X
        if (Y.tag = False)
            Visit ( Y )
```