

CMSC 132: Object-Oriented Programming II

Algorithm Strategies



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General Concepts

■ Algorithm strategy

- Approach to solving a problem
- May combine several approaches

■ Algorithm structure

- Iterative $\Rightarrow$ execute action in loop
- Recursive $\Rightarrow$ reapply action to subproblem(s)

■ Problem type

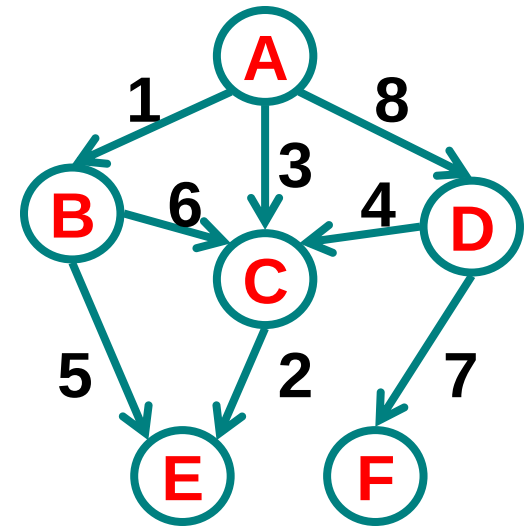
Problem Type

■ Satisfying

- Find any satisfactory solution
- Example → Find path from A to F

■ Optimization

- Find **best** solution (vs. cost metric)
- Example → Find **shortest** path from A to E



Some Algorithm Strategies

- **Recursive algorithms**
- **Backtracking algorithms**
- **Divide and conquer algorithms**
- **Dynamic programming algorithms**
- **Greedy algorithms**
- **Brute force algorithms**
- **Branch and bound algorithms**
- **Heuristic algorithms**

Recursive Algorithm

- Based on reapplying algorithm to subproblem
- Approach
 1. Solves **base case(s)** directly
 2. Recurs with a simpler subproblem
 3. May need to combine solution(s) to subproblems

Backtracking Algorithm

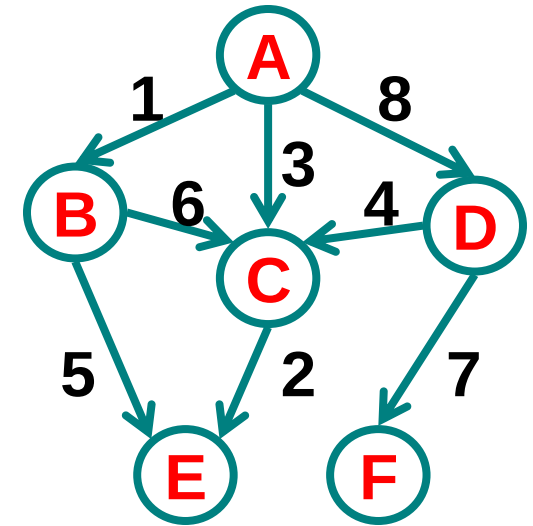
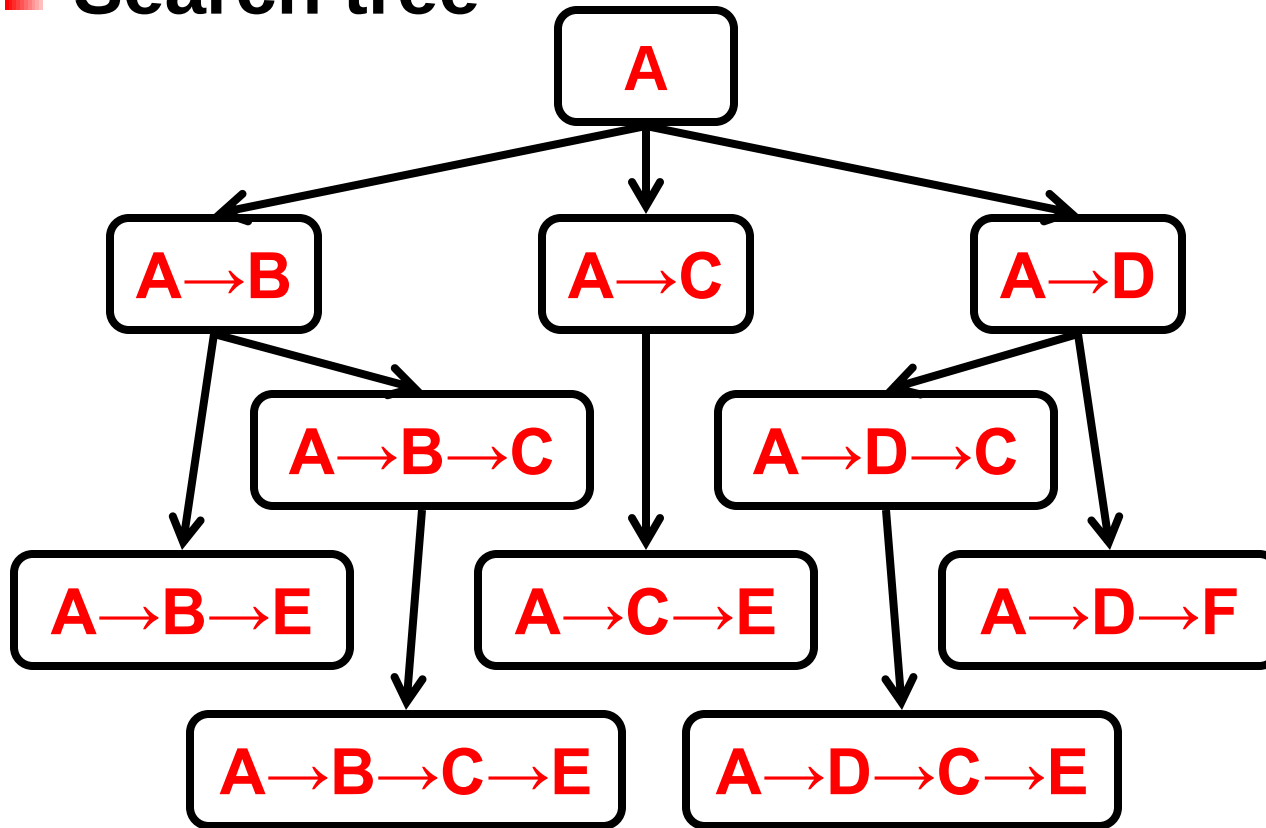
- Based on **depth-first** recursive search
- Approach
 1. Tests whether solution has been found
 2. If found solution, return it
 3. Else for each choice that can be made
 - a) Make that choice
 - b) Recur
 - c) If recursion returns a solution, return it
 4. If no choices remain, return failure
- Tree of alternatives → **search tree**

Backtracking Algorithm – Reachability

- Find path in graph from A to F
 1. Start with currentNode = A
 2. If currentNode has edge to F, return path
 3. Else select neighbor node X for currentNode
 - Recursively find path from X to F
 - If path found, return path
 - Else repeat for different X
 - Return false if no path from any neighbor X

Backtracking Algorithm – Path Finding

■ Search tree



Backtracking Algorithm – Map Coloring

- Color a map using four colors so adjacent regions do not share the same color.
- Coloring map of countries
 - If all countries have been colored return success
 - Else for each color c of four colors and country n
 - If country n is not adjacent to a country that has been colored c
 - Color country n with color c
 - Recursively color country $n+1$
 - If successful, return success
 - Return failure
- Map from wikipedia –
http://upload.wikimedia.org/wikipedia/commons/thumb/a/a5/Map_of_USA_with_state_names.svg/650px-Map_of_USA_with_state_names.svg.png

Divide and Conquer

- **Based on dividing problem into subproblems**
- **Approach**
 1. **Divide problem into smaller subproblems**
 - **Subproblems must be of same type**
 - **Subproblems do not need to overlap**
 2. **Solve each subproblem recursively**
 3. **Combine solutions to solve original problem**
- **Usually contains two or more recursive calls**

Divide and Conquer – Sorting

■ Quicksort

- Partition array into **two parts** around pivot
- Recursively quicksort each part of array
- Concatenate solutions

■ Mergesort

- Partition array into **two parts**
- Recursively mergesort each half
- Merge two sorted arrays into single sorted array

Dynamic Programming Algorithm

- Based on remembering past results
- Approach
 1. Divide problem into smaller subproblems
 - Subproblems must be of same type
 - Subproblems must **overlap**
 2. Solve each subproblem recursively
 - May simply look up solution (if previously solved)
 3. Combine solutions to solve original problem
 4. Store solution to problem
- Generally applied to optimization problems

Fibonacci Algorithm

■ Fibonacci numbers

- $\text{fibonacci}(0) = 1$

- $\text{fibonacci}(1) = 1$

- $\text{fibonacci}(n) = \text{fibonacci}(n-1) + \text{fibonacci}(n-2)$

■ Recursive algorithm to calculate $\text{fibonacci}(n)$

- If n is 0 or 1, return 1

- Else compute $\text{fibonacci}(n-1)$ and $\text{fibonacci}(n-2)$

- Return their sum

■ Simple algorithm $\Rightarrow$ exponential time $O(2^n)$

Dynamic Programming – Fibonacci

- **Dynamic programming version of fibonacci(n)**
 - If n is 0 or 1, return 1
 - Else solve fibonacci(n-1) and fibonacci(n-2)
 - Look up value if previously computed
 - Else recursively compute
 - Find their sum and store
 - Return result
- **Dynamic programming algorithm $\Rightarrow O(n)$ time**
 - Since solving fibonacci(n-2) is just looking up value

Dynamic Programming – Shortest Path

Dijkstra's Shortest Path Algorithm

$S = \emptyset$

$C[X] = 0$

$C[Y] = \infty$ for all other nodes

while (not all nodes in S)

 find node K not in S with smallest $C[K]$

 add K to S

 for each node M not in S adjacent to K

$C[M] = \min (C[M] , C[K] + \text{cost of } (K,M))$

Stores results of
smaller subproblems



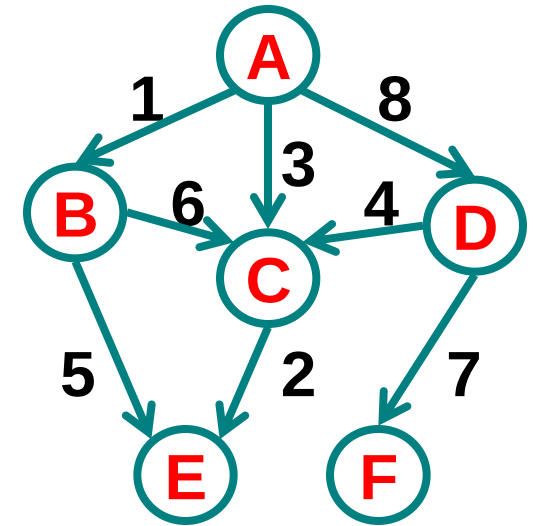
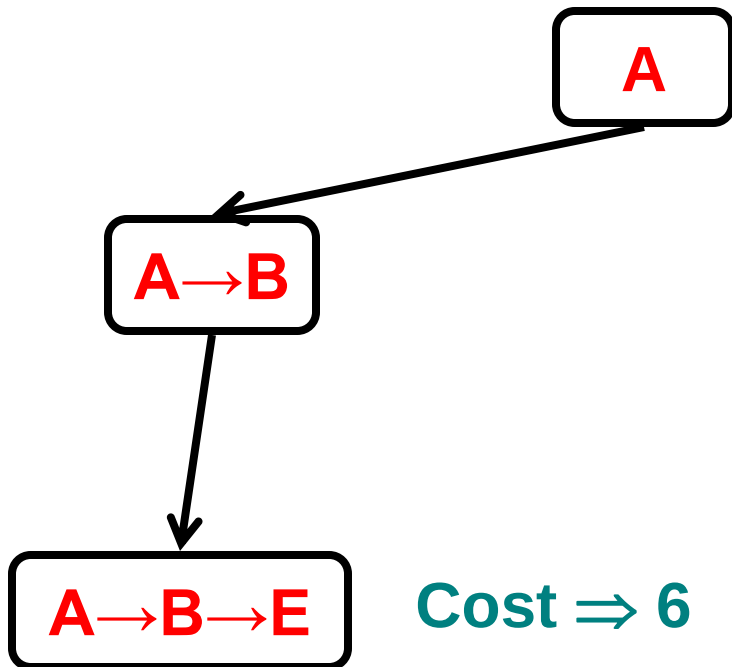
Greedy Algorithm

- Based on trying best current (local) choice
- Approach
 - At each step of algorithm
 - Choose best local solution
- Avoid backtracking, exponential time $O(2^n)$
- Hope local optimum lead to **global** optimum
- Example: Coin System
 - Coins – 30 20 15 1
 - Find minimum number of coins for 40
 - Greedy Algorithm fails

Greedy Algorithm – Shortest Path (A to E)

■ Example

■ Choose lowest-cost neighbor



■ Does not yield overall (global) shortest path

Greedy Algorithm – MST

Kruskal's Minimal Spanning Tree Algorithm

sort edges by weight (from least to most)

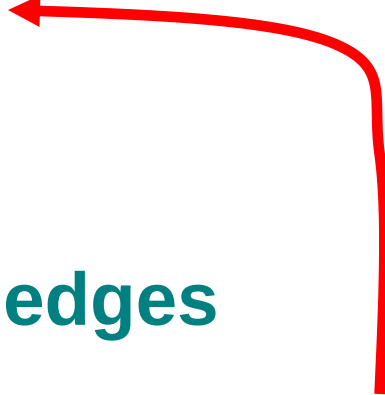
tree = $\emptyset$

for each edge (X,Y) in order

if it does not create a cycle

add (X,Y) to tree

stop when tree has N-1 edges



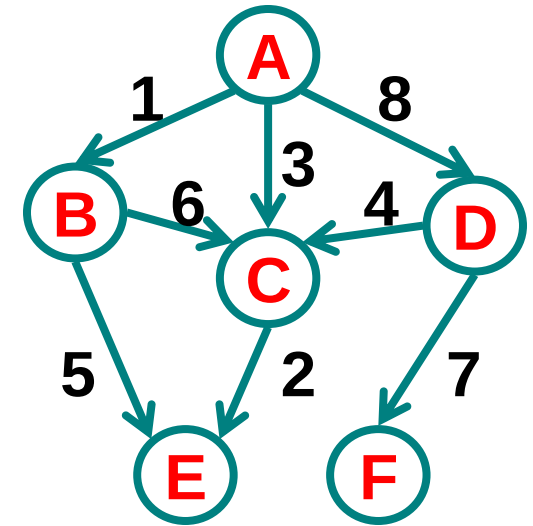
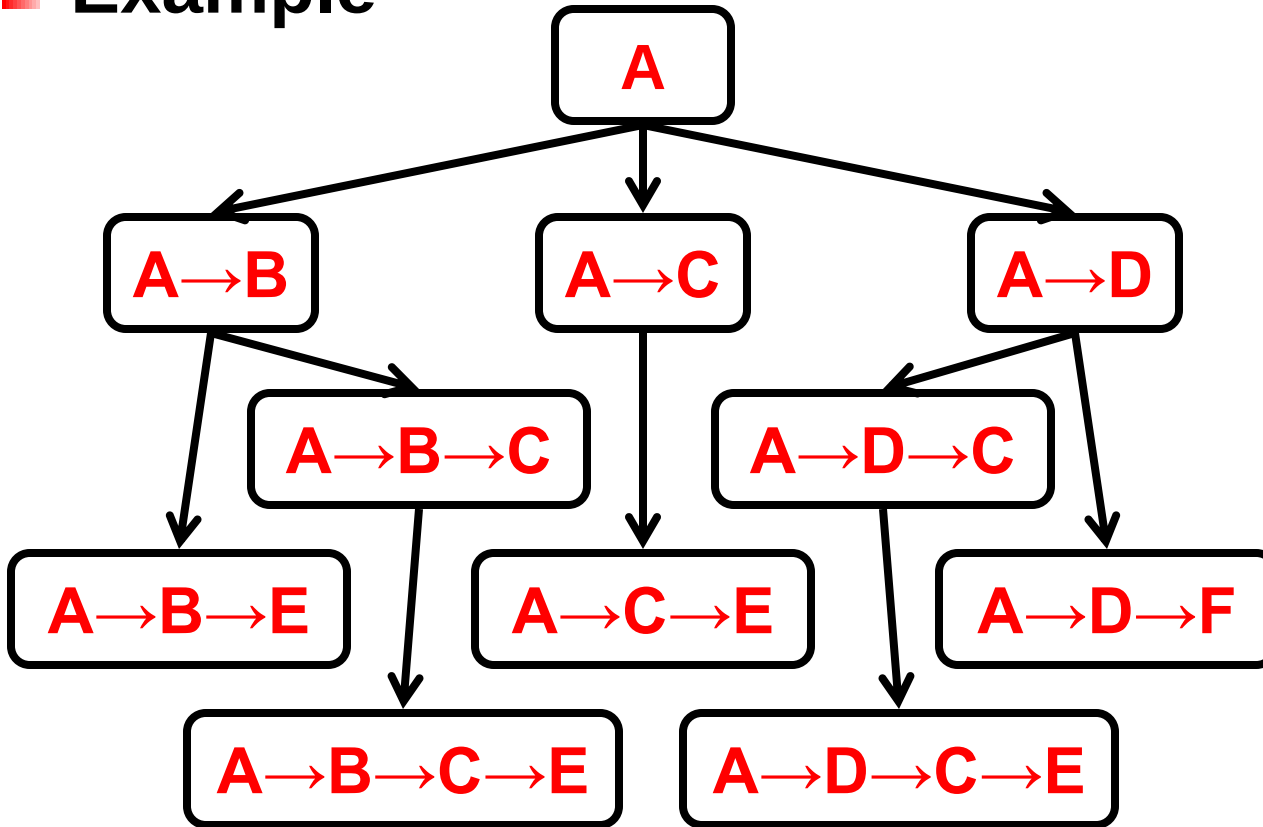
Picks best
local solution
at each step

Brute Force Algorithm

- Based on trying all possible solutions
- Approach
 - Generate and evaluate possible solutions until
 - Satisfactory solution is found
 - Best solution is found (if can be determined)
 - All possible solutions found
 - Return best solution
 - Return failure if no satisfactory solution
- Generally most expensive approach

Brute Force Algorithm – Shortest Path

■ Example



■ Examines all paths in graph

Brute Force Algorithm – TSP

■ Traveling Salesman Problem (TSP)

- Given weighted undirected graph (map of cities)
- Find lowest cost path visiting all nodes (cities) once
- No known polynomial-time general solution

■ Brute force approach

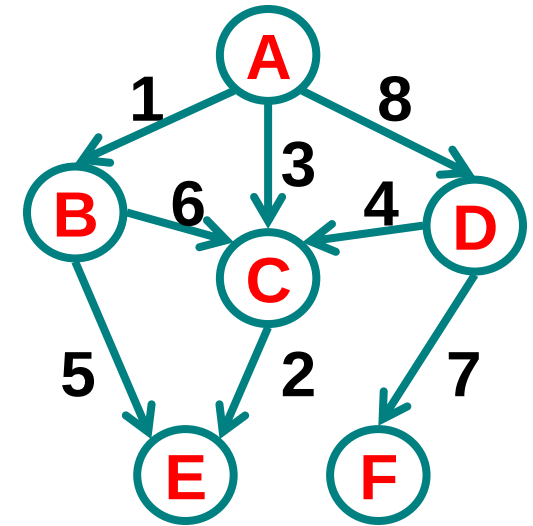
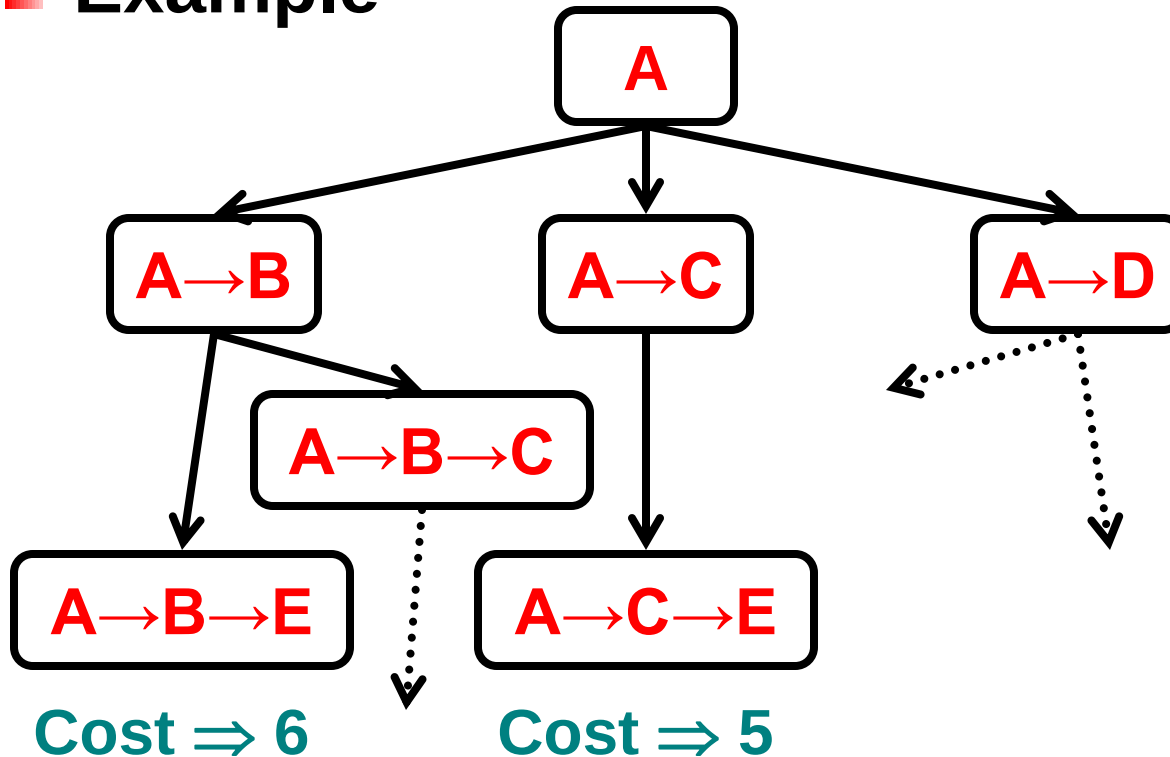
- Find all possible paths using recursive backtracking
- Calculate cost of each path
- Return lowest cost path
- Complexity $O(n!)$

Branch and Bound Algorithm

- Based on limiting search using current solution
- Approach
 - Track best current solution found
 - Eliminate (**prune**) partial solutions that can not improve upon best current solution
- Reduces amount of backtracking
 - Not guaranteed to avoid exponential time $O(2^n)$

Branch & Bound Alg. – Shortest Path

■ Example



■ Pruned paths beginning with A→B→C & A→D

Branch and Bound – TSP

- **Branch and bound algorithm for TSP**
 - Find possible paths using recursive backtracking
 - Track cost of best current solution found
 - Stop searching path **if cost > best current solution**
 - Return lowest cost path
- **If good solution found early, can reduce search**
- **May still require exponential time $O(2^n)$**

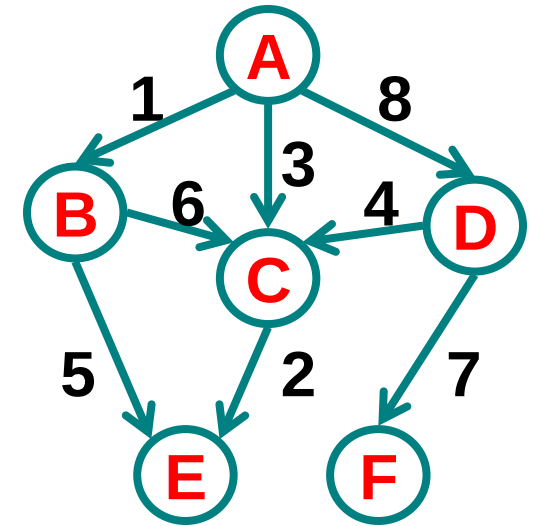
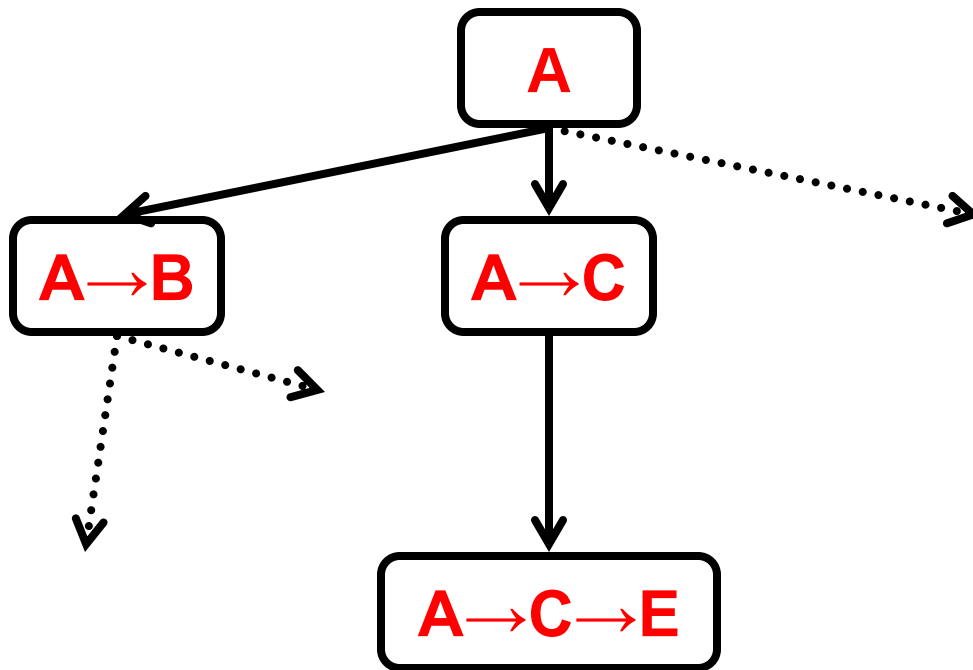
Heuristic Algorithm

- Based on trying to guide search for solution
- Heuristic $\Rightarrow$ “rule of thumb”
- Approach
 - Generate and evaluate possible solutions
 - Using “rule of thumb”
 - Stop if satisfactory solution is found
- Can reduce complexity
- Not guaranteed to yield best solution

Heuristic – Shortest Path

■ Example

■ Try only edges with cost < 5



■ Worked...in this case

Heuristic Algorithm – TSP

- **Heuristic algorithm for TSP**
 - Find possible paths using recursive backtracking
 - Search 2 lowest cost edges at each node first
 - Calculate cost of each path
 - Return lowest cost path from first 100 solutions
- **Not guaranteed to find best solution**
- **Heuristics used frequently in real applications**

Summary

- **Wide range of strategies**
- **Choice depends on**
 - **Properties of problem**
 - **Expected problem size**
 - **Available resources**