

CMSC 132: Object-Oriented Programming II

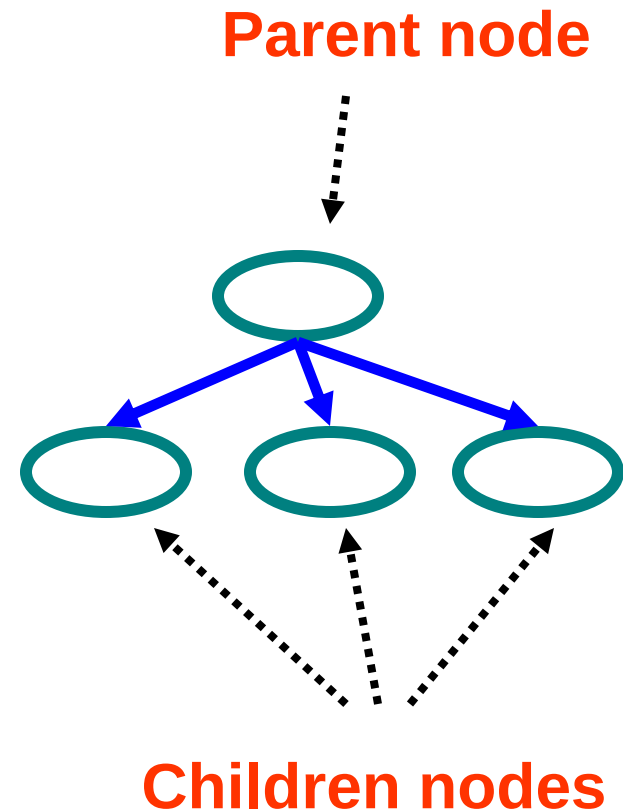


Trees & Binary Search Trees

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Trees

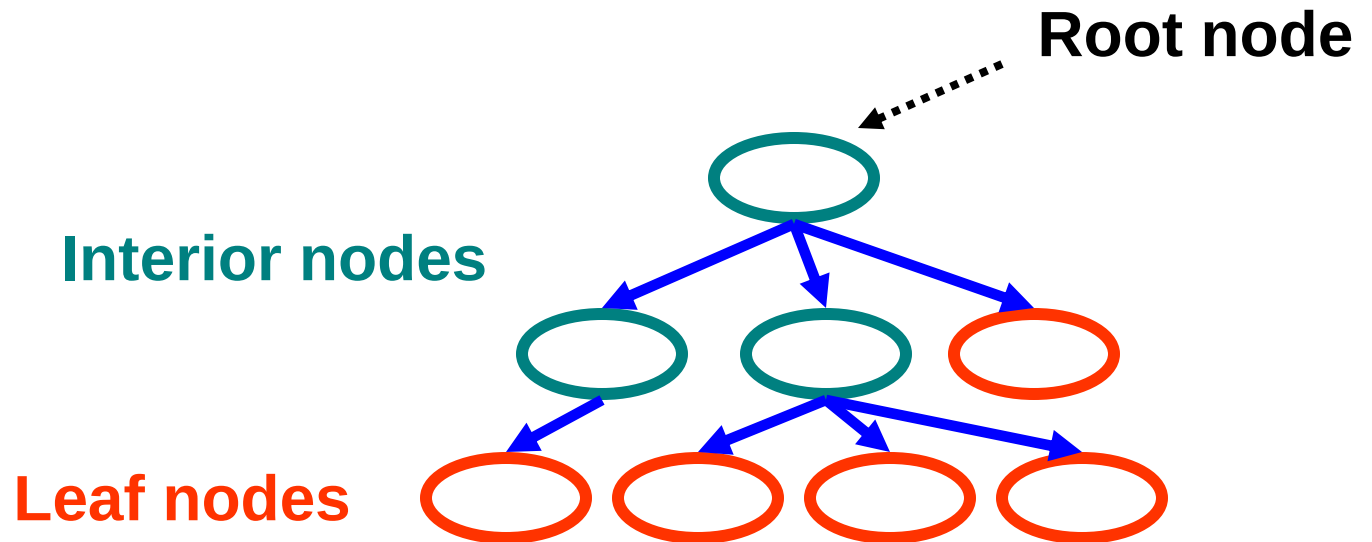
- Trees are hierarchical data structures
 - One-to-many relationship between elements
- Tree node / element
 - Contains data
 - Referred to by only 1 (**parent**) node
 - Contains links to any number of (**children**) nodes



Trees

■ Terminology

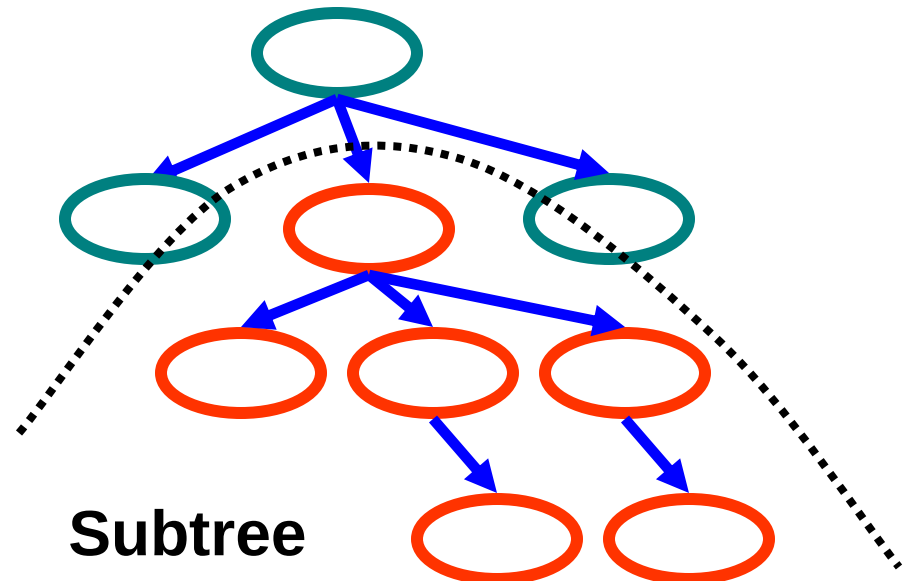
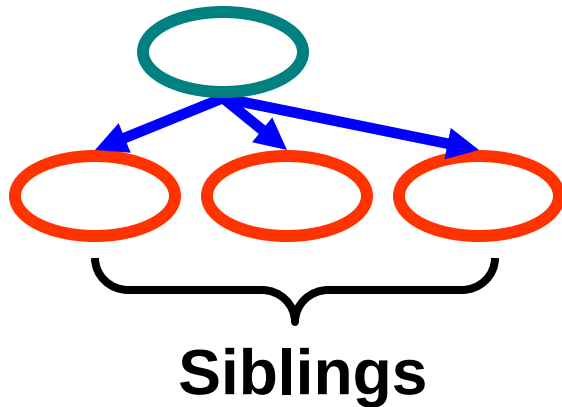
- Root $\Rightarrow$ node with no parent
- Leaf $\Rightarrow$ all nodes with no children
- Interior $\Rightarrow$ all nodes with children



Trees

■ Terminology

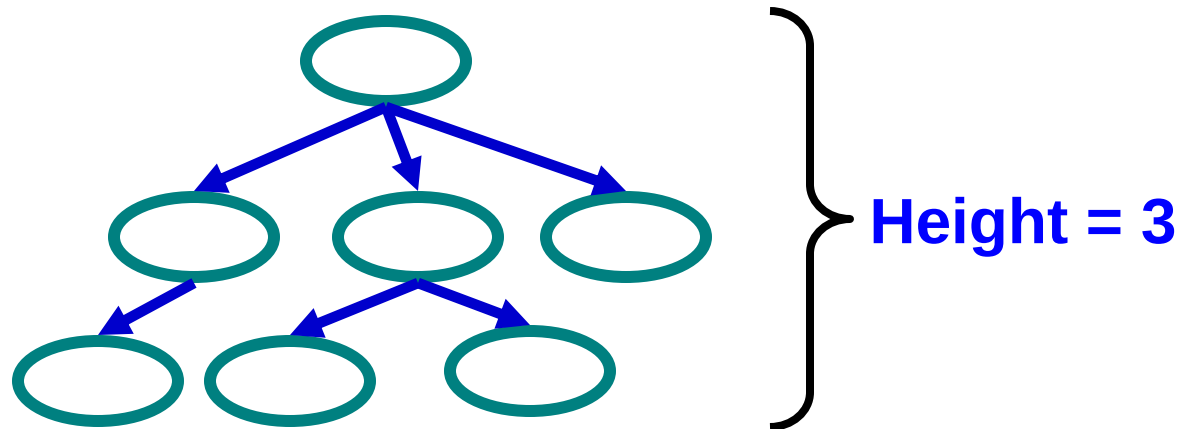
- Sibling $\Rightarrow$ node with same parent
- Descendent $\Rightarrow$ children nodes & their descendants
- Subtree $\Rightarrow$ portion of tree that is a tree by itself
 $\Rightarrow$ a node and its descendants



Trees

■ Terminology

- Level $\Rightarrow$ is a measure of a node's distance from root
- Definition of level
 - If node is the root of the tree, its level is 1
 - Else, the node's level is $1 + \text{its parent's level}$
- Height (depth) $\Rightarrow$ max level of any node in tree

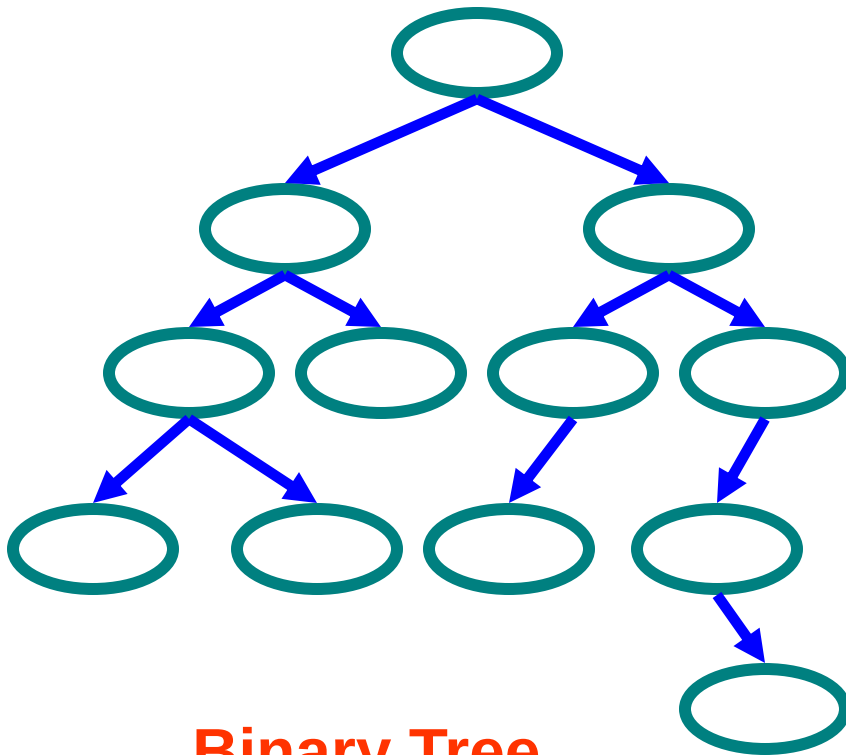


Binary Trees

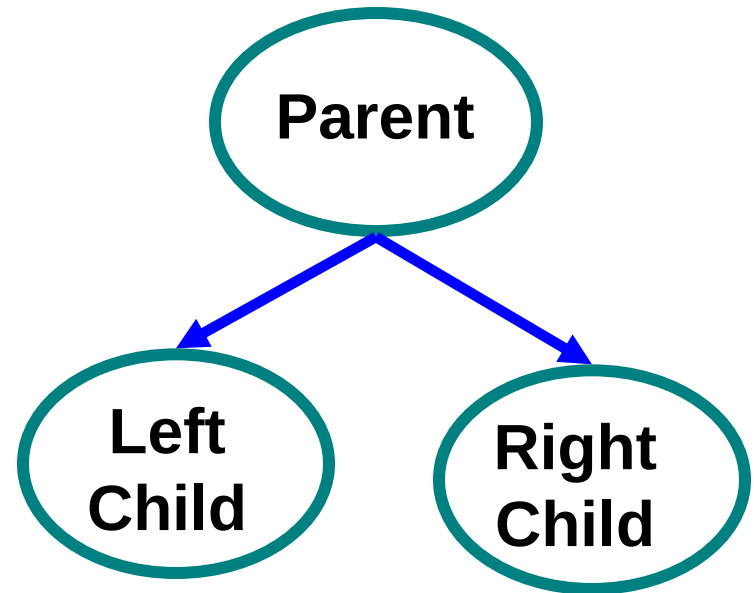
■ Binary tree

■ Tree with 0–2 children per node

■ Left & right child / subtree



Binary Tree



Tree Traversal

- Often we want to

1. Find all nodes in tree
2. Determine their relationship

- Can do this by

1. Walking through the tree in a prescribed order
2. Visiting the nodes as they are encountered

- Process is called **tree traversal**

Tree Traversal

■ Goal

- Visit every node in binary tree

■ Approaches

■ Depth first

- Preorder $\Rightarrow$ parent before children
- Inorder $\Rightarrow$ left child, parent, right child
- Postorder $\Rightarrow$ children before parent

- Breadth first $\Rightarrow$ closer nodes first

Tree Traversal Methods

■ Pre-order

1. Visit **node** // first
2. Recursively visit left subtree
3. Recursively visit right subtree

■ In-order

1. Recursively visit left subtree
2. Visit **node** // second
3. Recursively right subtree

■ Post-order

1. Recursively visit left subtree
2. Recursively visit right subtree
3. Visit **node** // last

Tree Traversal Methods

■ Breadth-first

```
BFS(Node n) {  
    Queue Q = new Queue();  
    Q.enqueue(n);           // insert node into Q  
    while ( !Q.empty()) {  
        n = Q.dequeue();    // remove next node  
        if ( !n.isEmpty()) {  
            visit(n);       // visit node  
            Q.enqueue(n.Left()); // insert left subtree in Q  
            Q.enqueue(n.Right()); // insert right subtree in Q  
        }  
    }  
}
```

Tree Traversal Examples

■ Pre-order (prefix)

■ $+ \times 2 3 / 8 4$

■ In-order (infix)

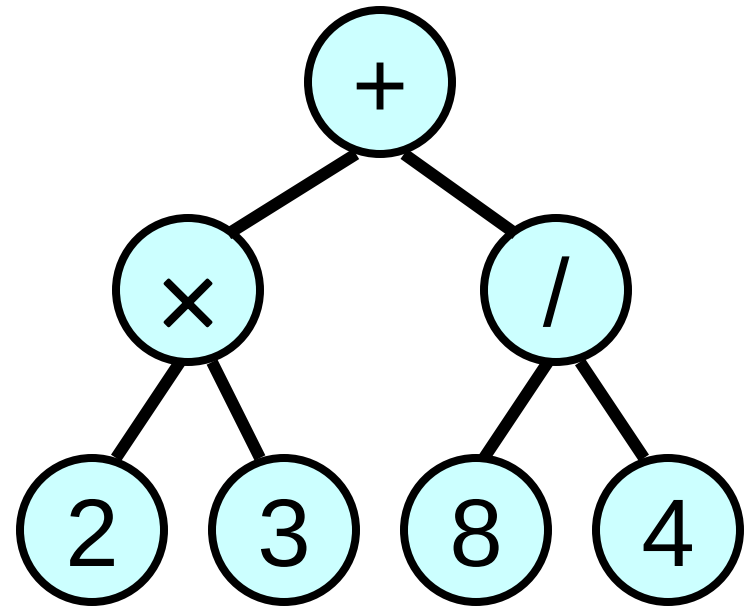
■ $2 \times 3 + 8 / 4$

■ Post-order (postfix)

■ $2 3 \times 8 4 / +$

■ Breadth-first

■ $+ \times / 2 3 8 4$



Expression tree

Binary Tree Implementation

- Using a class to represent a Node

```
Class Node {  
    KeyType key;  
    Node left, right;    // null if empty  
}
```

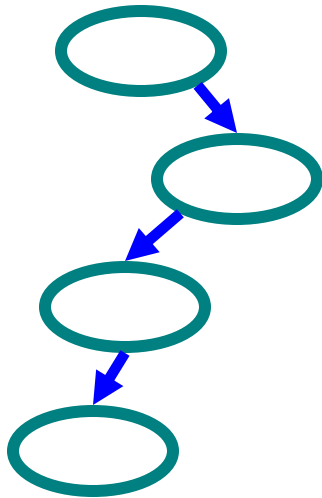
Node root = null; // Empty Tree

- Using a Polymorphic Binary Tree
 - We will talk about this implementation later on

Types of Binary Trees

■ Degenerate

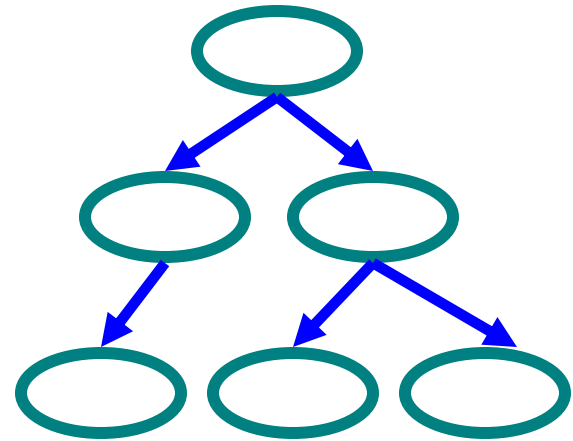
- Mostly 1 child / node
- Height = $O(n)$
- Similar to linear list



**Degenerate
binary tree**

■ Balanced

- Mostly 2 child / node
- Height = $O(\log(n))$
- $2^{\text{Height}} - 1 = n$ (# of nodes)
- Useful for searches



**Balanced
binary tree**

Binary Search Trees

- Key property

- Value at node

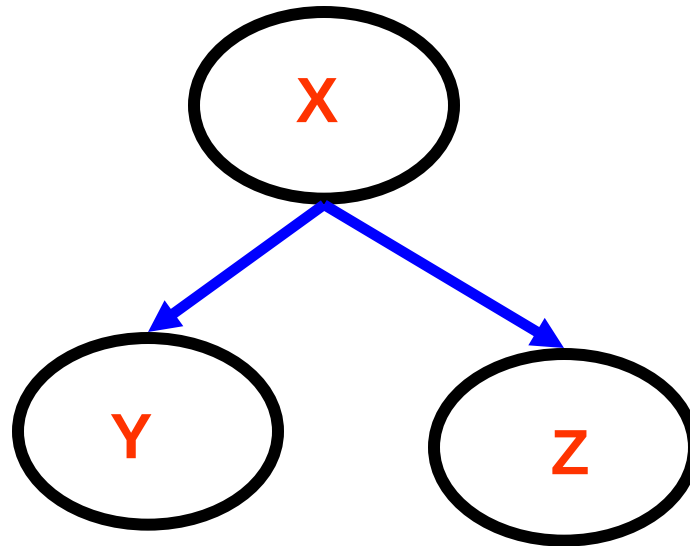
- Smaller values in left subtree

- Larger values in right subtree

- Example

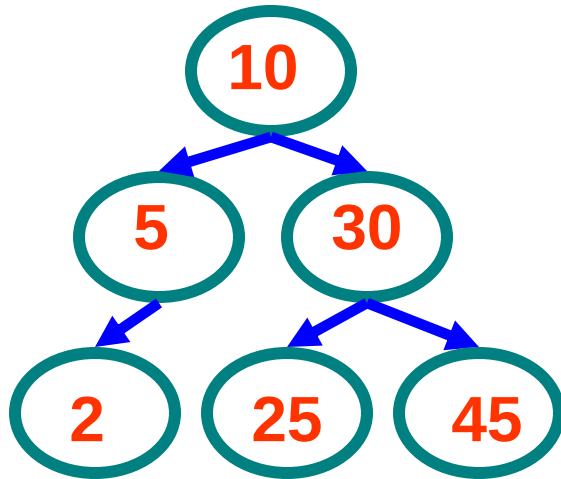
- $X > Y$

- $X < Z$

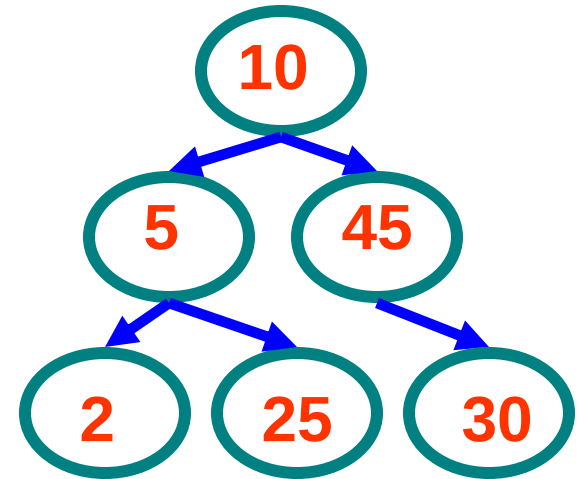
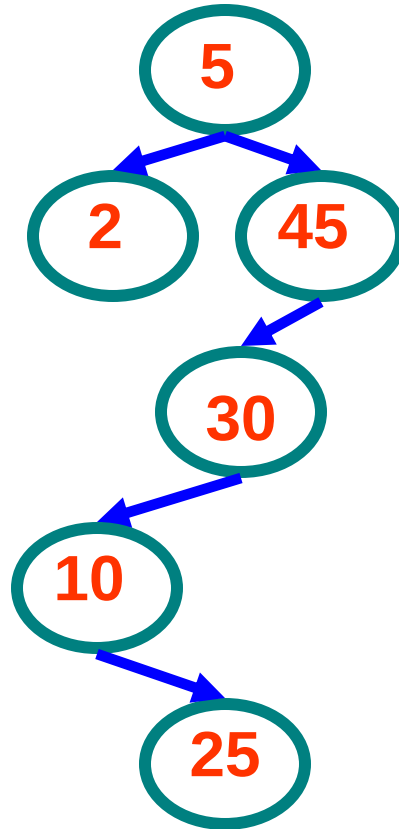


Binary Search Trees

■ Examples



Binary
search trees



Non-binary
search tree

Tree Traversal Examples

■ Pre-order

■ 44, 17, 32, 78,
50, 48, 62, 88

■ In-order

■ 17, 32, 44, 48,
50, 62, 78, 88

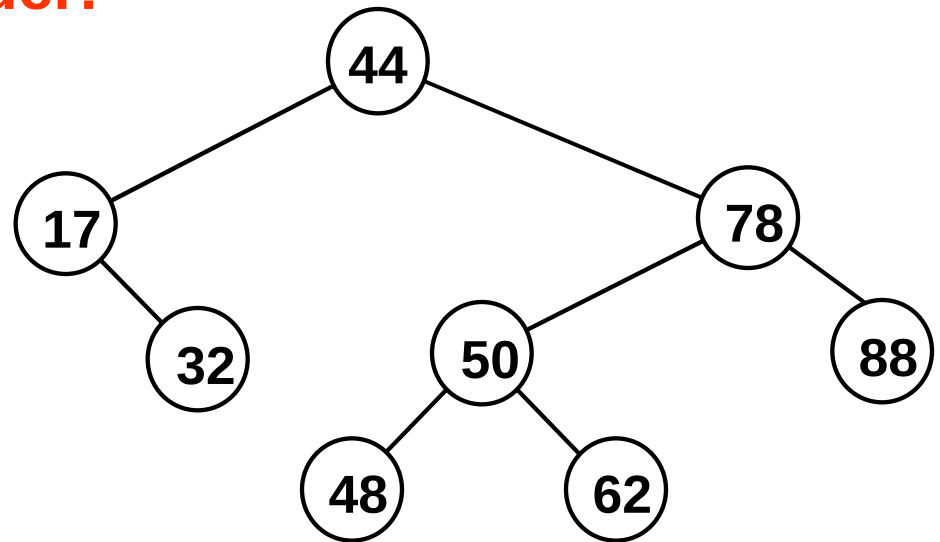
■ Post-order

■ 32, 17, 48, 62,
50, 88, 78, 44

■ Breadth-first

■ 44, 17, 78, 32,
50, 88, 48, 62

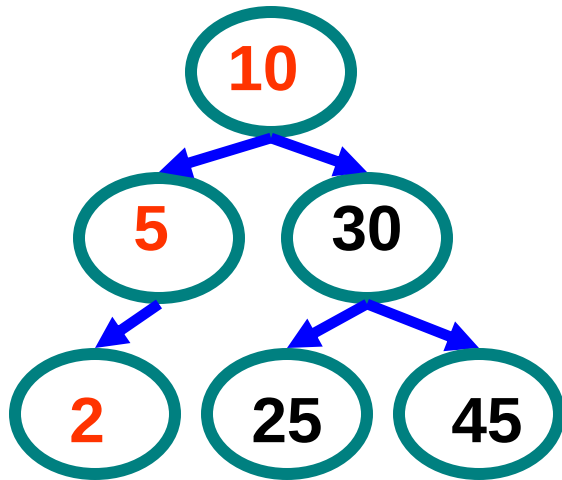
Sorted
order!



Binary search tree

Example Binary Searches

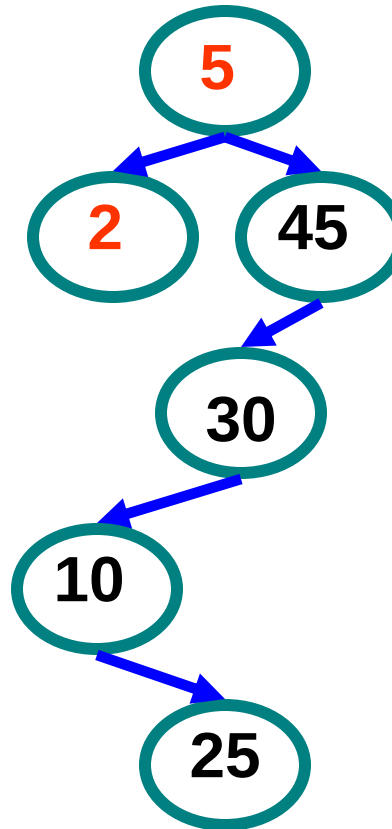
■ Find (2)



$2 < 10$, left

$2 < 5$, left

$2 = 2$, found

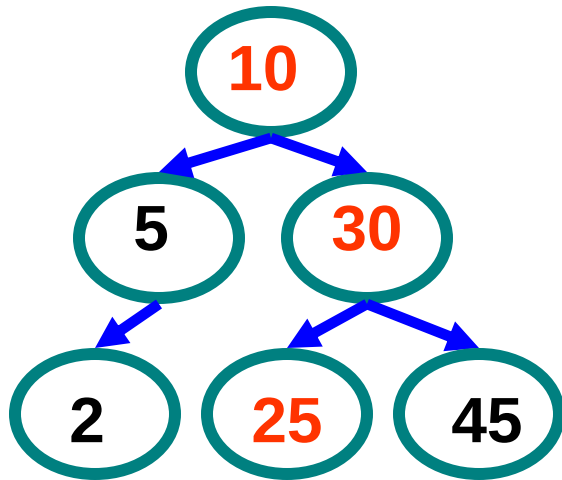


$2 < 5$, left

$2 = 2$, found

Example Binary Searches

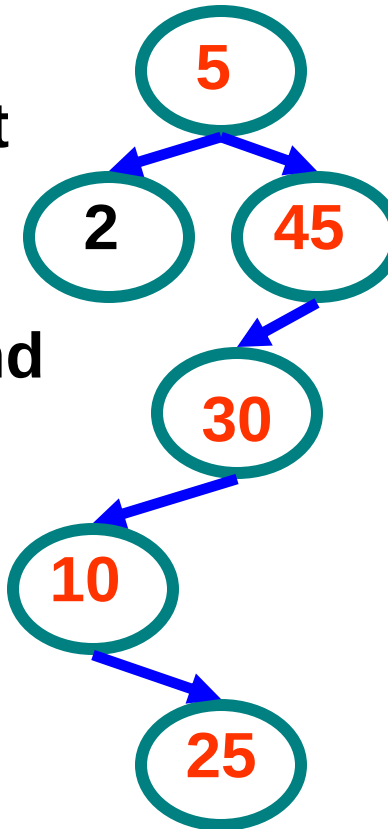
■ Find (25)



$25 > 10$, right

$25 < 30$, left

$25 = 25$, found



$25 > 5$, right

$25 < 45$, left

$25 < 30$, left

$25 > 10$, right

$25 = 25$, found

Binary Search Properties

■ Time of search

- Proportional to height of tree

- Balanced binary tree

 - $O(\log(n))$ time

- Degenerate tree

 - $O(n)$ time

 - Like searching linked list / unsorted array

■ Requires

- Ability to compare key values

Binary Search Tree Construction

- **How to build & maintain binary trees?**
 - **Insertion**
 - **Deletion**
- **Maintain key property (invariant)**
 - **Smaller values in left subtree**
 - **Larger values in right subtree**

Binary Search Tree – Insertion

■ Algorithm

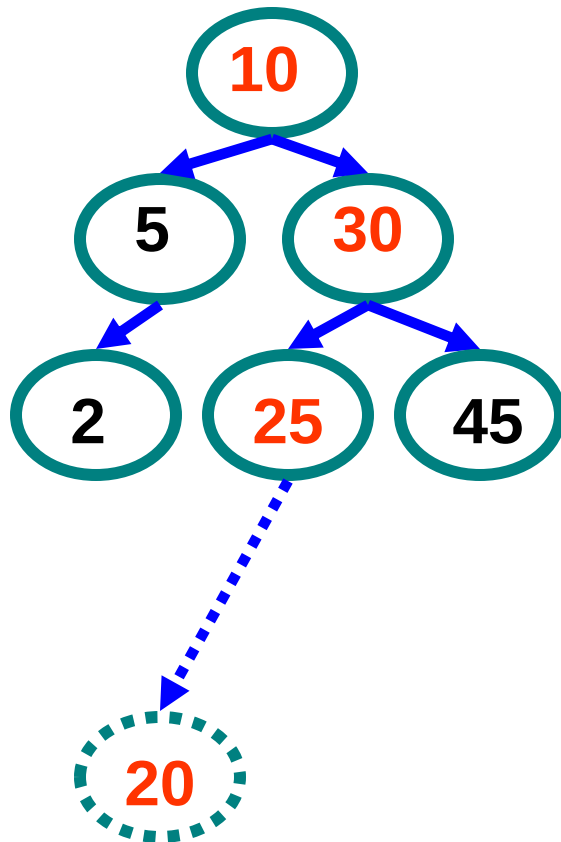
1. Perform search for value X
2. Search will end at node Y (if X not in tree)
3. If $X < Y$, insert new leaf X as new left subtree for Y
4. If $X > Y$, insert new leaf X as new right subtree for Y

■ Observations

- $O(\log(n))$ operation for balanced tree
- Insertions may unbalance tree

Example Insertion

■ Insert (20)



$20 > 10$, right

$20 < 30$, left

$20 < 25$, left

Insert 20 on left

Binary Search Tree – Deletion

■ Algorithm

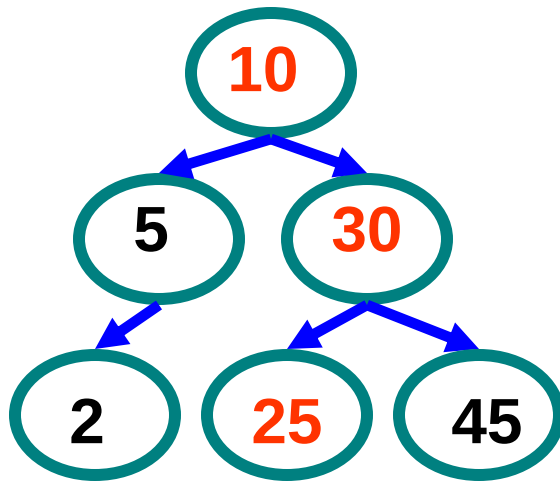
1. Perform search for value X
2. If X is a leaf, delete X
3. Else // must delete internal node
 - a) Replace with largest value Y on left subtree
OR smallest value Z on right subtree
 - b) Delete replacement value (Y or Z) from subtree

■ Observation

- $O(\log(n))$ operation for balanced tree
- Deletions may unbalance tree

Example Deletion (Leaf)

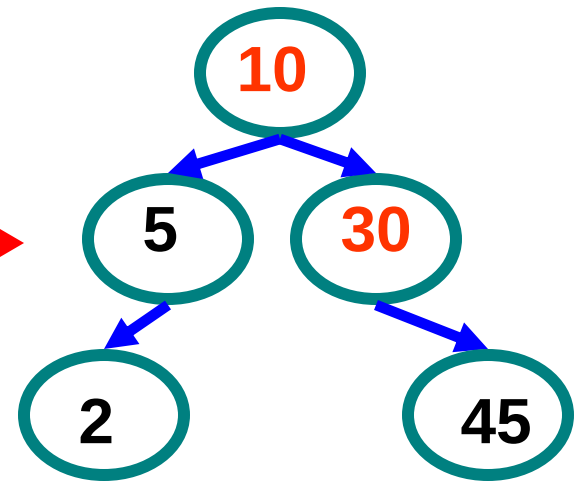
■ Delete (25)



$25 > 10$, right

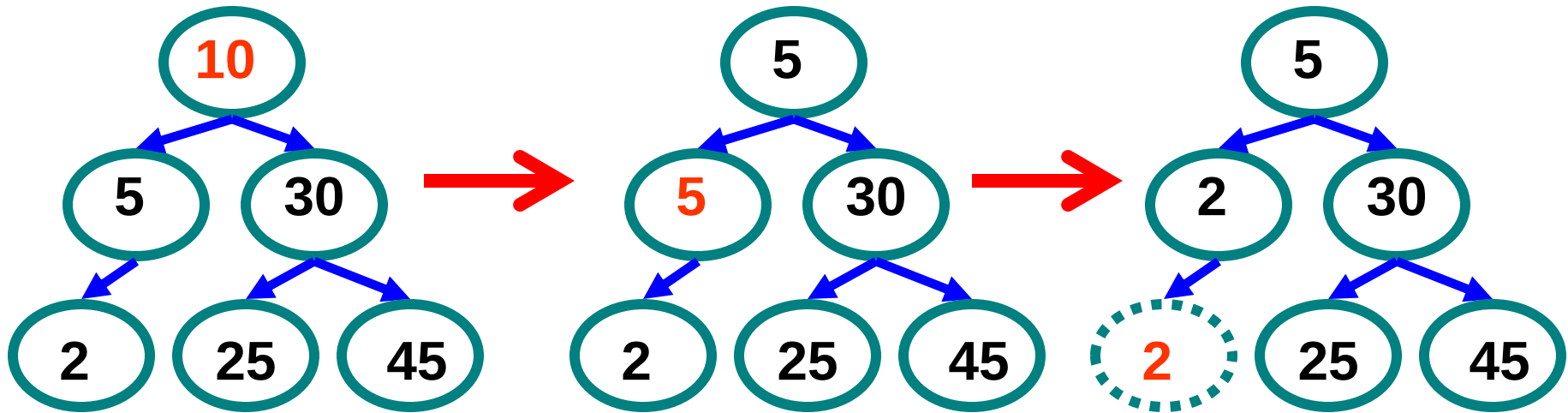
$25 < 30$, left

$25 = 25$, delete



Example Deletion (Internal Node)

■ Delete (10)



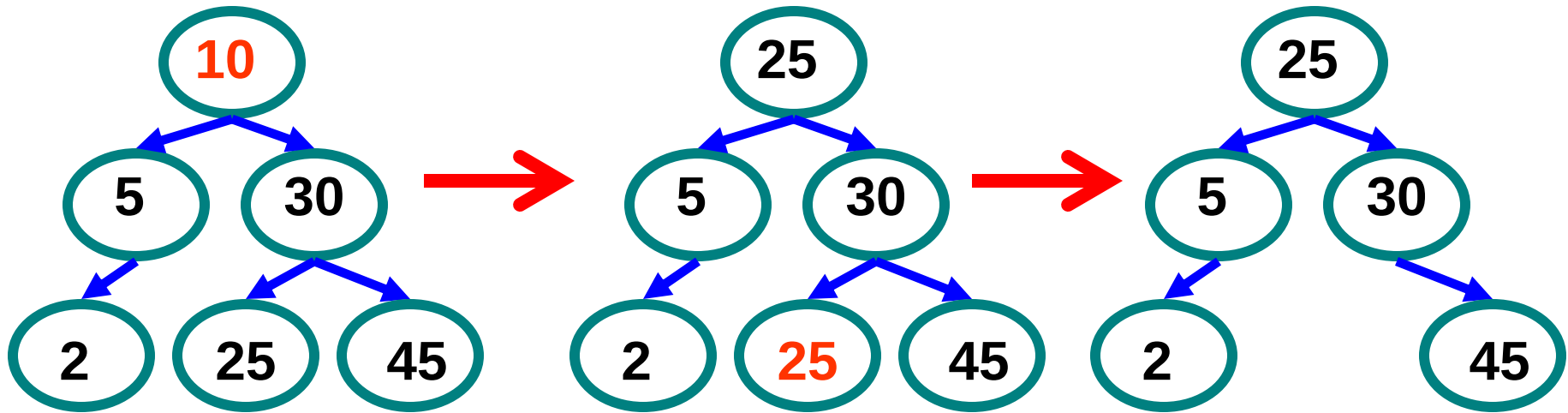
Replacing 10
with **largest**
value in left
subtree

Replacing 5
with **largest**
value in left
subtree

Deleting leaf

Example Deletion (Internal Node)

■ Delete (10)



Replacing 10
with **smallest**
value in right
subtree

Deleting leaf

Resulting tree

Building Maps w/ Search Trees

- Binary Search trees often used to implement **maps**
 - Each non-empty node contains
 - Key
 - Value
 - Left and right child
- Need to be able to compare keys
 - Generic type `<K extends Comparable<K>>`
 - Denotes any type K that can be compared to K's

BST (Binary Search Tree) Implementation

- Implementing Tree using traditional approach
- Based on the BST definition below let's see how to implement typical BST Operations (constructor, add, print, find, isEmpty, isFull, size, height, etc.)

```
public class BinarySearchTree <K extends Comparable<K>, V> {  
    private class Node {  
        private K key;  
        private V data;  
        private Node left, right;  
        public Node(K key, V data) {  
            this.key = key;  
            this.data = data;  
        }  
    }  
    private Node root;  
}
```

- See code distribution [BinaryTreeCode.zip](#)

BST Testing

- **How can we test the correctness of BST Methods?**
- **What is the best approach?**

Polymorphic Binary Search Trees

- Second approach to implement BST
- What do we mean by polymorphic?
- Implement two subtypes of Tree
 1. EmptyTree
 2. NonEmptyTree
- Use EmptyTree to represent the empty tree
 - Rather than null
- Invoke methods on tree nodes
 - Without checking for null (IMPORTANT!)

Polymorphic Binary Tree Implementation

Interface Tree {

Tree insert (Value data1) { ... }

}

Class EmptyTree implements Tree {

Tree insert (Value data1) { ... }

}

Class NonEmptyTree implements Tree {

Value data;

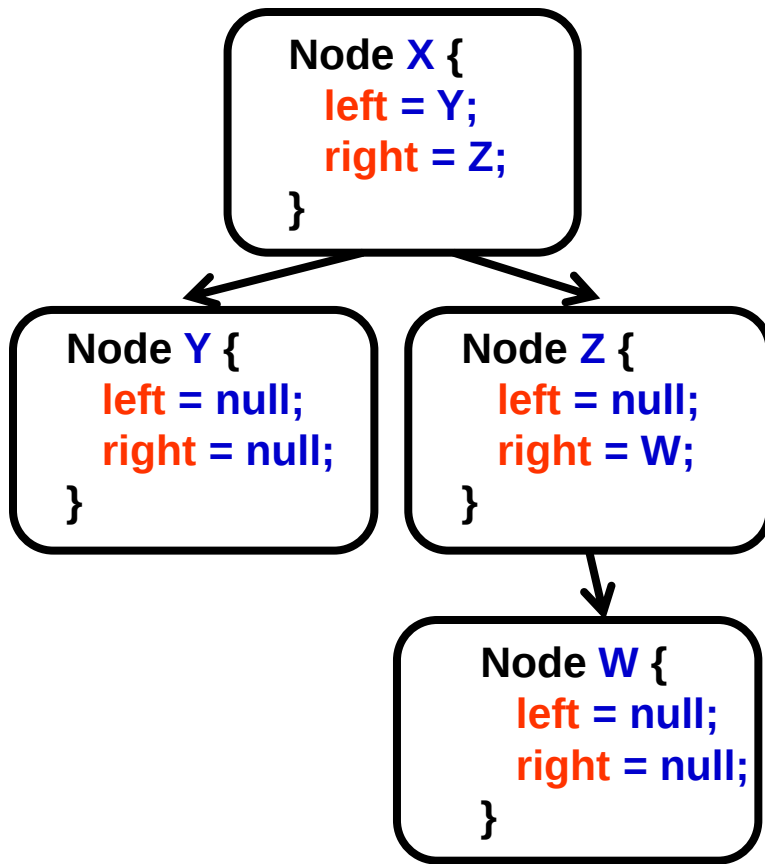
Tree left, right; // Either Empty or NonEmpty

Tree insert (Value data1) { ... }

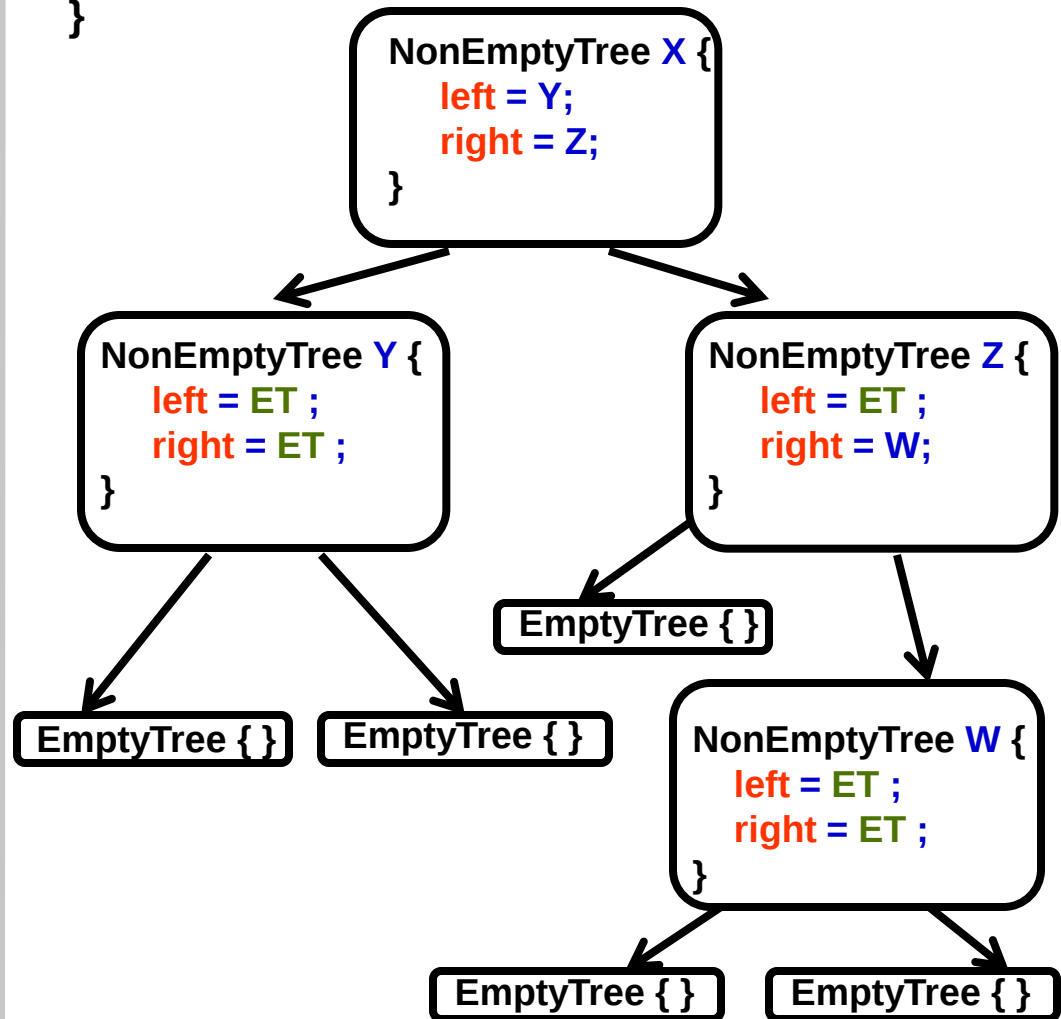
}

Standard vs. Polymorphic Binary Tree

```
Class Node {  
    Node left, right;  
}
```



```
Class EmptyTree {}  
Class NonEmptyTree {  
    Tree left, right;  
}
```



Singleton Design Pattern

■ Definition

- One instance of a class or value accessible globally

■ Where to use & benefits

- Ensure unique instance by defining class final
- Access to the instance only via methods provided

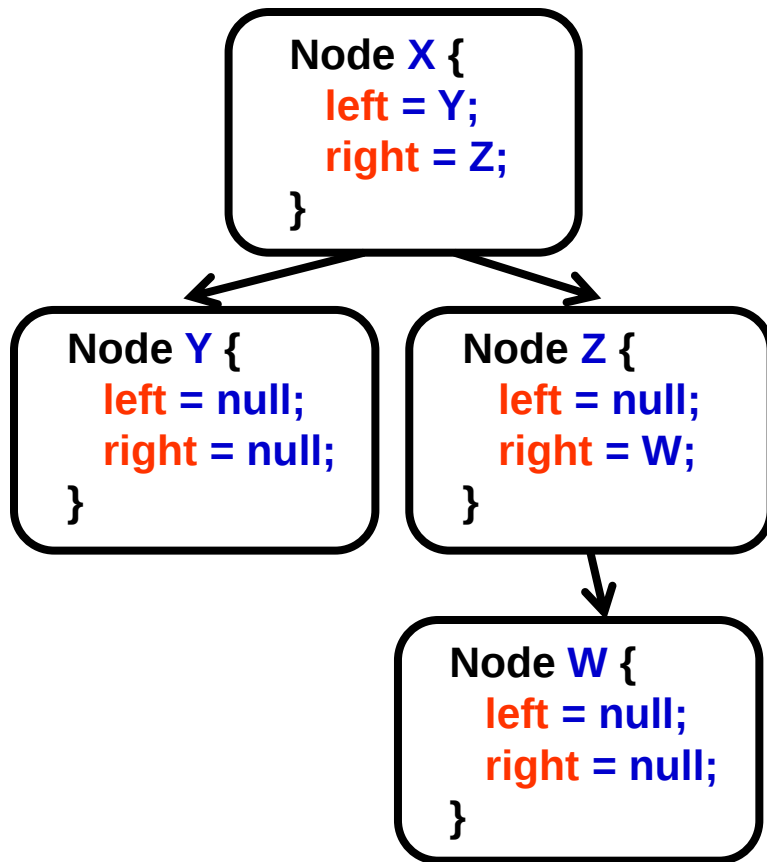
■ EmptyTree class will be a singleton class

Singleton Example

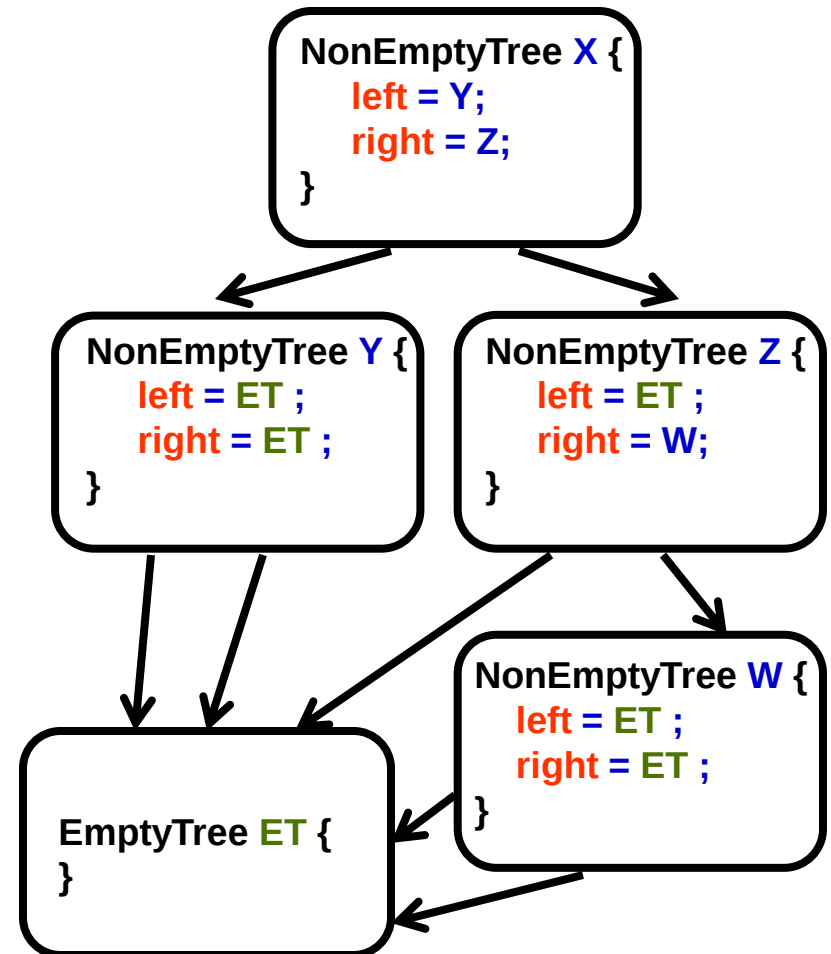
```
public final class MySingleton {  
    // declare the unique instance of the class  
    private static MySingleton uniq = new MySingleton();  
    // private constructor only accessed from this class  
    private MySingleton() { ... }  
    // return reference to unique instance of class  
    public static MySingleton getInstance() {  
        return uniq;  
    }  
}
```

Using Singleton EmptyTree

```
Class Node {  
    Node left, right;  
}
```



```
Class EmptyTree {}  
Class NonEmptyTree {  
    Tree left, right;  
}
```



Polymorphic List Implementation

- **Let's see a polymorphic list implementation**
- **See code distribution `PolymorphicListCode.zip`**