

CMSC330 Fall 2009 Example Quiz #3

Name _____

Discussion Time (circle one): 10am 11am 12pm 1pm 2pm 3pm

Instructions

- You have 25 minutes for this quiz.
- For partial credit, show all of your work and clearly indicate your answers.
- Write neatly. Credit cannot be given for illegible answers.

1. (9 pts) Lambda calculus

(3 pts) Find all free (unbound) variables in the following λ -expression

a. $(\lambda a. c \ b) \ \lambda b. a$

(3 pts each) Evaluate the following λ -expressions as much as possible

b. $(\lambda x. \lambda y. y \ x) \ a \ b$

c. $(\lambda z. z \ x) \ (\lambda y. y \ x)$

2. (16 pts) Lambda calculus encodings

Prove the following using the appropriate λ -calculus encodings, given:

$$1 = \lambda f. \lambda y. f \ y$$

$$2 = \lambda f. \lambda y. f \ (f \ y)$$

$$3 = \lambda f. \lambda y. f \ (f \ (f \ y))$$

$$4 = \lambda f. \lambda y. f \ (f \ (f \ (f \ y)))$$

$$M * N = \lambda x. (M \ (N \ x))$$

$$Y = \lambda f. (\lambda x. f \ (x \ x)) \ (\lambda x. f \ (x \ x))$$

$$\text{succ} = \lambda z. \lambda f. \lambda y. f \ (z \ f \ y)$$

a. (10 pts) Show that $2 * 2 \Rightarrow^* 4$ // $2 * 2$ reduces to 4 using encodings

b. (6 pts) Show that $(Y \ \text{succ}) \ x \Rightarrow^* \text{succ} \ (Y \ \text{succ}) \ x$ // using encoding for Y
// you do not need to expand succ
// you may assume $((\lambda x. \text{succ} \ (x \ x)) \ (\lambda x. \text{succ} \ (x \ x))) \Rightarrow (Y \ \text{succ})$