CMSC 330 Spring 2011
Practice Problems 3 (SOLUTIONS)

1. OCaml and Functional Programming
   a. Define functional programming
      Programs are expression evaluations
   b. Define imperative programming
      Programs change the value of variables
   c. Define higher-order functions
      Functions can be passed as arguments and returned as results
   d. Describe the relationship between type inference and static types
      Variable has a fixed type that can be inferred by looking at how variable is used in the code
   e. Describe the properties of OCaml lists
      Entity containing 0 or more elements of the same type. Type of list is determined by type of element.
   f. Describe the properties of OCaml tuples
      Entity containing 2 or more elements of possibly different types. Type of tuple is determined by type and number of elements.
   g. Define pattern variables in OCaml
      Variables making up patterns used by "match"
   h. Describe the usage of "_" in OCaml
      Pattern variable that can match anything but does not add binding
   i. Describe polymorphism
      Function that can take different types for same formal parameter
   j. Write a polymorphic OCaml function
      let f x = x   // 'a -> 'a, x can be of any type
   k. Describe variable binding
      A variable (symbol) is associated with a value in an expression (or environment)
   l. Describe scope
      Portion of program where variable binding is visible
   m. Describe lexical scoping
      Variable binding determined by nearest scope in text of program
   n. Describe dynamic scoping
      Variable binding determined by nearest runtime function invocation
   o. Describe environment
      Collection of variable bindings
   p. Describe closure
      Function code + environment pair, may be invoked as function
   q. Describe currying
      Functions consume one argument at a time, returning closures until all arguments are consumed

2. OCaml Types & Type Inference

Give the type of the following OCaml expressions:

```
a.  []                                            // 'a list
b.  1::[]                                         // int list
c.  1::2::[]                                      // int list
d.  [1;2;3]                                       // int list
e.  [[1];[1]]                                     // int list list
f.  (1)                                           // int
g.  (1,"bar")                                     // int * string
h.  ([1,2], ["foo","bar"])            // (int * int) list * (string * string) list
i.  [(1,2,"foo");(3,4,"bar")]                     // (int * int * string) list
j.  let f x = 1                                   // 'a -> int
k.  let f (x) = x *. 3.14                         // float -> float
l.  let f (x,y) = x                               // 'a * 'b -> 'a
m.  let f (x,y) = x+y                             // int * int -> int
n.  let f (x,y) = (x,y)                           // 'a * 'b -> 'a * 'b
o.  let f (x,y) = [x,y]                           // 'a * 'b -> ('a * 'b) list
p.  let f x y = 1                                 // 'a -> 'b -> int
q.  let f x y = x*y                               // int -> int -> int
r.  let f x y = x::y                              // 'a -> 'a list -> 'a list
s.  let f x = match x with [] -> 1                // 'a list -> int
t.  let f x = match x with (y,z) -> y+z           // int * int -> int
u.  let f (x::_) -> x                             // 'a list -> 'a
v.  let f (_::y) = y                              // 'a list -> 'a list
w.  let f (x::y::_) = x+y                         // int list -> int
x.  let f = fun x -> x + 1                        // int -> int
y.  let rec x = fun y -> x y                      // 'a -> 'b
z.  let rec f x = if (x = 0) then 1 else 1+f (x-1)   // int -> int
aa. let f x y z = x+y+z in f 1 2 3                // int
bb. let f x y z = x+y+z in f 1 2                  // int -> int
cc. let f x y z = x+y+z in f                      // int -> int -> int -> int
dd. let rec f x = match x with                    // 'a list -> int
        [] -> 0
        | (_::t)  -> 1 + f t
ee. let rec f x = match x with                    // int list -> int
        [] -> 0
        | (h::t)  -> h + f t
ff. let rec f = function                          // int list -> int
        [] -> 0
        | (h::t)  -> h + (2*(f t))
gg. let rec func (f, l1, l2) = match l1 with    // ('a -> 'b) * 'a list * 'a list -> 'b list
        [] -> []
        | (h1::t1) -> match l2 with
            [] -> [f h1]
            |(h2::t2) -> [f h1; f h2]
```

3. OCaml Types & Type Inference

   Write an OCaml expression with the following types:

   a. int list // **[1]**
   b. int * int // **(1,1)**
   c. int -> int // **let f x = x+1**
   d. int * int -> int // **let f (x,y) = x+y**
   e. int -> int -> int // **let f x y = x+y**
   f. int -> int list -> int list // **let f x y = (x+1)::y**
   g. int list list -> int list // **let f (x::_) = 1::x**
   h. ‘a -> ‘a // **let f x = x**
   i. ‘a * ‘b -> ‘a // **let f (x,y) = x**
   j. ‘a -> ‘b -> ‘a // **let f x y = x**
   k. ‘a -> ‘b -> ‘b // **let f x y = y**
   l. ‘a list * ‘b list -> (‘a * ‘b) list // **let f (x::_,y::_) = [(x,y)]**
   m. int -> (int -> int) // **let f x y = x+y**
   n. (int -> int) -> int // **let f x = 1+(x 1)**
   o. (int -> int) -> (int -> int) -> int // **let f x y = 1+(x 1)+(y 1)**
   p. (’a -> ’b) * (’c * ’c -> ’a) * ’c -> ’b // **let f (x, y, z) = (x (y (z,z)))**

4. OCaml Programs

   What is the value of the following OCaml expressions? If an error exists, describe the error.

   a. if 1<2 then 3 else 4 // **3**
   b. let x = 1 in 2 // **2**
   c. let x = 1 in x+1 // **2**
   d. let x = 1 in x ; x+1 // **2**
   e. let x = (1, 2) in x ; x+1
      // **error: x has type int*int but used with int**
   f. (let x = (1, 2) in x) ; x+1 // **error: unbound value x**
   g. let x = 1 in let y = x in y // **1**
   h. let x = 1 let y = 2 in x+y // **syntax error: missing “in”**
   i. let x = 1 in let x = x+1 in let x = x+1 in x // **3**
   j. let x = x in let x = x+1 in let x = x+1 in x // **error: unbound value x**
   k. let rec x y = x in 1 // **error: x has type ‘a -> ‘b but used with ‘b**
   l. let rec x y = y in 1 // **1**
   m. let rec x y = y in x 1 // **1**
   n. let x y = fun z -> z+1 in x // **fun y -> (fun z -> z+1)**
   o. let x y = fun z -> z+1 in x 1 // **fun z -> z+1**
   p. let x y = fun z -> z+1 in x 1 1 // **2**
   q. let x y = fun z -> x+1 in x 1 // **error: unbound value x**
   r. let rec x y = fun z -> x+1 in x 1
      // **error: x has type ‘a -> ‘b -> ‘c but used with int**

   s. let rec x y = fun z -> x+y in x 1
      // **error: x has type ‘a -> ‘b -> ‘c but used with int**
   t. let rec x y = fun z -> x y in x 1

// **error: x has type ‘a -> ‘b but used with ‘b**

u. let rec x y = fun z -> x z in x 1 // **error: x has type ‘a -> ‘b but used with ‘b**

v. let x y = y 1 in 1 // **1**

w. let x y = y 1 in x // **fun y -> (y 1)**

x. let x y = y 1 in x 1 // **error: 1 has type int but used with int -> ‘a**

y. let x y = y 1 in x fun z -> z + 1 // **syntax error at “x fun”**

z. let x y = y 1 in x (fun z -> z + 1) // **2**

aa. let a = 1 in let f x y z = x+y+z+a in f 1 2 3 // **7**

bb. let a = 1 in let f x y z = x+y+z+a in f 1 2 -3 // **error: (f 1 2) has type int -> int but used with int**

5. OCaml Programming

```
let rec map f l = match l with
        [] -> []
        | (h::t) -> (f h)::(map f t)
;;
let rec fold f a l = match l with
        [] -> a
        | (h::t) -> fold f (f a h) t
;;
```

a. Write an OCaml function named *fib* that takes an int *x*, and returns the Fibonacci number for *x*. Recall that fib(0) = 0, fib(1) = 1, fib(2) = 1, fib(3) = 2.

```
let rec fib x =
        if (x = 0) then 0
        else if (x = 1) then 1
        else (fib (x-1) + fib (x-2))
;;
```

b. Write a function *find_suffixes* which applied to a list *lst* returns a list of all the suffixes of *lst*. For instance, suffixes [1;2;5] = [[1;2;5] ; [2;5] ; [5]]

```
let rec suffix_helper (x, r) =
  match x with
    [] -> r
   | (h::t) -> (suffix_helper (t, (h::t)::r))
;;
let suffixes x = List.rev (suffix_helper (x, []))
;;
```

c. Write an OCaml function named *map_odd* which takes a function *f* and a list *lst*, applies the function to every other element of the list, starting with the first element, and returns the result in a new list.

```
let rec map_odd f l = match l with
        [] -> []
        | (x1::[]) -> [f x1]
        | (x1::x2::t) -> (f x1)::(map_odd f t)
;;
```

d. Use *map_odd* and *fib* applied to the list [1;2;3;4;5;6;7] to calculate the Fibonacci numbers for 1, 3, 5, and 7.

```
map_odd fib [1;2;3;4;5;6;7]  ;;
```

e. Using *map*, write a function *triple* which applied to a list of ints *lst* returns a list with all elements of *lst* tripled in value.

```
let triple x = map (fun x -> 3*x) x ;;
```

f. Using *fold*, write a function *all_true* which applied to a list of booleans *lst* returns true only if all elements of *lst* are true.

```
let all_true lst = fold (fun a x -> (x = true) && (a = true)) true lst ;;
```

g. Using *fold* and anonymous helper functions, write a function *product* which applied to a list of ints *lst* returns the product of all the elements in *lst*.

```
let product x = fold (fun a y -> a*y) 1 x ;;
```

h. Using *fold* and anonymous helper functions, write a function *find_min* which applied to a list of ints *lst* returns the smallest element in *lst*.

```
let find_min x = fold (fun a y -> min a y) max_int x ;;
```

i. Using the *fold* function and anonymous helper functions, write a function *count_vote* which applied to a list of booleans *lst* returns a tuple (x,y) where x is the number of true elements and y is the number of false elements.

```
let count_vote x = fold (fun (y,n) v ->
            if (v) then (y+1,n) else (y,n+1)) (0,0) x
;;
```

j. Using the function *count_vote*, write a function *majority* which applied to a list of booleans *lst* returns true if 1/2 or more elements of *lst* are true.

```
let majority x = match (count_vote x) with (y,n) -> (y >= n) ;;
```

6. OCaml Polymorphic Types

Consider a OCaml module Bst that implements a binary search tree:

```
module Bst = struct
  type bst =
     Empty
     | Node of int * bst * bst

  let empty = Empty            (* empty binary search tree          *)

  let is_empty = function      (* return true for empty bst         *)
     Empty -> true
     | Node (_, _, _) -> false

  let rec insert n = function  (* insert n into binary search tree  *)
     Empty -> Node (n, Empty, Empty)
   | Node (m, left, right) ->
       if m = n then Node (m, left, right)
       else if n < m then Node(m, (insert n left), right)
       else Node(m, left, (insert n right))

  (* Implement the following functions
          val min : bst -> int
          val remove : int -> bst -> bst
          val fold : ('a -> int -> 'a) -> 'a -> bst -> 'a
          val size : bst -> int
        *)
     let rec min =             (* return smallest value in bst      *)
     let rec remove n t =      (* tree with n removed               *)
     let rec fold f a t =      (* apply f to nodes of t in inorder  *)
     let size t =              (* # of non-empty nodes in t         *)
  end
```

a. Is insert tail recursive? Explain why or why not.

   No, since the return value for recursive call to insert cannot be used as the return value of the original call to insert. The return value is used to create a Node data type first, and the Node value is returned.

b. Implement min as a tail-recursive function. Raise an exception for an empty bst. Any reasonable exception is fine.

```
let rec min = function
   Empty -> (raise (Failure "min"))
 | Node (m, left, right) ->
     if (is_empty left) then m
     else min left
```

c. Implement remove. The result should still be a binary search tree.

```
let rec remove n = function
   Empty -> Empty
 | Node (m, left, right) ->
    if m = n then (
     if (is_empty left) then right
     else if (is_empty right) then left
     else let x = min right in
           Node(x, left, remove x right)
    //           OR
    // else let x = max left in
    //        Node(x, remove x left, right)
    )
    else if n < m then Node(m, (remove n left), right)
    else Node(m, left, (remove n right))
```

d. Implement fold as an inorder traversal of the tree so that the code

    List.rev (fold (fun a m -> m::a) [] t)

will produce an (ordered) list of values in the binary search tree.

```
let rec fold f a n = match n with
   Empty -> a
 | Node (m, left, right) -> fold f (f (fold f a left) m) right
```

e. Implement size using fold.

```
let size t = fold (fun a m -> a+1) 0 t
```

7. Recursive Descent Parser in OCaml

The example OCaml recursive descent parser 15-parseArith_fact.ml employs a number of shortcuts. For instance, the function parseS handles the grammar rules for

$S \rightarrow T + S \mid T$

directly instead of first applying left factoring:

$S \rightarrow T A \qquad A \rightarrow + S \mid \text{epsilon}$

However, we can still identify where code corresponding to parseA was inserted directly in the code for parseS, in the comments below:

```
let rec parseS lr =                 (* parseS *)
  let x = parseT lr in              (* S → T A *)
  match !lr with                    (* parseA *)
    | ('+'::t) ->                   (* if lookahead = First( + S ) *)
      lr := t;                      (* A → + S *)
      Sum (x,parseS lr)
    | _ -> x                        (* A → epsilon *)
```

Similarly, the function parseF handles the grammar rules for

F -> U ! | U

directly instead of rewriting the grammar, creating the following productions:

F -> ? B -> ?

You must identify where code corresponding to parseB was inserted directly in the code for parseF in the comments below:

```
let rec parseF lr =                 (* parseF *)
 let rec fHelper lr tmp =
  match !lr with                    (* parseB *)
    | ('!'::t) ->                   (* 1: if lookahead = First( ? ) *)
      lr := t;                      (* 2: ? → ? *)
      Fact (fHelper lr tmp)
    | _ -> tmp                      (* 3: ? →  ? *)
 in let x = parseU lr in (fHelper lr x)  (* 4: ? →  ? *)
```

a. What rule should have been applied to the productions for F?

   Eliminate left recursion
   (e.g., change A → A B | C to A → C N
   N → B N | epsilon)

b. What productions for F & B would be created by applying the rule?

   F → U B
   B → ! B | epsilon

c. What sentential form should appear in place of ? in comment 1?

   ! B

d. What production should appear in place of ? in comment 2?

   B → ! B

e. What production should appear in place of ? in comment 3?

   B → epsilon

f. What production should appear in place of ? in comment 4?

   F → U B