CMSC330 Spring 2011 Quiz #4 - SOLUTIONS

Name _____

Discussion Time (circle one): **9am 10am 11am 12pm 1pm 2pm**

Instructions

- Do not start this test until you are told to do so!
- You have 15 minutes for this quiz.
- This is a closed book test. No notes or other aids are allowed.
- For partial credit, show all of your work and clearly indicate your answers.
- Write neatly. Credit cannot be given for illegible answers.

1. (6 pts) Beta-reduction
   Write down what each of the following reduces to via a *single* beta-reduction step.

   a. (λ x. (x x)) (λ x. (x x))

      RESULT = (λ x. (x x)) (λ x. (x x))

   b. (λ f. λ x. (f x)) (λ y. x)

      RESULT = λ w. ((λ y. x) w) Note need for alpha-conversion when substituting for f.

   c. (λ x. λ y. x) 0 1

      RESULT = (λ y. 0) 1 Note one step of beta-reduction can only process one argument

2. (5 pts) Arithmetic
   Suppose you are given a lambda expression **`pred`** that computes the predecessor of a Church numeral **`n`**. That is:

$$pred\ n = \begin{matrix} 0 & if\ n = 0 \\ n-1 & otherwise \end{matrix}$$

   Using pred and other operators in class, give a function that, given Church numerals M and N, computes the Church numeral representation of M – N, where the result is 0 if N > M. Your function should have the following form: λ M. λ N. E for some E. Recall that Church numeral N has form λ f. λ x. f (f … (f x)), where f is applied N times to x.

   λ M. λ N. (N pred) M (i.e. compute the precedessor of M N times)

3. (4 pts) Free variables
   For each of the following expressions, write "yes" if variable x has at least one free (i.e. unbound) occurrence within the term and "no" if it does not.

   a. λ x. x **ANSWER = no**

   b. x (λ x. x) **ANSWER = yes**

   c. λ f. (λ x. f (x x)) (λ x. f (x x)) **ANSWER = no**

   d. (λ x.y) (λ y. x) **ANSWER = yes**