

CMSC330 Spring 2011 Quiz #4

Name _____

Discussion Time (circle one): 9am 10am 11am 12pm 1pm 2pm

Instructions

- Do not start this test until you are told to do so!
- You have 15 minutes for this quiz.
- This is a closed book test. No notes or other aids are allowed.
- For partial credit, show all of your work and clearly indicate your answers.
- Write neatly. Credit cannot be given for illegible answers.

1. (6 pts) Beta-reduction

Write down what each of the following reduces to via a *single* beta-reduction step.

a. $(\lambda x. (x x)) (\lambda x. (x x))$

RESULT =

b. $(\lambda f. \lambda x. (f x)) (\lambda y. x)$

RESULT =

c. $(\lambda x. \lambda y. x) 0 1$

RESULT =

2. (5 pts) Arithmetic

Suppose you are given a lambda expression **pred** that computes the predecessor of a Church numeral **n**. That is:

$$\text{pred } n = \begin{array}{ll} 0 & \text{if } n = 0 \\ n - 1 & \text{otherwise} \end{array}$$

Using **pred** and other operators in class, give a function that, given Church numerals **M** and **N**, computes the Church numeral representation of **M – N**, where the result is 0 if **N > M**. Your function should have the following form: $\lambda M. \lambda N. E$ for some **E**. Recall that Church numeral **N** has form $\lambda f. \lambda x. f (f \dots (f x))$, where **f** is applied **N** times to **x**.

3. (4 pts) Free variables

For each of the following expressions, write “yes” if variable x has at least one free (i.e. unbound) occurrence within the term and “no” if it does not.

a. $\lambda x. x$ **ANSWER =**

b. $x (\lambda x. x)$ **ANSWER =**

c. $\lambda f. (\lambda x. f(x x)) (\lambda x. f(x x))$ **ANSWER =**

d. $(\lambda x. y) (\lambda y. x)$ **ANSWER =**