# Bottom-up Parsing

## Roadmap (Where are we?)

*We set out to study parsing*

- Predictive top-down parsing
  - → FIRST, FOLLOW, FIRST+
  - → The LL(1) condition
  - → Table-driven LL(1) parsers
- Bottom-up parsing
  - → Finding reductions
  - → Shift-reduce parsers



## Parsing Techniques: Bottom-up parsers

*LR(1), operator precedence*

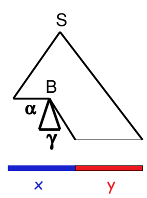
1 input symbol lookahead

construct rightmost derivation (backwards)

input: read left-to-right

rule $B ::= \gamma$

$$S \Rightarrow^*_{rm} \alpha B y \Rightarrow_{rm} \alpha \gamma y \Rightarrow^*_{rm} x\ y$$






## Parsing Techniques: Bottom-up parsers

*LR(1), operator precedence*

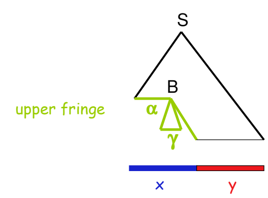
1 input symbol lookahead

construct rightmost derivation (backwards)

input: read left-to-right

rule $B ::= \gamma$

$$S \Rightarrow^*_{rm} \alpha B y \Rightarrow_{rm} \alpha \gamma y \Rightarrow^*_{rm} x\ y$$






## Parsing Techniques: Bottom-up parsers

*LR(1), operator precedence*

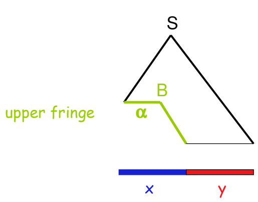
1 input symbol lookahead

construct rightmost derivation (backwards)

input: read left-to-right

rule $B ::= \gamma$

$$S \Rightarrow^*_{rm} \alpha B y \Rightarrow_{rm} \alpha \gamma y \Rightarrow^*_{rm} x\ y$$






## Bottom-up Parsing

A bottom-up parser builds a derivation by working from the input sentence back toward the start symbol *S*

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \textit{sentence}$$

bottom-up

To reduce $\gamma_i$ to $\gamma_{i-1}$ match some *rhs* $\beta$ against $\gamma_i$ then replace $\beta$ with its corresponding *lhs*, *A*. *(assuming the production $A \rightarrow \beta$)*

In terms of the parse tree, this is working from leaves to root

- Nodes with no parent in a partial tree form its *upper fringe*
- Since each replacement of $\beta$ with *A* shrinks the upper fringe, we call it a *reduction*.

The parse tree need not be built, it can be simulated

|*parse tree nodes*| = |*words*| + |*reductions*|

## Finding Reductions

Consider the simple grammar

| | | | |
|---|---|---|---|
| 1 | *Goal* | $\rightarrow$ | a *A B* e |
| 2 | *A* | $\rightarrow$ | *A* b c |
| 3 | | \| | b |
| 4 | *B* | $\rightarrow$ | d |

And the input string abbcde

| *Sentential Form* | *Next Reduction Prod'n* | *Pos'n* |
|---|---|---|
| abbcde | 3 | 2 |
| a *A* bcde | 2 | 4 |
| a *A* de | 4 | 3 |
| a *A B* e | 1 | 4 |
| *Goal* | — | — |

*The trick is scanning the input and finding the next reduction*
*The mechanism for doing this must be efficient*



## Finding Reductions (Handles)

The parser must find a substring $\beta$ of the tree's frontier that
*matches some production $A \rightarrow \beta$ that occurs as one step in the rightmost derivation*

Informally, we call this substring $\beta$ a *handle*

Formally,
A *handle* of a right-sentential form $\gamma$ is a pair $\langle A\rightarrow\beta, k\rangle$ where $A\rightarrow\beta \in P$ and $k$ is the position in $\gamma$ of $\beta$'s rightmost symbol.
If $\langle A\rightarrow\beta, k\rangle$ is a handle, then replacing $\beta$ at $k$ with $A$ produces the right sentential form from which $\gamma$ is derived in the rightmost derivation.

Because $\gamma$ is a right-sentential form, the substring to the right of a handle contains only terminal symbols

$\Rightarrow$ the parser doesn't need to scan past the handle (only lookahead)



## Example (a very busy slide)

| | | | |
|---|---|---|---|
| 1 | *Goal* | $\rightarrow$ | *Expr* |
| 2 | *Expr* | $\rightarrow$ | *Expr + Term* |
| 3 | | \| | *Expr - Term* |
| 4 | | \| | *Term* |
| 5 | *Term* | $\rightarrow$ | *Term * Factor* |
| 6 | | \| | *Term / Factor* |
| 7 | | \| | *Factor* |
| 8 | *Factor* | $\rightarrow$ | number |
| 9 | | \| | id |
| 10 | | \| | ( *Expr* ) |

*The expression grammar*

| *Prod'n.* | *Sentential Form* | *Handle* |
|---|---|---|
| — | *Goal* | — |
| 1 | *Expr* | 1,1 |
| 3 | *Expr – Term* | 3,3 |
| 5 | *Expr – Term * Factor* | 5,5 |
| 9 | *Expr – Term* * <id,y> | 9,5 |
| 7 | *Expr – Factor* * <id,y> | 7,3 |
| 8 | *Expr* – <num,2> * <id,y> | 8,3 |
| 4 | *Term* – <num,2> * <id,y> | 4,1 |
| 7 | *Factor* – <num,2> * <id,y> | 7,1 |
| 9 | <id,x> – <num,2> * <id,y> | 9,1 |

*Handles for rightmost derivation of* x – 2 * y

*This is the inverse of Figure 3.9 in EaC*



## Finding Reductions (Handles)

Critical Insight *(Theorem)*
*If G is unambiguous, then every right-sentential form has a unique handle.*

If we can find those handles, we can build a derivation !

Sketch of Proof:
1. *G* is unambiguous $\Rightarrow$ rightmost derivation is unique
2. $\Rightarrow$ a unique production $A \rightarrow \beta$ applied to derive $\gamma_i$ from $\gamma_{i-1}$
3. $\Rightarrow$ a unique position $k$ at which $A\rightarrow\beta$ is applied
4. $\Rightarrow$ a unique handle $\langle A\rightarrow\beta, k\rangle$

This all follows from the definitions



## Handle-pruning, Bottom-up Parsers

The process of discovering a handle & reducing it to the appropriate left-hand side is called *handle pruning*

Handle pruning forms the basis for a bottom-up parsing method

To construct a rightmost derivation
$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow w$

Apply the following simple algorithm
*for $i \leftarrow n$ to 1 by -1*
*Find the handle $\langle A_i \rightarrow \beta_i, k_i\rangle$ in $\gamma_i$*
*Replace $\beta_i$ with $A_i$ to generate $\gamma_{i-1}$*

This takes *2n* steps



## Handle-pruning, Bottom-up Parsers

One implementation technique is the *shift-reduce parser*

```
push INVALID
token ← next_token( )
repeat until (top of stack = Goal and token = EOF)
    if the top of the stack is a handle A→β
      then      // reduce β to A
          pop |β| symbols off the stack
          push A onto the stack
      else if (token ≠ EOF)
          then // shift
              push token
              token ← next_token( )
          else    // need to shift, but out of input
            report an error
```

**How do errors show up?**
- **failure to find a handle**
- **hitting EOF & needing to shift (final else clause)**

**Either generates an error**

**Figure 3.11 in EAC**

## Back to x - 2 * y

| Stack | Input | Handle | Action |
|---|---|---|---|
| $ | id - num * id | *none* | shift |
| $ id | - num * id | | |

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce



## Back to x - 2 * y

| Stack | Input | Handle | Action |
|---|---|---|---|
| $ | id - num * id | *none* | shift |
| $ id | - num * id | 9,1 | red. 9 |
| $ *Factor* | - num * id | 7,1 | red. 7 |
| $ *Term* | - num * id | 4,1 | red. 4 |
| $ *Expr* | - num * id | | |

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce



## Back to x - 2 * y

| Stack | Input | Handle | Action |
|---|---|---|---|
| $ | id - num * id | *none* | shift |
| $ id | - num * id | 9,1 | red. 9 |
| $ *Factor* | - num * id | 7,1 | red. 7 |
| $ *Term* | - num * id | 4,1 | red. 4 |
| $ *Expr* | - num * id | *none* | shift |
| $ *Expr* - | num * id | *none* | shift |
| $ *Expr* - num | * id | | |

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce



## Back to x - 2 * y

| Stack | Input | Handle | Action |
|---|---|---|---|
| $ | id - num * id | *none* | shift |
| $ id | - num * id | 9,1 | red. 9 |
| $ *Factor* | - num * id | 7,1 | red. 7 |
| $ *Term* | - num * id | 4,1 | red. 4 |
| $ *Expr* | - num * id | *none* | shift |
| $ *Expr* - | num * id | *none* | shift |
| $ *Expr* - num | * id | 8,3 | red. 8 |
| $ *Expr* - *Factor* | * id | 7,3 | red. 7 |
| $ *Expr* - *Term* | * id | | |

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce



## Back to x - 2 * y

| Stack | Input | Handle | Action |
|---|---|---|---|
| $ | id - num * id | *none* | shift |
| $ id | - num * id | 9,1 | red. 9 |
| $ *Factor* | - num * id | 7,1 | red. 7 |
| $ *Term* | - num * id | 4,1 | red. 4 |
| $ *Expr* | - num * id | *none* | shift |
| $ *Expr* - | num * id | *none* | shift |
| $ *Expr* - num | * id | 8,3 | red. 8 |
| $ *Expr* - *Factor* | * id | 7,3 | red. 7 |
| $ *Expr* - *Term* | * id | *none* | shift |
| $ *Expr* - *Term* * | id | *none* | shift |
| $ *Expr* - *Term* * id | | | |

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce



## Back to x - 2 * y

| Stack | Input | Handle | Action |
|---|---|---|---|
| $ | id - num * id | *none* | shift |
| $ id | - num * id | 9,1 | red. 9 |
| $ *Factor* | - num * id | 7,1 | red. 7 |
| $ *Term* | - num * id | 4,1 | red. 4 |
| $ *Expr* | - num * id | *none* | shift |
| $ *Expr* - | num * id | *none* | shift |
| $ *Expr* - num | * id | 8,3 | red. 8 |
| $ *Expr* - *Factor* | * id | 7,3 | red. 7 |
| $ *Expr* - *Term* | * id | *none* | shift |
| $ *Expr* - *Term* * | id | *none* | shift |
| $ *Expr* - *Term* * id | | 9,5 | red. 9 |
| $ *Expr* - *Term* * *Factor* | | 5,5 | red. 5 |
| $ *Expr* - *Term* | | 3,3 | red. 3 |
| $ *Expr* | | 1,1 | red. 1 |
| $ *Goal* | | *none* | accept |

5 shifts +
9 reduces +
1 accept

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce

## Back to x – 2 * y

| Stack | Input | Handle | Action |
|---|---|---|---|
| $ | id – num * id | *none* | shift |
| $ id | – num * id | 9,1 | red. 9 |
| $ *Factor* | – num * id | 7,1 | red. 7 |
| $ *Term* | – num * id | 4,1 | red. 4 |
| $ *Expr* | – num * id | *none* | shift |
| $ *Expr* – | num * id | *none* | shift |
| $ *Expr* – num | * id | 8,3 | red. 8 |
| $ *Expr* – *Factor* | * id | 7,3 | red. 7 |
| $ *Expr* – *Term* | * id | *none* | shift |
| $ *Expr* – *Term* * | id | *none* | shift |
| $ *Expr* – *Term* * id | | 9,5 | red. 9 |
| $ *Expr* – *Term* * *Factor* | | 5,5 | red. 5 |
| $ *Expr* – *Term* | | 3,3 | red. 3 |
| $ *Expr* | | 1,1 | red. 1 |
| $ *Goal* | | *none* | accept |

shift here

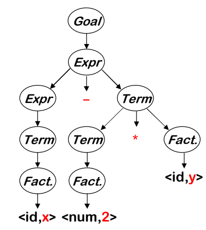
reduce here

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce



## Back to x – 2 * y

| Stack | Input | Action |
|---|---|---|
| $ | id – num * id | shift |
| $ id | – num * id | red. 9 |
| $ *Factor* | – num * id | red. 7 |
| $ *Term* | – num * id | red. 4 |
| $ *Expr* | – num * id | shift |
| $ *Expr* – | num * id | shift |
| $ *Expr* – num | * id | red. 8 |
| $ *Expr* – *Factor* | * id | red. 7 |
| $ *Expr* – *Term* | * id | shift |
| $ *Expr* – *Term* * | id | shift |
| $ *Expr* – *Term* * id | | red. 9 |
| $ *Expr* – *Term* * *Factor* | | red. 5 |
| $ *Expr* – *Term* | | red. 3 |
| $ *Expr* | | red. 1 |
| $ *Goal* | | accept |






## Shift-reduce Parsing

*Shift reduce parsers are easily built and easily understood*

A shift-reduce parser has just four actions

- *Shift* — next word is shifted onto the stack
- *Reduce* — right end of handle is at top of stack
  - Locate left end of handle within the stack
  - Pop handle off stack & push appropriate *lhs*
- *Accept* — stop parsing & report success
- *Error* — call an error reporting/recovery routine

*Accept & Error* are simple
*Shift* is just a push and a call to the scanner
*Reduce* takes |*rhs*| pops & 1 push
*If handle-finding requires state, put it in the stack ⇒ 2x work*

Handle finding is key
- handle is on stack
- finite set of handles
- ⇒ use a DFA !



## An Important Lesson about Handles

To be a handle, a substring of a sentential form $\gamma$ must have two properties:
- → It must match the right hand side $\beta$ of some rule $A \rightarrow \beta$
- → There must be some rightmost derivation from the goal symbol that produces the sentential form $\gamma$ with $A \rightarrow \beta$ as the last production applied

- Simply looking for right hand sides that match strings is not good enough
- Critical Question: How can we know when we have found a handle without generating lots of different derivations?
  - → Answer: *LR(1)* parsers build a DFA that runs over the stack & finds them. One token look-ahead may be used to determine the next action (shift or reduce) in each state of the DFA. *LR(0)* parsers: no look-ahead.



## LR(1) Parsers

- LR(1) parsers are table-driven, shift-reduce parsers that use a limited right context (1 token) for handle recognition
- LR(1) parsers recognize languages that have an LR(1) grammar

*Informal definition:*
A grammar is LR(1) if, given a rightmost derivation

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \textit{sentence}$$

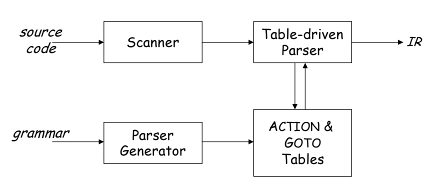
We can
1. *isolate the handle of each right-sentential form* $\gamma_i$, and
2. *determine the production by which to reduce,*

by scanning $\gamma_i$ from left-to-right, going at most 1 symbol beyond the right end of the handle of $\gamma_i$



## LR(1) Parsers

A table-driven LR(1) parser looks like




Tables <u>*can*</u> be built by hand
However, this is a perfect task to automate

## LR(1) Skeleton Parser

```
stack.push(INVALID); stack.push(s0);
not_found = true;
token = scanner.next_token();
do while (not_found) {
        s = stack.top();
        if ( ACTION[s,token] ==  "reduce A→β" ) then {
                stack.popnum(2*|β|); // pop 2*|β| symbols
        s = stack.top();
        stack.push(A);
        stack.push(GOTO[s,A]);
        }
        else if ( ACTION[s,token] == "shift si" ) then {
                stack.push(token); stack.push(si);
                token ← scanner.next_token();
        }
        else if ( ACTION[s,token] == "accept"
                                & token == EOF )
                then not_found = false;
        else report a syntax error and recover;
}
report success;
```

*The skeleton parser*

- uses ACTION & GOTO tables
- does |*words*| shifts
- does |derivation| reductions
- does 1 accept
- detects errors by failure of 3 other cases