Classical Optimizations

Overview

- Last lecture
  - → Code generation
    - High-level languages
  - → Optimization
    - Goals, approaches
- This lecture
  - → Classical optimizations
    - Common subexpression elimination
    - Forward substitution / copy propagation
    - Loop invariant code motion
    - Strength reduction
    - Loop test elimination, induction variable elimination
    - Constant propagation
    - Dead code elimination
  - → Basic block construction
  - → Basic block DAG construction

Example Code

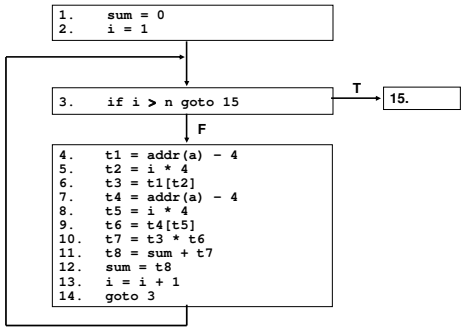
Fortran Source Code:

```
        .
        .
        .
        sum = 0
        do 10 i = 1, n
10      sum = sum + a(i) * a(i)
        .
        .
        .
```


3-address code

1.	sum = 0	sum = 0
2.	i = 1	init for loop and check limit
3.	if i > n goto 15	
4.	t1 = addr(a) – 4	a[i]
5.	t2 = i * 4	
6.	t3 = t1[t2]	
7.	t4 = addr(a) – 4	a[i]
8.	t5 = i * 4	
9.	t6 = t4[t5]	
10.	t7 = t3 * t6	a[i] * a[i]
11.	t8 = sum + t7	increment sum
12.	sum = t8	
13.	i = i + 1	
14.	goto 3	Incr. loop counter back to loop check
15.		

Control Flow Graph (CFG)



Common Subexpression Elimination

```
1.      sum = 0
2.      i = 1
3.      if i > n goto 15
4.      t1 = addr(a) – 4
5.      t2 = i * 4
6.      t3 = t1[t2]
7.      t4 = addr(a) – 4
8.      t5 = i * 4                    common subexpression
9.      t6 = t4[t5]                       elimination
10.     t7 = t3 * t6
10a.    t7 = t3 * t3
11.     t8 = sum + t7
12.     sum = t8
13.     i = i + 1
14.     goto 3
15.
```

Forward Substitution (Copy Propagation)

```
1.      sum = 0
2.      i = 1
3.      if i > n goto 15
4.      t1 = addr(a) - 4
5.      t2 = i * 4
6.      t3 = t1[t2]
10a.    t7 = t3 * t3
11.     t8 = sum + t7
12.     sum = t8                    forward substitution
12a.    sum = sum + t7
13.     i = i + 1
14.     goto 3
15.
```


Invariant Code Motion

```
1.      sum = 0
2.      i = 1
2a.     t1 = addr(a) - 4
3.      if i > n goto 15
4.      t1 = addr(a) - 4            invariant code motion
5.      t2 = i * 4
6.      t3 = t1[t2]
10a.    t7 = t3 * t3
12a.    sum = sum + t7
13.     i = i + 1
14.     goto 3
15.
```


Strength Reduction

```
1.      sum = 0
2.      i = 1
2a.     t1 = addr(a) - 4
2b.     t2 = i * 4
3.      if i > n goto 15
5.      t2 = i * 4                  strength reduction
6.      t3 = t1[t2]
10a.    t7 = t3 * t3
12a.    sum = sum + t7
12b.    t2 = t2 + 4
13.     i = i + 1
14.     goto 3
15.
```


Loop Test Adjustment

```
1.      sum = 0
2.      i = 1
2a.     t1 = addr(a) - 4
2b.     t2 = i * 4
2c.     t9 = n * 4
3.      if i > n goto 15            loop test adjustment
3a.     if t2 > t9 goto 15
6.      t3 = t1[t2]
10a.    t7 = t3 * t3
12a.    sum = sum + t7
12b.    t2 = t2 + 4
13.     i = i + 1
14.     goto 3a
15.
```


Induction Variable Elimination

```
1.      sum = 0
2.      i = 1
2a.     t1 = addr(a) - 4
2b.     t2 = i * 4
2c.     t9 = n * 4
3a.     if t2 > t9 goto 15
6.      t3 = t1[t2]
10a.    t7 = t3 * t3
12a.    sum = sum + t7
12b.    t2 = t2 + 4
13.     i = i + 1                   induction variable elimination
14.     goto 3a
15.
```


Constant Propagation

```
1.      sum = 0
2.      i = 1
2a.     t1 = addr(a) - 4
2b.     t2 = i * 4                  constant propagation
2d.     t2 = 4
2c.     t9 = n * 4
3a.     if t2 > t9 goto 15
6.      t3 = t1[t2]
10a.    t7 = t3 * t3
12a.    sum = sum + t7
12b.    t2 = t2 + 4
14.     goto 3a
15.
```

Dead Code Elimination

```
1.      sum = 0
2.      i = 1                                  dead code elimination
2a.     t1 = addr(a) - 4
2d.     t2 = 4
2c.     t9 = n * 4
3a.     if t2 > t9 goto 15
6.      t3 = t1[t2]
10a.    t7 = t3 * t3
12a.    sum = sum + t7
12b.    t2 = t2 + 4
14.     goto 3a
15.
```

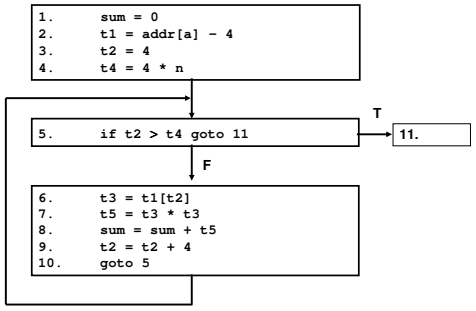

Final Optimized Code

```
1.      sum = 0
2.      t1 = addr(a) - 4
3.      t2 = 4
4.      t4 = n * 4
5.      if t2 > t4 goto 11
6.      t3 = t1[t2]
7.      t5 = t3 * t3
8.      sum = sum + t5
9.      t2 = t2 + 4
10.     goto 5
11.
```

unoptimized: 8 temps, 11 stmts in innermost loop
optimized: 5 temps, 5 stmts in innermost loop

1 index addressing	2 index addressing
1 multiplication	3 multiplications
2 additions	2 additions & 2 subtractions
1 jump	1 jump
1 test	1 test
	1 copy

CFG of Final Optimized Code



Basic Block Construction

- Find leader statements
  - First program statement
  - Targets of conditional or unconditional gotos
  - Any statement following a conditional goto
- (∀x ∈ leaders) construct Bx, basic block headed by x: include all statements following x until next leader or end of program is reached.
- At end of algorithm, any statements not in some basic block are unreachable from program entry and are therefore, dead code.

3-address code example

leader

```
1.    sum = 0
2.    i = 1
3.    if i > n goto 15
4.    t1 = addr(a) - 4
5.    t2 = i * 4
6.    t3 = t1[t2]
7.    t4 = addr(a) - 4
8.    t5 = i * 4
9.    t6 = t4[t5]
10.   t7 = t3 * t6
11.   t8 = sum + t7
12.   sum = t8
13.   i = i + 1
14.   goto 3
15.
```


3-address code example

leader

```
1.    sum = 0
2.    i = 1
3.    if i > n goto 15
4.    t1 = addr(a) - 4
5.    t2 = i * 4
6.    t3 = t1[t2]
7.    t4 = addr(a) - 4
8.    t5 = i * 4
9.    t6 = t4[t5]
10.   t7 = t3 * t6
11.   t8 = sum + t7
12.   sum = t8
13.   i = i + 1
14.   goto 3
15.
```

3-address code example

leader

basic block

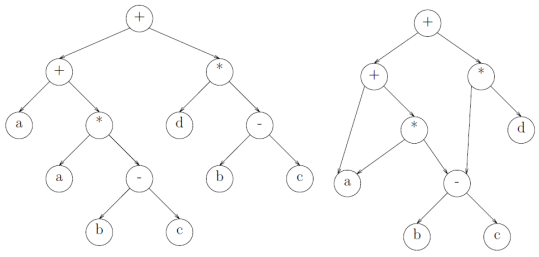
B1
```
1.    sum = 0
2.    i = 1
```
B3
```
3.    if i > n goto 15
```
B4
```
4.    t1 = addr(a) – 4
5.    t2 = i * 4
6.    t3 = t1[t2]
7.    t4 = addr(a) – 4
8.    t5 = i * 4
9.    t6 = t4[t5]
10.   t7 = t3 * t6
11.   t8 = sum + t7
12.   sum = t8
13.   i = i + 1
14.   goto 3
```
B15
```
15.
```


Common Subexpression Elimination

- Common subexpression (CSE)
  - → Portion of expressions repeated multiple times
  - → If computes same value, can reuse previously computed value
- Directed acyclic graph (DAG)
  - → Program representation that exposes CSEs
  - → Unlike tree, nodes can have multiple parents
  - → No cycles allowed
- Building expression DAGs
  - → Maintain hash table for leaves, expressions
  - → Reuse nodes found in hash table when building DAG

Directed Acyclic Graph Example

- Consider the following expression
  - → a + a * (b – c) + (b – c) * d
  - → Representations
    - Tree
    - Directed acyclic graph



Directed Acyclic Graphs

- What about assignment?
  - → Identical subexpressions may have different values
  - → Must ensure each value has unique node in DAG
- Renaming
  - → Add subscripts to variables
  - → Increment subscript for LHS after assignment
  - → Variable references use new subscript
- Example
  - → Original
    - a = a + b
    - c = a + b
  - → After renaming
    - $a_1 = a_0 + b_0$
    - $c_0 = a_1 + b_0$

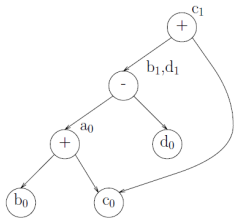
Directed Acyclic Graphs – Renaming Example

→ Original

```
a = b + c
b = a – d
c = b + c
d = a – d
```

→ After renaming

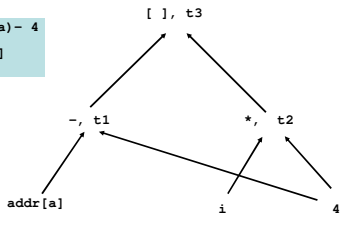
$a_0 = b_0 + c_0$
$b_1 = a_0 – d_0$
$c_1 = b_1 + c_0$
$d_1 = a_0 – d_0$



- Notes
  - → LHS of assignment becomes label for node
  - → Node may have multiple labels if expression is reused

Basic Block DAG Construction

B4
```
4.      t1 = addr(a)– 4
5.      t2 = i * 4
6.      t3 = t1[t2]
    . . .
```

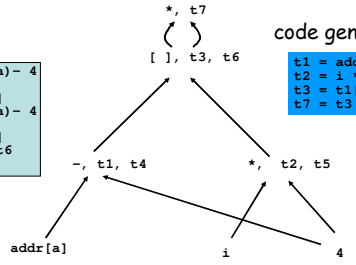


Reference: ASU, p.548

Basic Block DAG Construction

B4

```
4.      t1 = addr(a)- 4
5.      t2 = i * 4
6.      t3 = t1[t2]
7.      t4 = addr(a)- 4
8.      t5 = i * 4
9.      t6 = t4[t5]
10.     t7 = t3 * t6
    . . .
```



code generated:

```
t1 = addr[a]-4
t2 = i * 4
t3 = t1[t2]
t7 = t3 * t3
```

Reference: ASU, p.548

Basic Block DAG Construction

How to add a subexpression into a partially constructed DAG:

A = B + C

Is there a node already for B + C? <+,node(B),node(C)> defined?

- If so, add A to its list of labels.
- If not:
  - is there a node labeled B already? node(B) defined?
    If not, create a leaf labeled B.
  - Is there a node labeled C already? node(C) defined?
    If not, create a leaf labeled C.
  - Create a node labeled A, for +, with left child B and right child C. Create node(+) with children node(B), node(C)

How to do this? HASHING <op, node(opd1), node(opd2)>

Reference: ASU, p.548