Dataflow Analysis 2

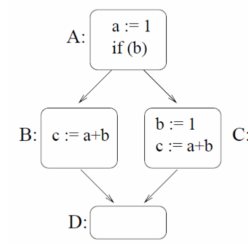
Data-flow Analysis

- Algorithm
  1. Find basic blocks, build CFG
  2. Find propagation functions for basic blocks
  3. Propagate information around CFG
  4. Propagate information into basic blocks
- Example

  Code

```
a = 1
if (b) then
  c = a+b
else
  b = 1
  c = a+b
...
```

  Control flow graph (CFG)



Live Variables

A use of a variable x is a statement where the value of x may be read.

A use of variable x is killed if x is reassigned a value;
the "killing" definition must occur.

$$LV(B) = \bigcup_{Bi \, \varepsilon \, SUCC(B)} (GEN(Bi) \cup [LV(Bi) - KILL(Bi)])$$

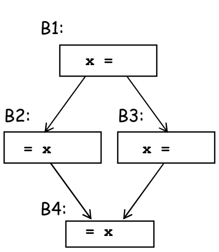
GEN(Bi) : All uses in Bi that are not killed by preceding definitions in Bi

KILL(Bi): All uses of variables x that are defined in Bi

Live Variables

- Universe of facts?
  - All possible subsets of variables in the program
- GEN and KILL sets for each basic block?
  - GEN = variables used in basic block AND not killed before reaching beginning of basic block
  - KILL = variables defined in basic block
- Initial values of LV(B) before propagation starts?
  - All variables in the program
    - Assumes variable is live until proven otherwise
    - The conservative assumption

Live Variables Example



Basic Block	GEN	KILL
B1		
B2		
B3		
B4		

$$LV(B) = \bigcup_{Bi \, \varepsilon \, SUCC(B)} (GEN(Bi) \cup [LV(Bi) - KILL(Bi)])$$

Available Expressions

An expression e is defined if its value is computed.

An expression e is killed if the values of any of its operands may have been changed.

$$AVAIL(B) = \bigcap_{Bi \, \varepsilon \, PRED(B)} (GEN(Bi) \cup [AVAIL(Bi) - KILL(Bi)])$$

GEN(Bi) : All definitions of expressions in Bi that are not subsequently killed in Bi

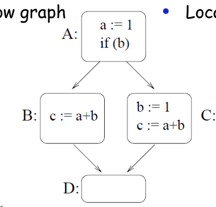
KILL(Bi): All expressions with operand variables defined in Bi

Available Expressions

- Universe of facts?
  - → All possible subsets of expressions in the program
- GEN and KILL sets for each basic block?
  - → GEN = expressions defined in basic block AND operands not defined before reaching end of basic block
  - → KILL = expressions whose operands are defined in basic block
- Initial values of AVAIL(B) before propagation starts?
  - → ∅
    - Assumes expression is not available until proven otherwise
    - The conservative assumption

Available Expressions Example

- Control flow graph



- Local information

Node	KILL	GEN
A	a+b	∅
B	∅	a+b
C	a+b	a+b
D	∅	∅

- Solution

$$\begin{aligned}
\mathrm{AVAIL}(A) &= \emptyset \\
\mathrm{AVAIL}(B) &= \mathrm{GEN}(A) \cup (\mathrm{AVAIL}(A) - \mathrm{KILL}(A)) = \emptyset \cup (\emptyset - \{\ a{+}b\ \}) = \emptyset \\
\mathrm{AVAIL}(C) &= \mathrm{GEN}(A) \cup (\mathrm{AVAIL}(A) - \mathrm{KILL}(A)) = \emptyset \cup (\emptyset - \{\ a{+}b\ \}) = \emptyset \\
\mathrm{AVAIL}(D) &= (\mathrm{GEN}(B) \cup (\mathrm{AVAIL}(B) - \mathrm{KILL}(B))) \cap \\
&\quad\ (\mathrm{GEN}(C) \cup (\mathrm{AVAIL}(C) - \mathrm{KILL}(C))) \\
&= (\{\ a{+}b\ \} \cup (\emptyset - \emptyset)) \cap (\{\ a{+}b\ \} \cup (\emptyset - \{\ a{+}b\ \})) = \{\ a{+}b\ \}
\end{aligned}$$

Very Busy Expressions

An expression e is defined if its value is used before any definition of its operands.

An expression e is killed if the values of any of its operands may have been changed before it is used

$$\boxed{VBE(B) = \bigcap_{Bi\ \varepsilon\ SUCC(B)} (\ GEN(Bi) \cup [\ VBE(Bi) - KILL(Bi)\ ]\ )}$$

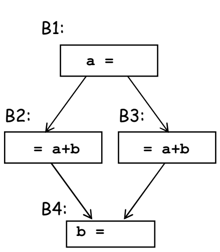
GEN(Bi) : All definitions of expressions in Bi that are used before they are killed in Bi

KILL(Bi): All expressions with operand variables defined in Bi

Very Busy Expressions

- Universe of facts?
  - → All possible subsets of expressions in the program
- GEN and KILL sets for each basic block?
  - → GEN = expressions used in basic block AND operands not defined before reaching beginning of basic block
  - → KILL = expressions whose operands are defined in basic block
- Initial values of VBE(B) before propagation starts?
  - → ∅
    - Assumes expression is not busy until proven otherwise
    - The conservative assumption

Very Busy Expressions Example



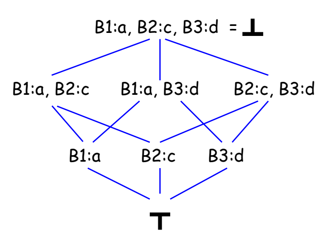
Basic Block	GEN	KILL
B1		
B2		
B3		
B4		

$$VBE(B) = \bigcap_{Bi\ \varepsilon\ SUCC(B)} (\ GEN(Bi) \cup [VBE(Bi) - KILL(Bi)\ ]\ )$$

Implementation Issues: Bit-vector Problems

The set of facts (universe of facts) can often be expressed as finite subsets of a finite base set. Such sets can be represented as bit-vectors.

Example: RD - { B1: a:=2, B2: c := d+2, B3: d := a-5 }



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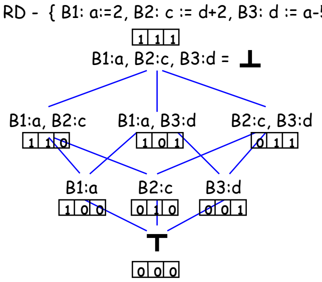


Implementation Issues: Bit-vector Problems

Meet operation is either bit-wise logical AND or bit-wise logical OR .

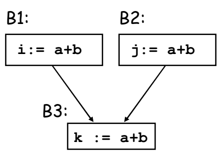
GEN and KILL sets can be expressed as single bit-vectors.

Bit-wise logical – is bit-wise negation followed by bit-wise AND .

Implementation steps:

1. - bit-vector construction/interpretation
2. - bit-vector CFG initialization (RD, GEN, and KILL vectors)
3. - bit-vector CFG propagation
4. - information post-processing (e.g.: DU/UD chains)

Bit-vector Problem?



- What does lattice look like?

  a+b

- GEN and KILL functions?

```
B1: a+b in GEN, KILL is empty
B2: a+b in GEN, KILL is empty
B3: a+b in GEN, KILL is empty
```

- Confluence (meet) operator $\wedge$**?**

  NOT bit-wise logical AND

- Initial value AVAIL(Bi)

  AVAIL(Bi) = ENTIRE SET (a+b)

Constant Propagation

A constant for a variable x is generated if it is assigned a constant value

A constant for variable x is killed if x is reassigned a different value, even if it is a constant value

$$CONST(B) = \bigcap_{Bi \in PRED(B)} (GEN(Bi) \cup [CONST(Bi) - KILL(Bi)])$$

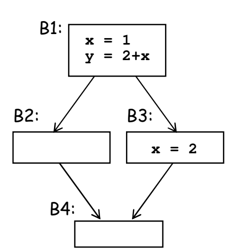
GEN(Bi) : Set of immediate constants (static) and dynamically encountered constants (dynamic)

KILL(Bi): All definitions of variables x defined in Bi

Very Busy Expressions

- Universe of facts?
  - → All possible subsets of variable * constant in the program
- GEN and KILL sets for each basic block?
  - → GEN = variable assigned constant value in basic block AND variable not redefined before reaching end of basic block
  - → KILL = variable defined to non-constant value in basic block
- Initial values of CONST(B) before propagation starts?
  - → Ø
    - Assumes variable is not constant until proven otherwise
    - The conservative assumption

Constant Propagation Example

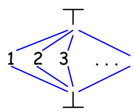


Basic Block	GEN	KILL
B1		
B2		
B3		
B4		

$$C(B) = \bigcap_{Bi \in PRED(B)} (GEN(Bi) \cup [C(Bi) - KILL(Bi)])$$

Non Bit-vector Problem

The set of facts (universe of facts) are set of pairs
(variable, value)
where value is from the following lattice:



Examples: (a,⊤), (b,5)

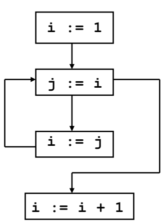
Note: Typically, constant propagation is only performed on integral values, not floating point values

Non Bit-vector Problem

The confluence (meet) operator $\wedge$ for the second component of a (variable, value) pair is defined as follows:

$\wedge$	$\top$	c_2	$\perp$
$\top$	$\top$	c_2	$\perp$
c_1	c_1	If $c_1 = c_2$ then c_1 else $\perp$	$\perp$
$\perp$	$\perp$	$\perp$	$\perp$

Non Bit-vector Problem



Remarks:

- optimistic constant propagation
- constant propagation typically done on UD/DU chains, but similar principles apply
- pessimistic constant propagation does not find any constants in our example
- intermediate results are not safe in optimistic approach

Dead Code Elimination

How to use DU/UD chains for dead code elimination?

Terminology:

UD(S, v): set of statements that contain definitions for variable v that reach the use of v in statement S

DU(S, v): set of statements that contain used of variable v that are reached by the definition of v in statement S

Dead Code Elimination

Two flavors of dead code: A statement can be considered dead code if

(1) it will never be executed, or

(2) it may be executed, but the result will never be used (including side effects)

The discussed algorithm uses UD chains.

Dead Code Elimination

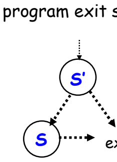
Terminology:

DEFS(S): set of variables that may be written in statement S.

USES(S): set of variables that may be read in statement S.

CONTROL(S): set of statements such that S is control dependent on these statements

S is control dependent on S': S' contains a control flow branch, and there are execution paths from S' to the program exit such that S is on one path, but not on the other.



Dead Code Elimination

Report all statements that are not on any path from the entry to exit node as dead code, and remove them

Initialize all statement as not useful

Initialize worklist with critical statements (print statements)

While worklist is not empty Do

Remove a statement from worklist, call it S; mark S useful

Mark all S' in CONTROL(S) useful and add them to worklist

Forall variables v in USES(S) add each S' in UD(S,v) to worklist EndForall

EndWhile

Report all unmarked statements as dead code