Due in class: Apr 13.

Problems are to be done by yourself (unless specified) with no help from another person. If you use any sources other than the textbooks for the class, you are to cite them. In any case, the writeup should be your own and not simply taken from another source.

(1) Given an undirected graph $G$ with positive and negative edge weights (no negative cycle), we wish to compute a shortest path from $s$ to $t$. Solve this problem by reducing it to min weight perfect matching? (If you cannot solve it by reducing to a min weight perfect matching problem, then solve it by another method for partial credit.)

(2) Imagine a two person game on a graph $G$. The players, First and Second play alternately with First playing first. At each step, the player whose turn it is chooses an edge that has not been chosen before. The only rule is that at each step, the set of chosen edges forms a simple path. The loser is the first player unable to choose an edge. Prove that if $G$ has a perfect matching then First can always force a win.

(3) Write out a linear program formulation for the maximum weight matching problem in a bipartite graph. Also write out the dual of this linear program.

(4) (You may discuss this with a friend.) There are three students and one single seat bicycle. These three students have to transport themselves from College Park to DC, a distance of 10 miles. We would like to minimize the arrival time of the last person at the destination. Student $j$ has a walking speed of $w_j$ miles/hour and a bicycling speed of $b_j$ miles/hour. Suppose $w_1 = 4, b_1 = 16$ and $w_2 = w_3 = 2$ and $b_2 = b_3 = 12$. Show how to obtain an optimal solution. Provide a strategy for solving this problem optimally in general as we change the $w_i$ and $b_i$ values.

(5) Suppose a company has the following vehicle requirements. Jan: 230 cars, Feb: 210 cars, Mar: 440 cars, Apr: 390 cars, May: 425 cars, Jun: 450 cars. They have the following lease options. A 3 month lease for 1700$, a 4 month lease for 2200$, a 5 month lease for 2600$.

Show how to formulate this problem as a linear (integer) programming problem, and then argue that this can be reduced to a min cost flow computation.