

CMSC 132:

OBJECT-ORIENTED PROGRAMMING II



Heaps & Priority Queues

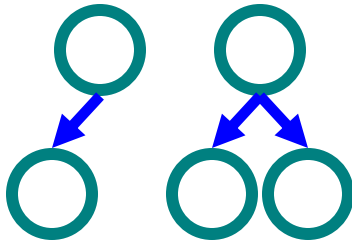
Department of Computer Science
University of Maryland, College Park

Complete Binary Trees

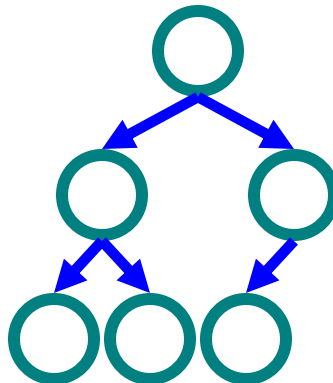
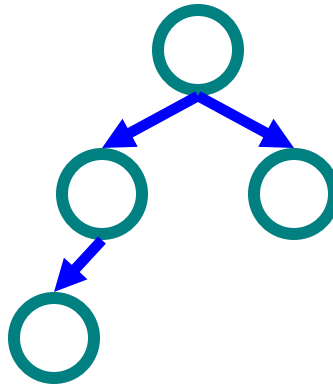
- An binary tree (height h) where
 - Perfect tree to level $h-1$
 - Leaves at level h are as far **left** as possible



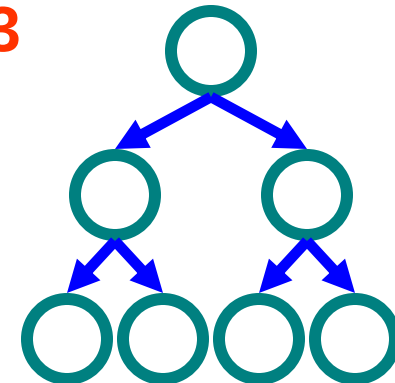
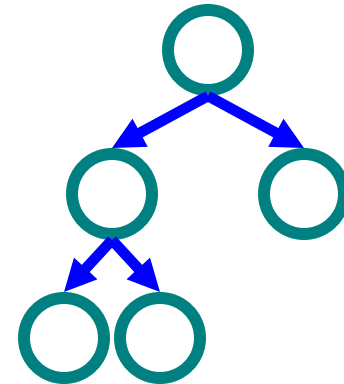
$h = 1$



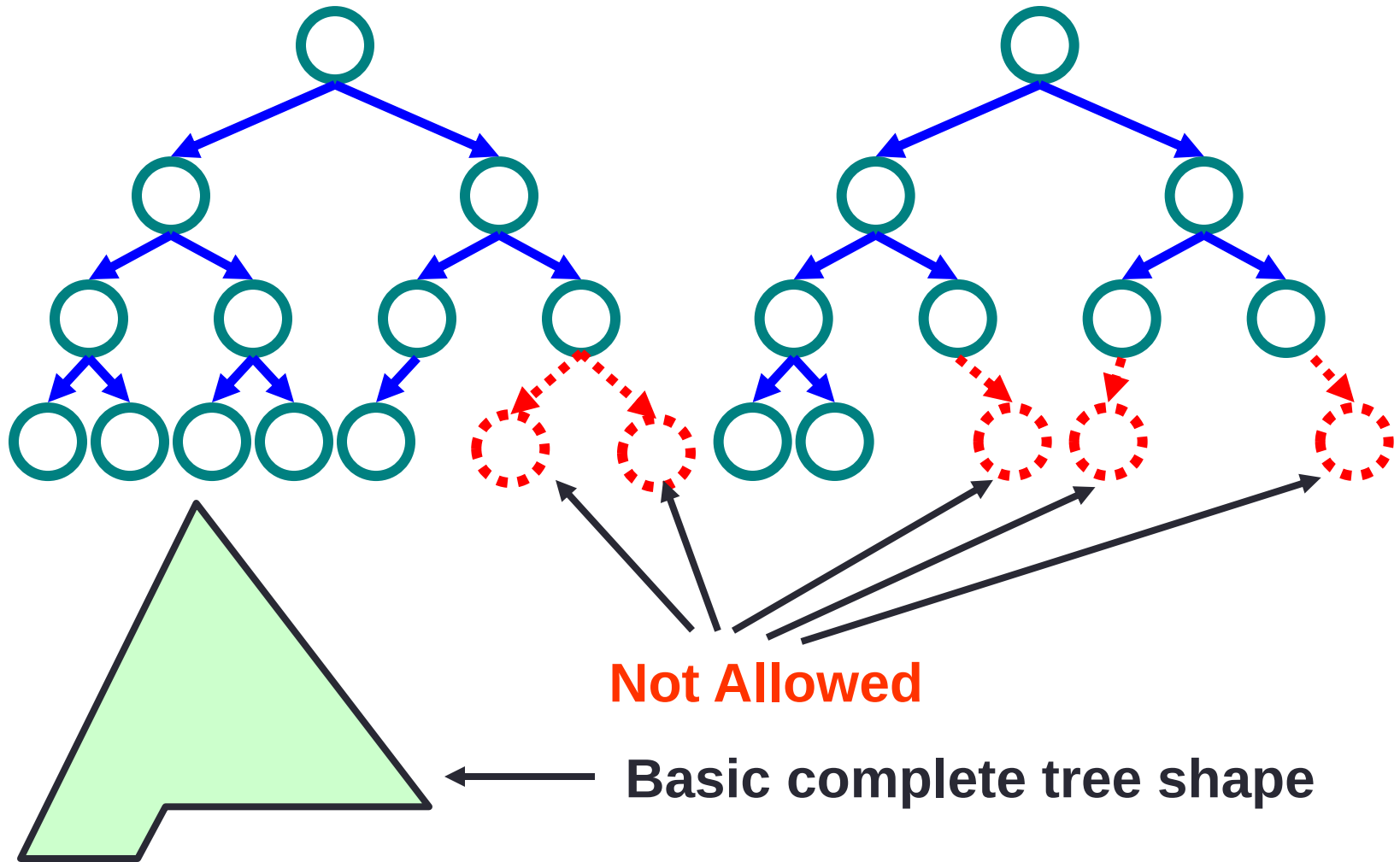
$h = 2$



$h = 3$

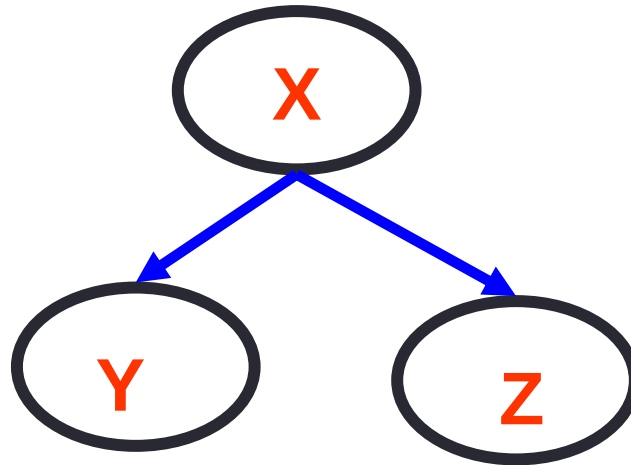


Complete Binary Trees

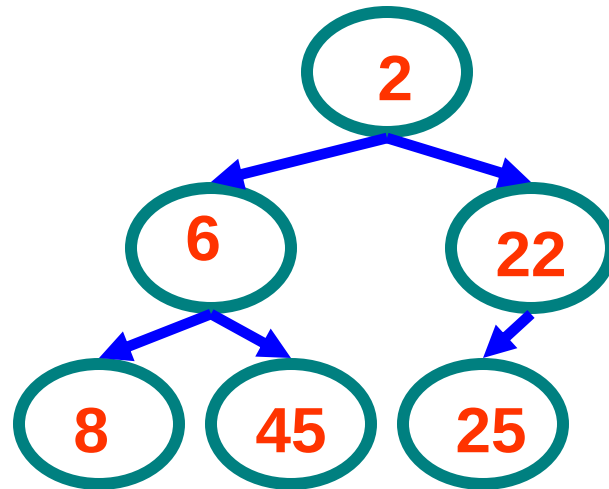
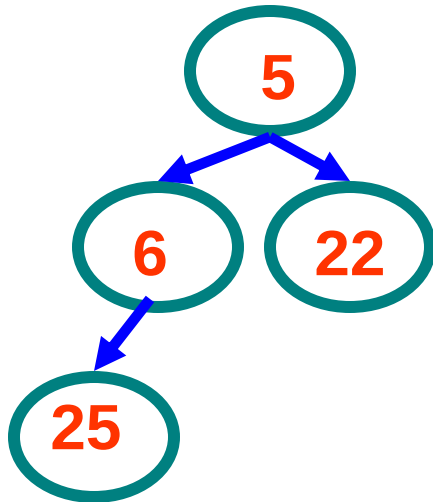
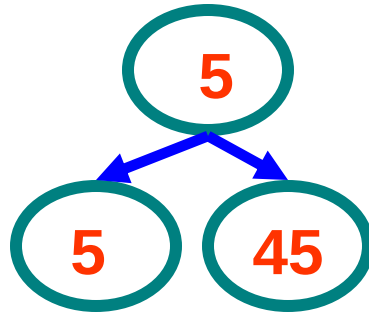


Heaps

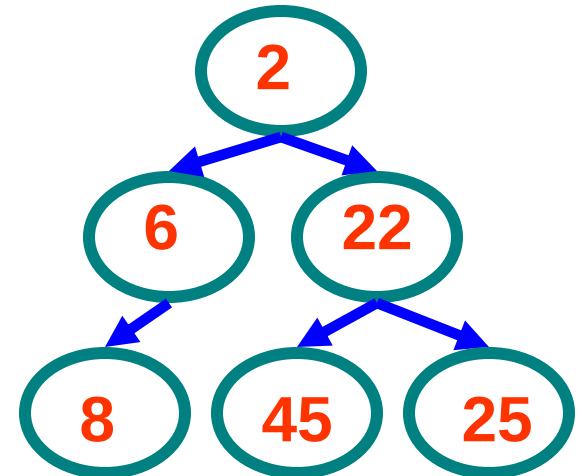
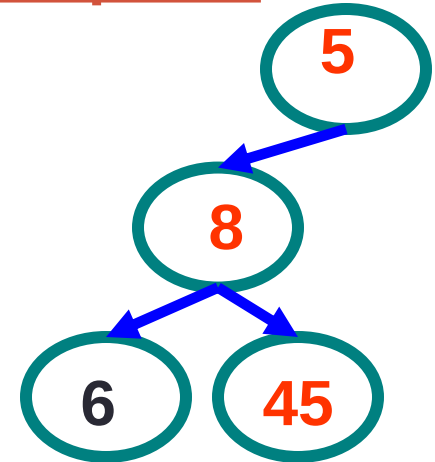
- **Two key properties**
 - Complete binary tree (shape property)
 - Value at node (value property)
 - **Minheap** → smaller than or equal to values in subtrees ($X \leq Y, X \leq Z$)
 - **Maxheap** → larger than or equal to values in subtrees ($X \geq Y, X \geq Z$)
- We will use minheap in our discussion



Heap (min) & Non-heap Examples



Heaps



Non-heaps

Heap Properties

- Heaps are **balanced trees**
 - Height = $\log_2(n) = O(\log(n))$
- Can find smallest/largest element easily
 - Always at top of heap!
 - Heap can track either min or max, but not both

Heap

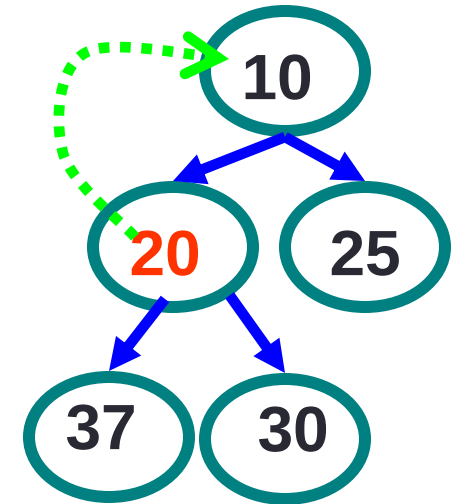
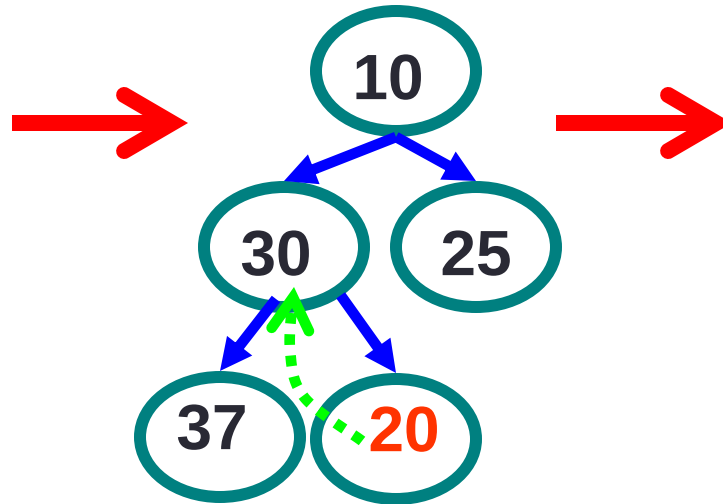
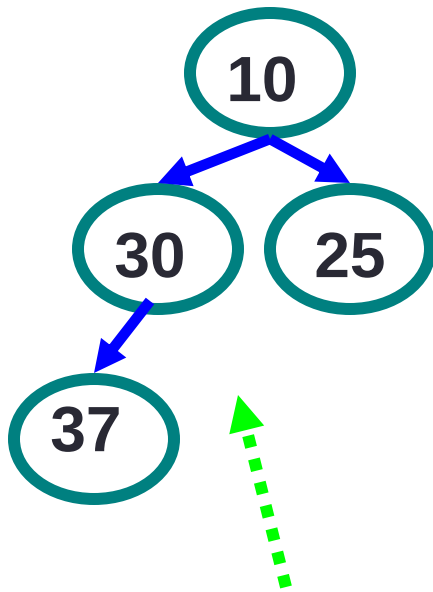
- Key operations
 - Insert (X)
 - getSmallest ()
- Key applications
 - Heapsort
 - Priority queue

Heap Operations – Insert(X)

- Algorithm
 - Add X to end of tree
 - While ($X < \text{parent}$)
 Swap X with parent // X bubbles up tree
- Complexity
 - # of swaps proportional to height of tree
 - $O(\log(n))$

Heap Insert Example

- Insert (20)



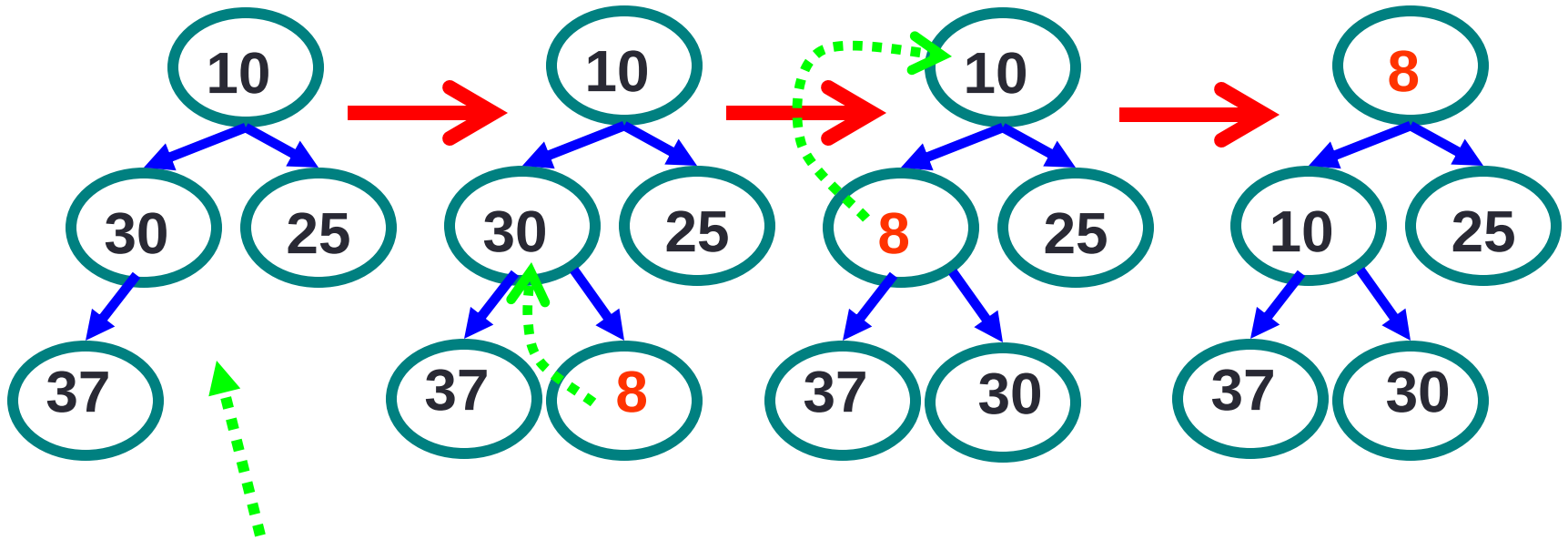
1) Insert to
end of tree

2) Compare to parent,
swap if parent key larger

3) Insert
complete

Heap Insert Example

- Insert (8)



1) Insert to
end of tree

2) Compare to parent,
swap if parent key larger

3) Insert
complete

Heap Operation – getSmallest()

- Algorithm

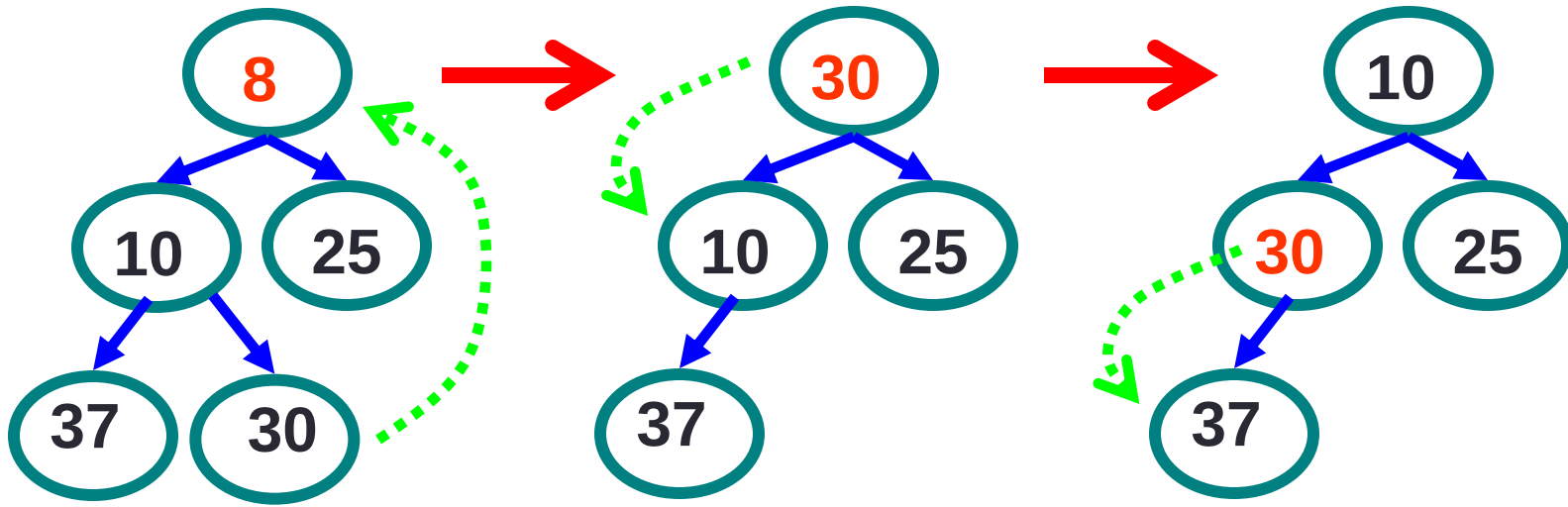
- Get smallest node at root
- Replace root with X at end of tree
- While ($X > \text{child}$)
 - Swap X with smallest child // X drops down tree
- Return smallest node

- Complexity

- # swaps proportional to height of tree
- $O(\log(n))$

Heap GetSmallest Example

- getSmallest ()



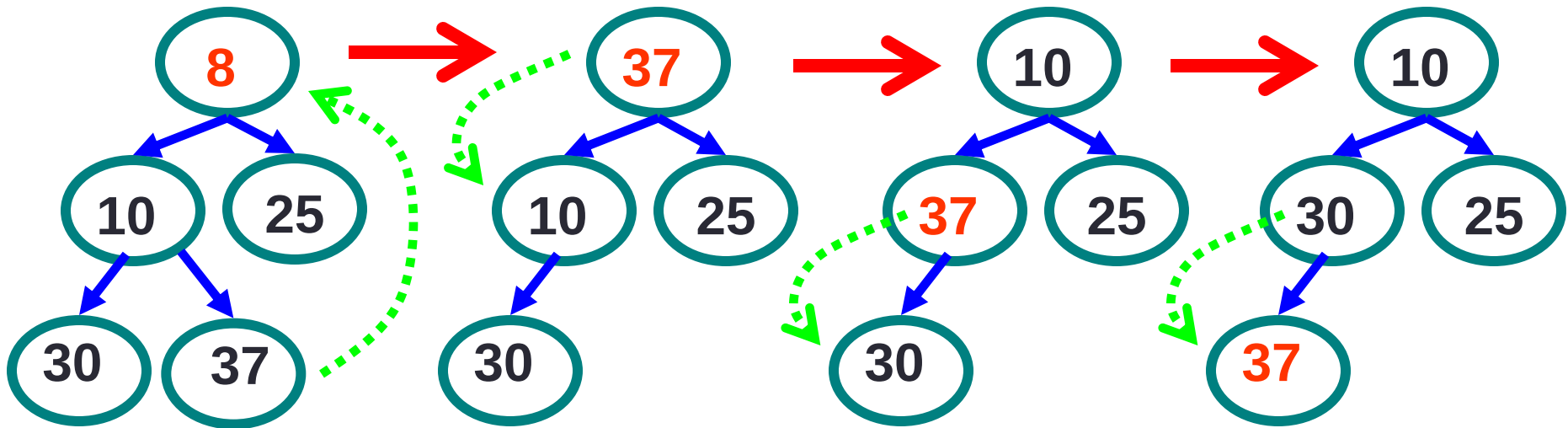
**1) Replace root
with end of tree**

**2) Compare node to
children, if larger swap
with smallest child**

**3) Repeat swap
if needed**

Heap GetSmallest Example

- getSmallest ()



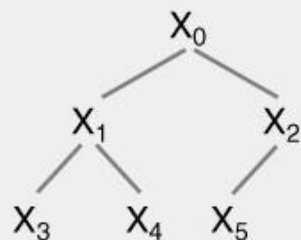
**1) Replace root
with end of tree**

**2) Compare node to
children, if larger swap
with smallest child**

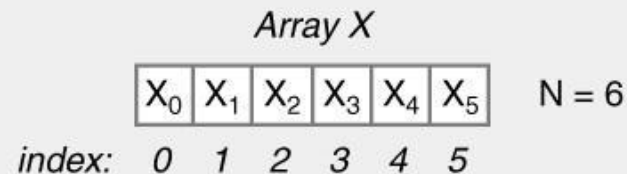
**3) Repeat swap
if needed**

Heap Implementation

- Can implement heap as array
 - Store nodes in array elements
 - Assign location (index) for elements using formula
- Observations
 - Compact representation
 - Edges are implicit (no storage required)
 - Works well for complete trees (no wasted space)



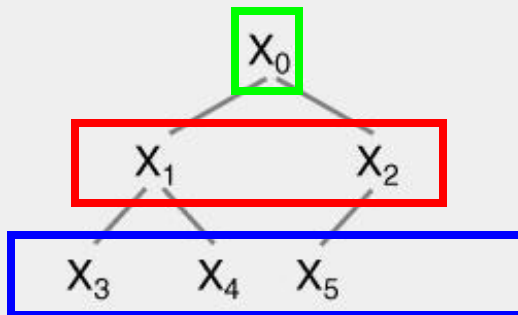
(a) Heap represented as a tree



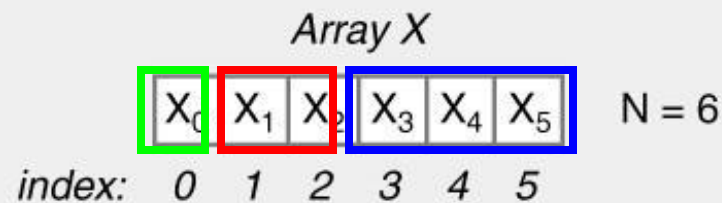
(b) Heap represented as an array

Heap Implementation

- $\lfloor \rfloor \rightarrow$ floor (e.g., $1.7 \rightarrow 1$, $2 \rightarrow 2$)
- Calculating node locations
 - Array index i starts at 0
 - $\text{Parent}(i) = \lfloor (i - 1) / 2 \rfloor$
 - $\text{LeftChild}(i) = 2 \times i + 1$
 - $\text{RightChild}(i) = 2 \times i + 2$



(a) Heap represented as a tree

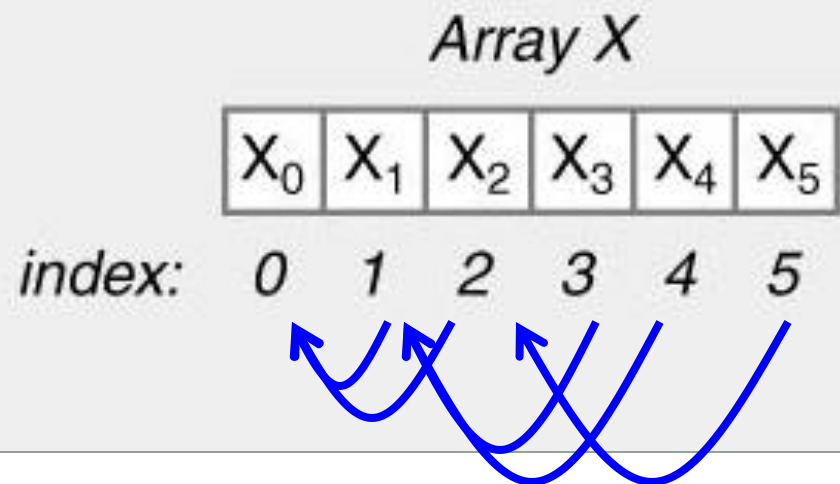
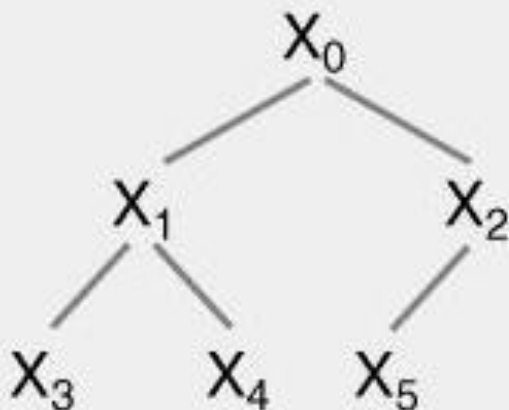


(b) Heap represented as an array

Heap Implementation

- Example

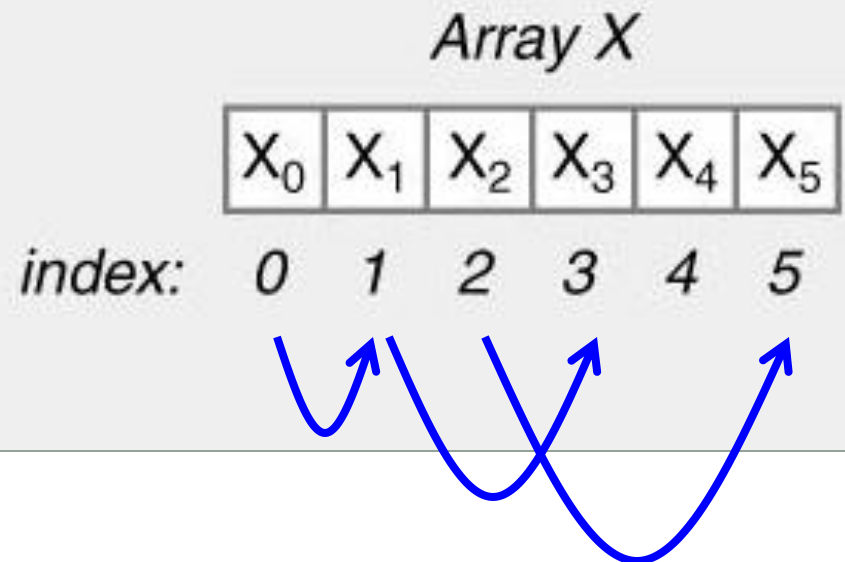
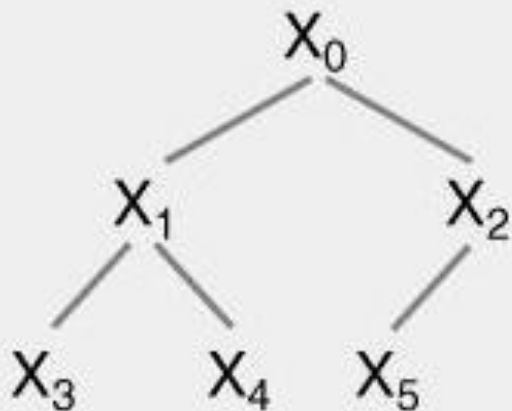
- $\text{Parent}(1) = \lfloor (1 - 1) / 2 \rfloor = \lfloor 0 / 2 \rfloor = 0$
- $\text{Parent}(2) = \lfloor (2 - 1) / 2 \rfloor = \lfloor 1 / 2 \rfloor = 0$
- $\text{Parent}(3) = \lfloor (3 - 1) / 2 \rfloor = \lfloor 2 / 2 \rfloor = 1$
- $\text{Parent}(4) = \lfloor (4 - 1) / 2 \rfloor = \lfloor 3 / 2 \rfloor = 1$
- $\text{Parent}(5) = \lfloor (5 - 1) / 2 \rfloor = \lfloor 4 / 2 \rfloor = 2$



Heap Implementation

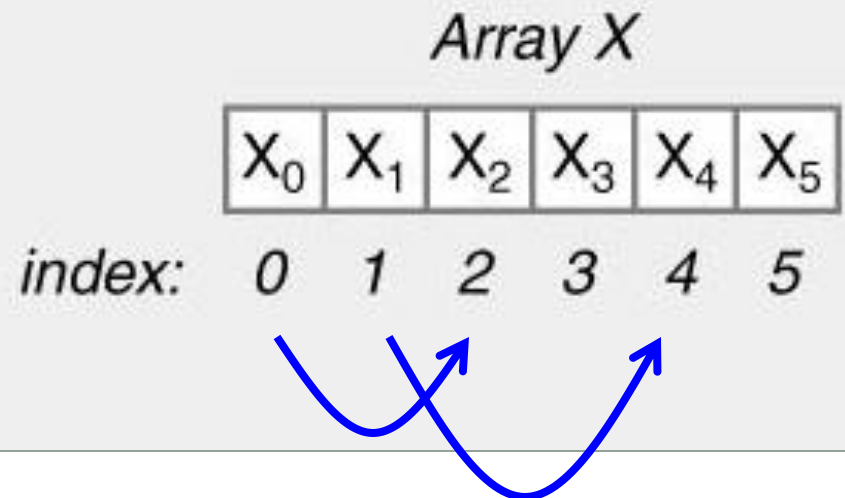
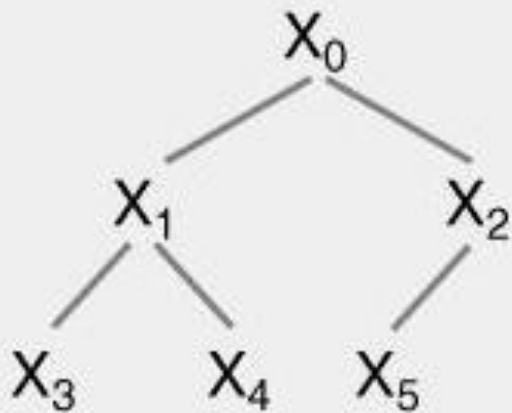
- Example

- $\text{LeftChild}(0) = 2 \times 0 + 1 = 1$
- $\text{LeftChild}(1) = 2 \times 1 + 1 = 3$
- $\text{LeftChild}(2) = 2 \times 2 + 1 = 5$



Heap Implementation

- Example
 - $\text{RightChild}(0) = 2 \times 0 + 2 = 2$
 - $\text{RightChild}(1) = 2 \times 1 + 2 = 4$



Heap Application – Heapsort

- Use heaps to sort values
 - Heap keeps track of smallest/largest element in heap
- Algorithm
 1. Create heap
 2. Insert values in heap
 3. Remove values from heap (in ascending/descending order)
- Complexity
 - $O(n\log(n))$

Heapsort Example

- Input
 - 11, 5, 13, 6, 1
- View heap during insert, removal
 - As tree
 - As array

Heapsort – Insert Values

(a) Insert 11

11

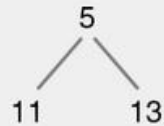
(b) Insert 5



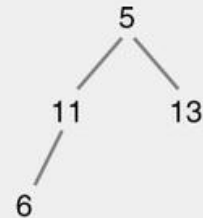
(c) Rebuild heap



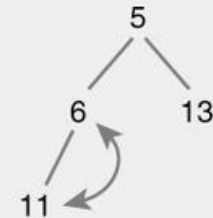
(d) Insert 13



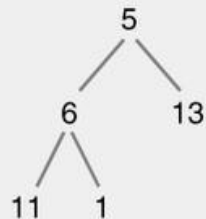
(e) Insert 6



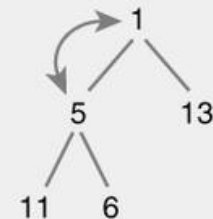
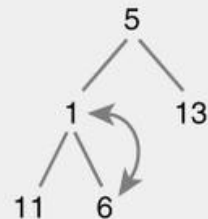
(f) Rebuild heap



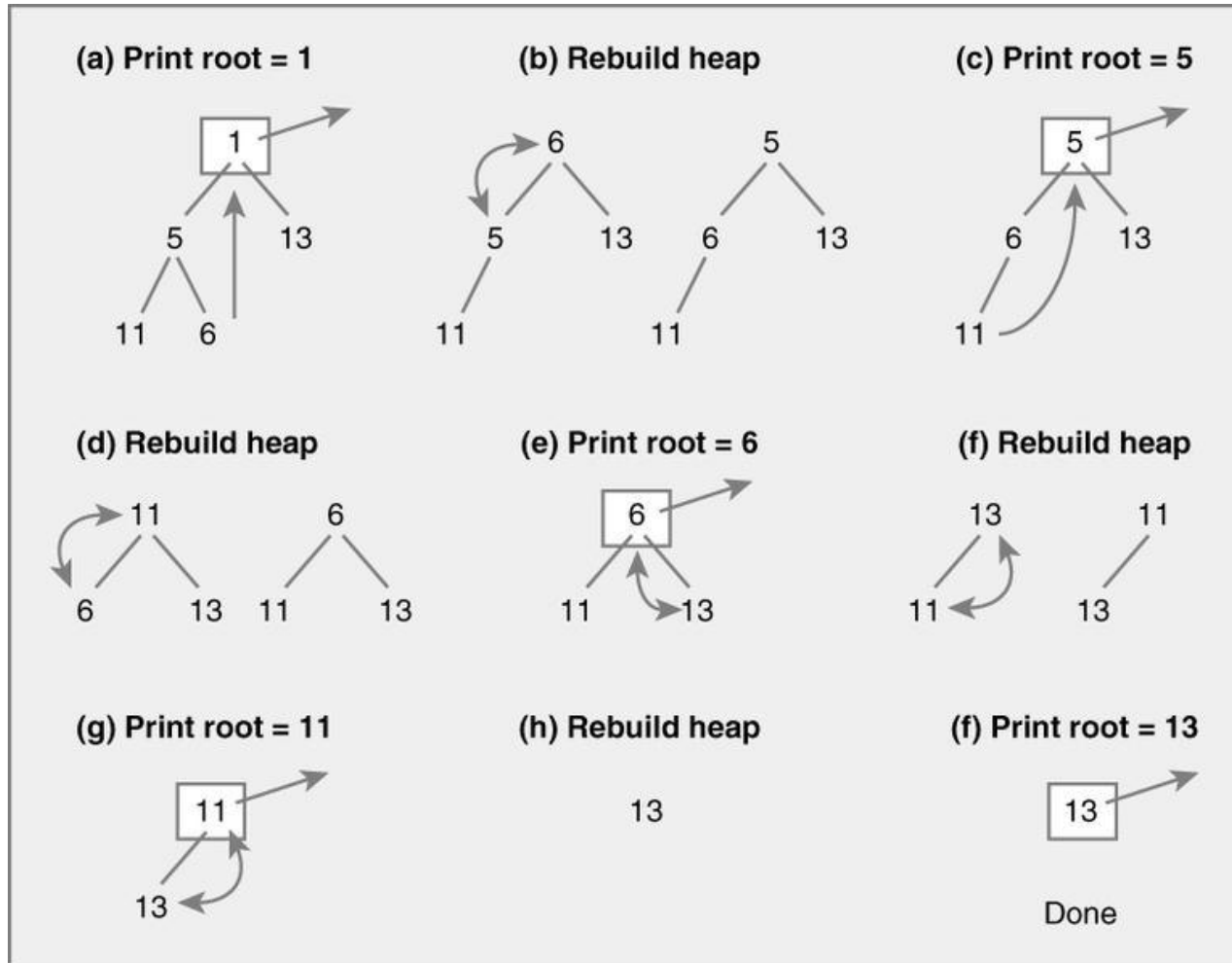
(g) Insert 1



(h) Rebuild heap



Heapsort – Remove Values



Heapsort – Insert in to Array 1

- Input
 - 11, 5, 13, 6, 1

Index =	0	1	2	3	4
Insert 11	11				

Heapsort – Insert in to Array 2

- Input
 - 11, 5, 13, 6, 1

Index =	0	1	2	3	4
Insert 5	11	5			
Swap	5	11			

Heapsort – Insert in to Array 3

- Input
 - 11, 5, 13, 6, 1

Index =	0	1	2	3	4
Insert 13	5	11	13		

Heapsort – Insert in to Array 4

- Input
 - 11, 5, 13, 6, 1

Index =	0	1	2	3	4
Insert 6	5	11	13	6	

Swap	5	6	13	11	
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...

Heapsort – Remove from Array 1

- Input

- 11, 5, 13, 6, 1

Index = 0 1 2 3 4

Remove root

1	5	13	11	6
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Replace

6	5	13	11	
----------	----------	-----------	-----------	--

Swap w/ child

5	6	13	11	
----------	----------	-----------	-----------	--

Heapsort – Remove from Array 2

- Input

- 11, 5, 13, 6, 1

Index = 0 1 2 3 4

Remove root

5	6	13	11	
---	---	----	----	--

Replace

11	6	13		
----	---	----	--	--

Swap w/ child

6	11	13		
---	----	----	--	--

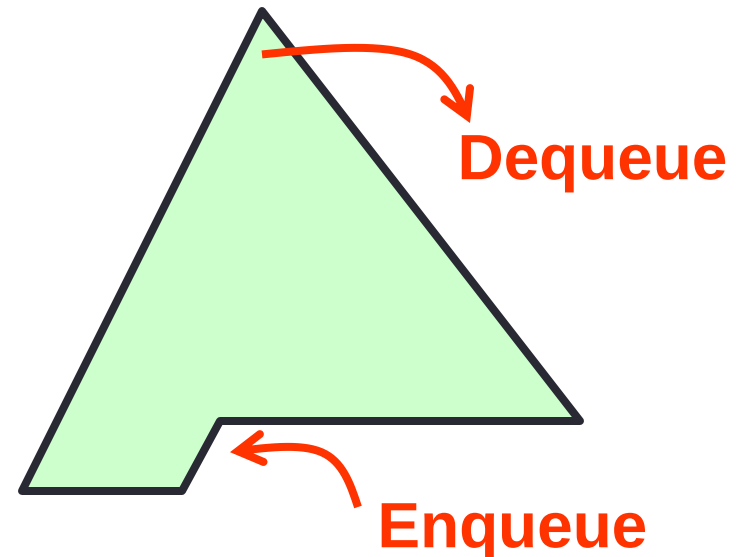
Heap Application – Priority Queue

- Queue
 - Linear data structure
 - First-in First-out (FIFO)
 - Implement as array / linked list



Heap Application – Priority Queue

- Priority queue
 - Elements are assigned **priority** value
 - Higher priority elements are taken out first
 - Implement as heap
 - Enqueue $\Rightarrow$ **insert()**
 - Dequeue $\Rightarrow$ **getSmallest()**



Priority Queue

- Properties
 - Lower value = higher priority
 - Heap keeps highest priority items in front
- Complexity
 - Enqueue $\Rightarrow$ `insert( )` = $O(\log(n))$
 - Dequeue $\Rightarrow$ `getSmallest( )` = $O(\log(n))$
 - For any heap

Heap vs. Binary Search Tree

- Binary search tree
 - Keeps values in sorted order
 - Find any value
 - $O(\log(n))$ for balanced tree
 - $O(n)$ for degenerate tree (worst case)
- Heap
 - Keeps smaller values in front
 - Find **minimum** value
 - $O(\log(n))$ for any heap

About Heap Implementation

- Implementing a heap ([Video I](#) / [Video II](#)) This videos illustrates the process a programmer (Prof. Bill Pugh in this case) goes through while implementing code. This video was filmed in Dr. Bill Pugh's lecture. Keep in mind that in this video some bugs might be present in the implementation as the testing phase has not been completed yet.