CMSC 132: OBJECT-ORIENTED PROGRAMMING II

Graph Implementation

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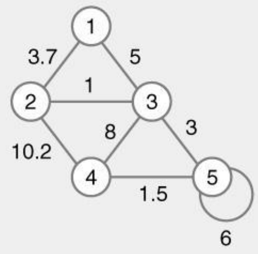
Graph Implementation

- How do we represent edges?
  - Adjacency matrix
    - 2D array of neighbors
  - Adjacency list
    - List of neighbors
  - Adjacency set / map
    - Set / map of neighbors
- Important for very large graphs
  - Affects efficiency / storage

Adjacency Matrix

- Representation
  - Single array for entire graph
  - Unweighted graph
    - Matrix elements ⇒ boolean
    - Let's see an example
  - Weighted graph
    - Matrix elements ⇒ weight
    - Let's see an example
  - Undirected graph
    - Only upper / lower triangle matrix needed
    - Since n_j, n_k implies n_k, n_j



	1	2	3	4	5
1	°				
2	3.7	°			
3	5.0	1.0	°		
4	°	10.2	8.0	°	
5	°	°	3.0	1.5	6.0

Adjacency List/Set

- Representation
  - For each node, store
    - List/Set of neighbors / successors
      - Linked list
      - Array list
  - For weighted graph
    - Also store weight for each edge
    - Using a Map is a good choice
  - For undirected graph with edge (a↔b)
    - Nodes a & b need to store each other as neighbor
  - For directed graph with edge (a→b)
    - Node a needs to store node b as neighbor

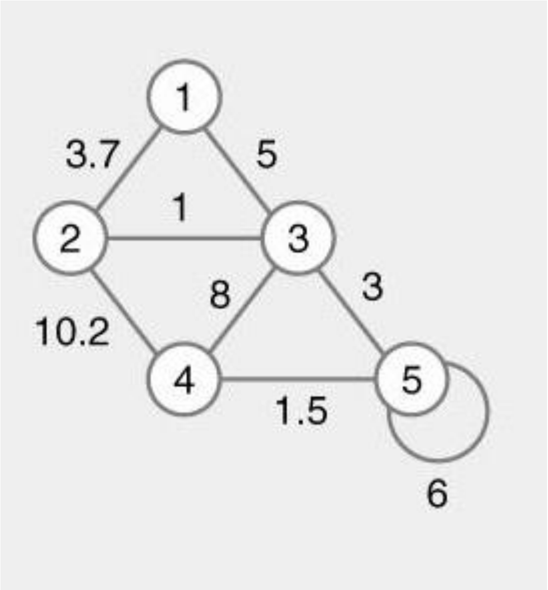
Adjacency List

- Example
  - Unweighted graph

```
node 1: {2, 3}
node 2: {1, 3, 4}
node 3: {1, 2, 4, 5}
node 4: {2, 3, 5}
node 5: {3, 4, 5}
```

  - Weighted graph

```
node 1: {2=3.7, 3=5}
node 2: {1=3.7, 3=1, 4=10.2}
node 3: {1=5, 2=1, 4=8, 5=3}
node 4: {2=10.2, 3=8, 5=1.5}
node 5: {3=3, 4=1.5, 5=6}
```



Adjacency Set / Map

- Representation
  - For each node, store
    - Set or map of neighbors / successors
  - For unweighted graph
    - Use set of neighbors
  - For weighted graph
    - Use map of neighbors, w/ value = weight of edge
  - For undirected graph with edge (a↔b)
    - Nodes a & b need to store each other as neighbor
  - For directed graph with edge (a→b)
    - Node a needs to store node b as neighbor

Graph Space Requirements

- Adjacency matrix
  - ½ N^2 entries (for graph with N nodes, E edges)
  - Many empty entries for large, sparse graphs
- Adjacency list
  - 2×E entries
- Adjacency set / map
  - 2×E entries
  - Space overhead per entry
    - Higher than for adjacency list

Graph Time Requirements

- Adjacency matrix
  - Can find individual edge (a,b) quickly
  - Examine entry in array Edge[a,b]
    - Constant time operation

- Adjacency list / set / map
  - Can find all edges for node (a) quickly
  - Iterate through collection of edges for a
    - On average E / N edges per node

Graph Time Requirements

- Average Complexity of operations
  - For graph with N nodes, E edges

Operation	Adj Matrix	Adj List	Adj Set/Map
Find edge	**O(1)**	**O(E/N)**	**O(1)**
Insert edge	**O(1)**	**O(E/N)**	**O(1)**
Delete edge	**O(1)**	**O(E/N)**	**O(1)**
Enumerate edges for node	**O(N)**	**O(E/N)**	**O(E/N)**

Choosing Graph Implementations

- Graph density
  - Ratio edges to nodes (dense vs. sparse)
- Graph algorithm
  - Neighbor based

```
For each node X in graph
    For each neighbor Y of X  // adj list faster if sparse
            doWork( )
```

  - Connection based

```
For each node X in …
    For each node Y in …
            if (X,Y) is an edge      // adj matrix faster if dense
              doWork( )
```