CMSC 132: OBJECT-ORIENTED PROGRAMMING II

Graphs & Graph Traversal

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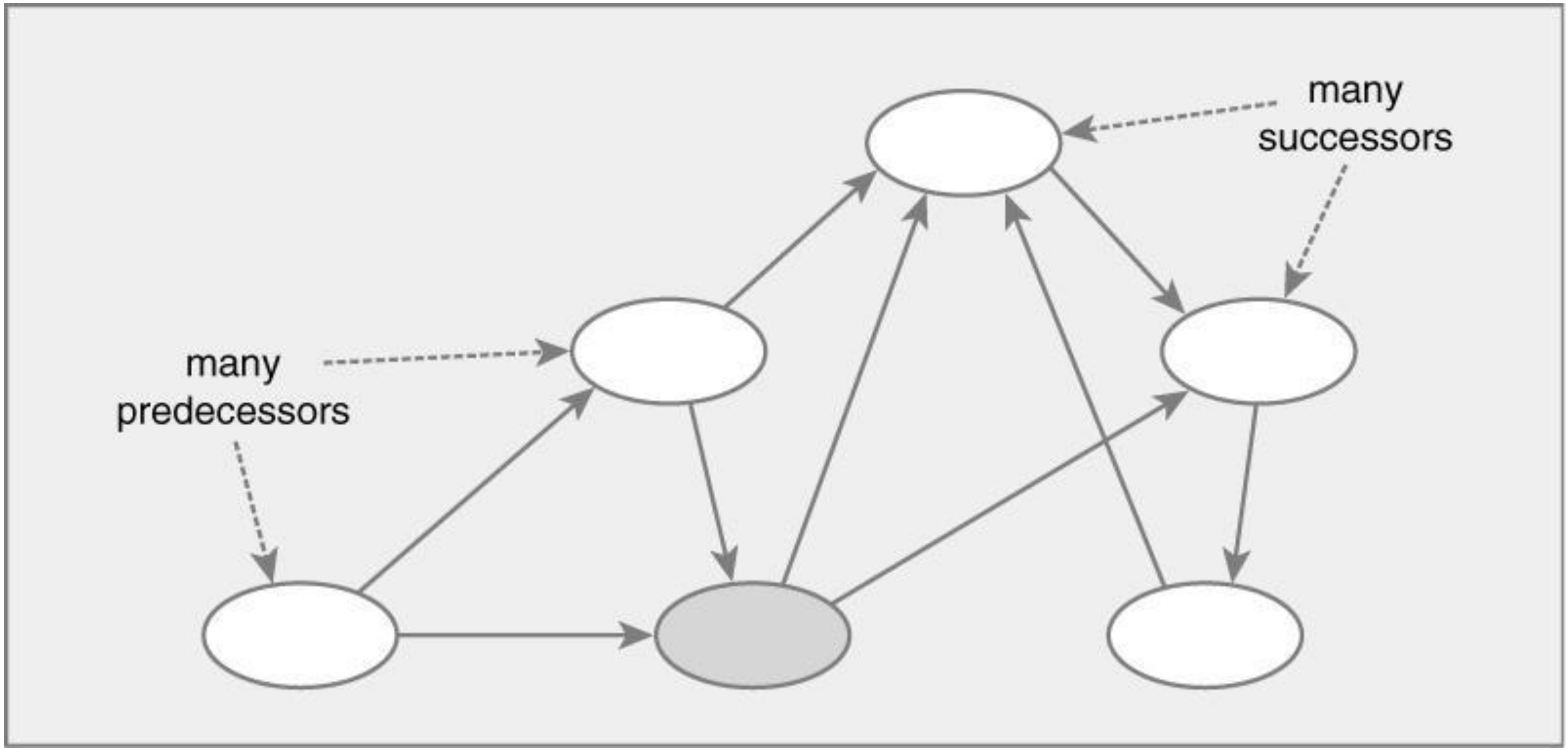
Graph Data Structures

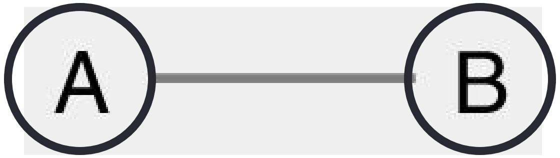
- Many-to-many relationship between elements
  - Each element has multiple predecessors
  - Each element has multiple successors



Graph Definitions

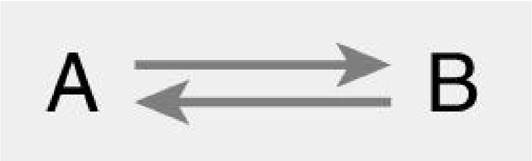
- Node
  - Element of graph
  - State
    - List of adjacent/neighbor/successor nodes

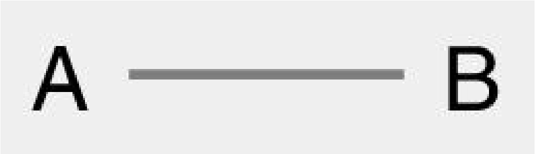
- Edge
  - Connection between two nodes
  - State
    - Endpoints of edge

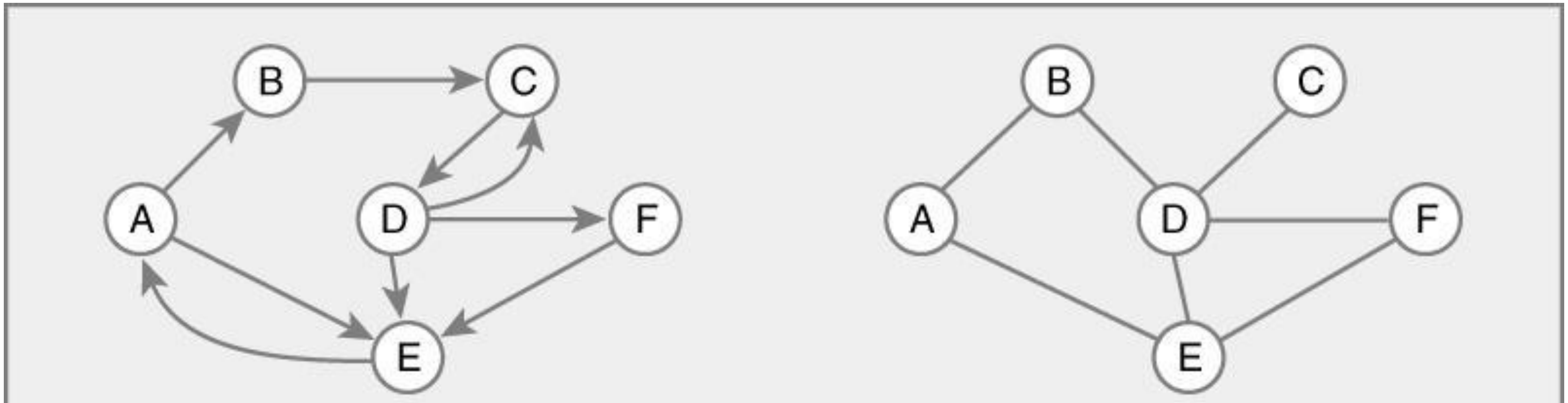


Graph Definitions

- Directed graph
  - Directed edges
- Undirected graph
  - Undirected edges





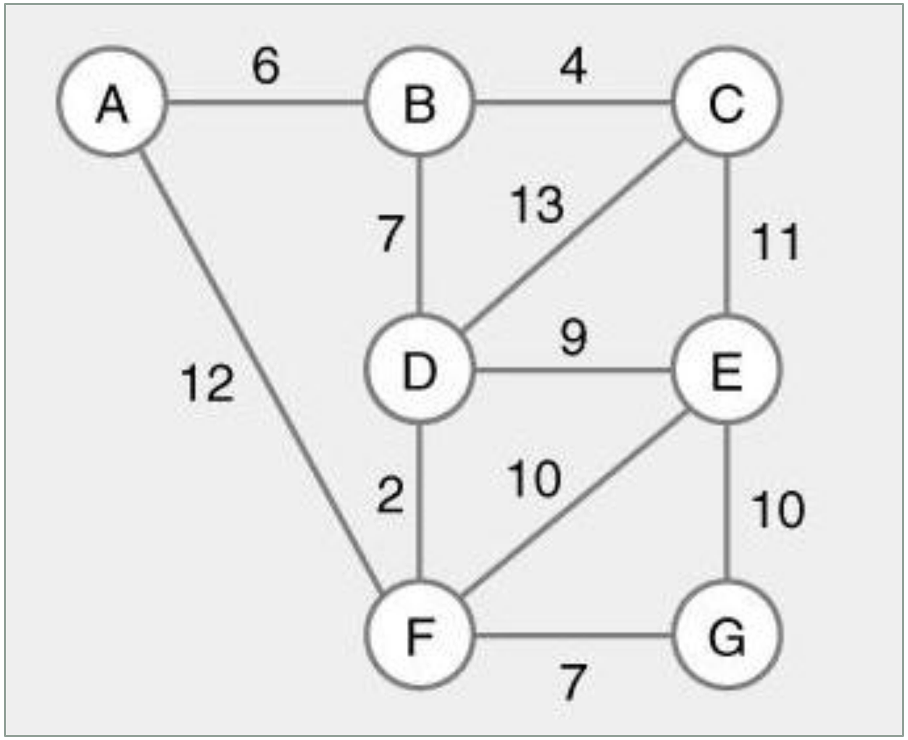


(a) Directed graph

(b) Undirected graph

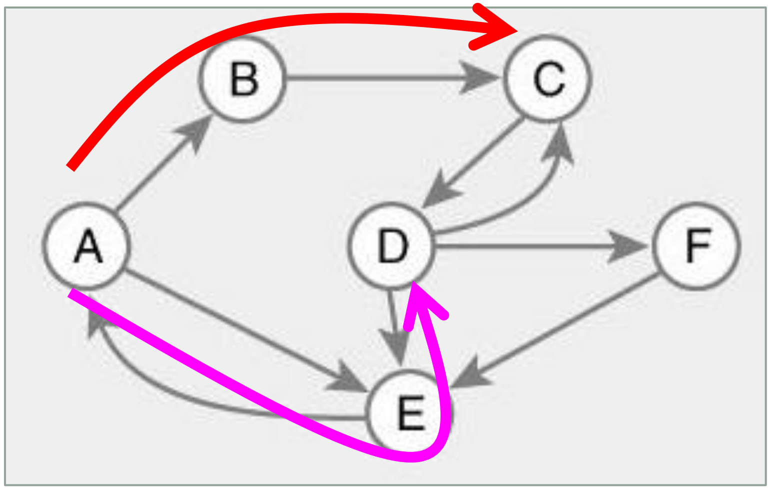
Graph Definitions

- Weighted graph
  - Weight (cost) associated with each edge



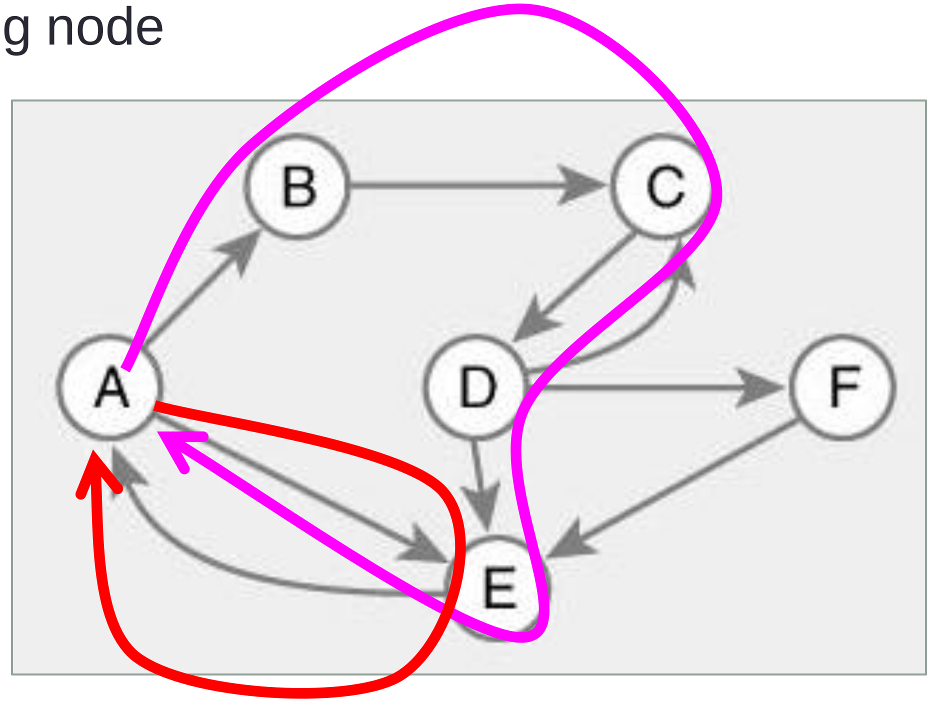
Graph Definitions

- Path
  - Sequence of nodes n_1, n_2, … n_k
  - Edge exists between each pair of nodes n_i , n_{i+1}
  - Example
    - A, B, C is a path
    - A, E, D is not a path



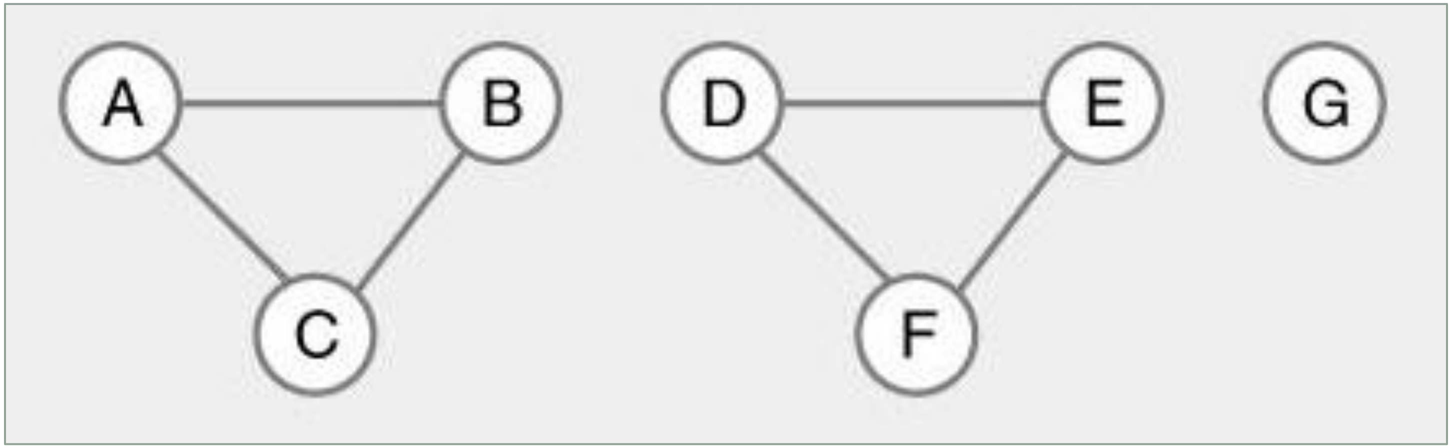
Graph Definitions

- Cycle
  - Path that ends back at starting node
  - Example
    - A, E, A
    - A, B, C, D, E, A
- Simple path
  - No cycles in path
- Acyclic graph
  - No cycles in graph



Graph Definitions

- Connected Graph
  - Every node in the graph is reachable from every other node in the graph
- Unconnected graph
  - Graph that has several disjoint components



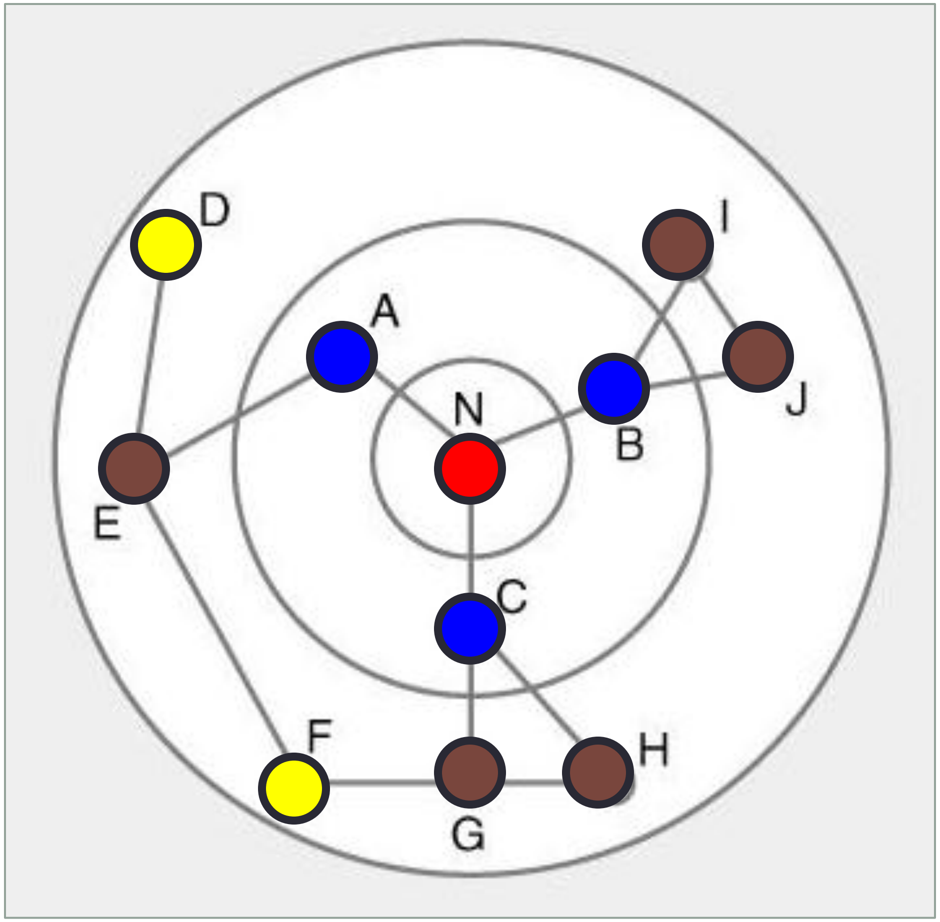
Unconnected graph

Graph Operations

- Traversal (search)
  - Visit each node in graph exactly once
  - Usually perform computation at each node
  - Two approaches
    - Breadth first search (BFS)
    - Depth first search (DFS)

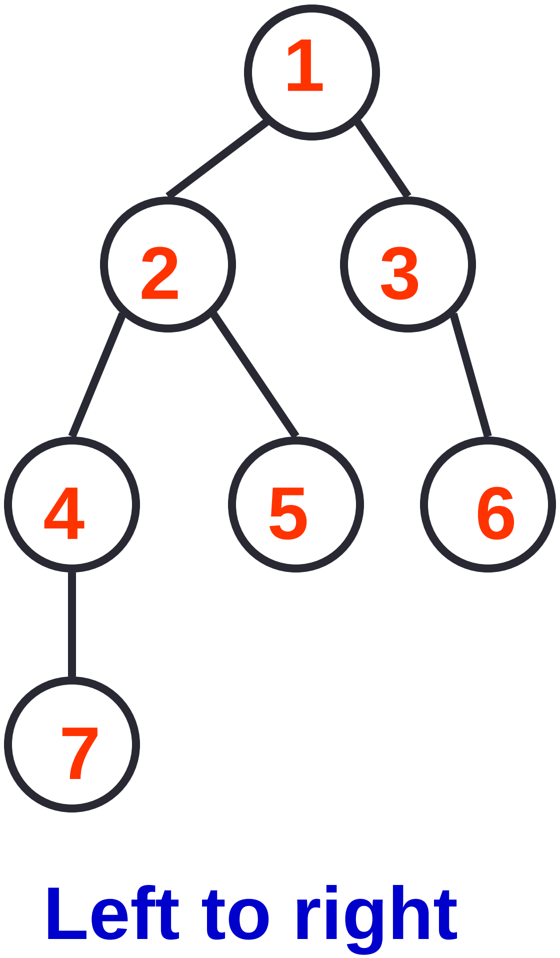
Breadth-first Search (BFS)

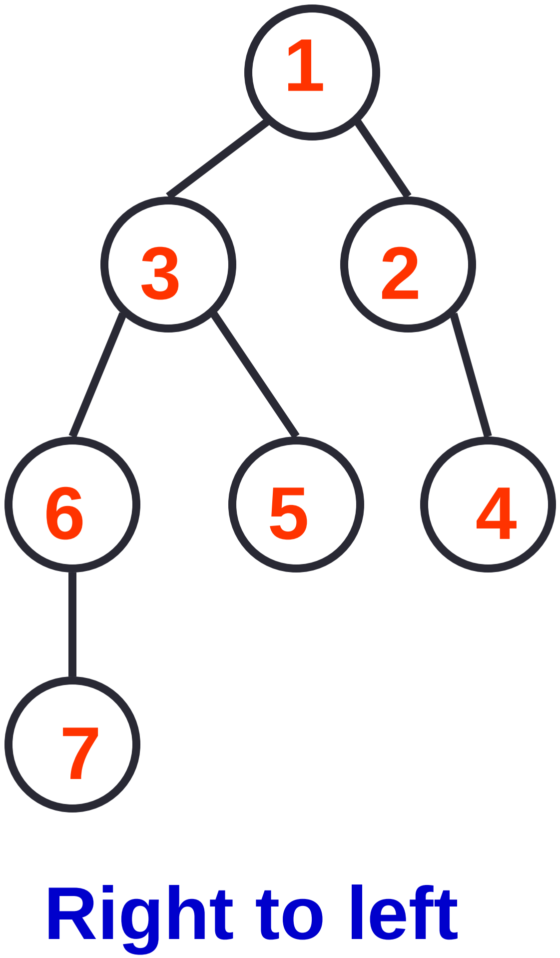
- Approach
  - Visit all neighbors of node first
  - View as series of expanding circles
  - Keep list of nodes to visit in queue
- Example traversal
  1. n
  2. a, c, b
  3. e, g, h, i, j
  4. d, f

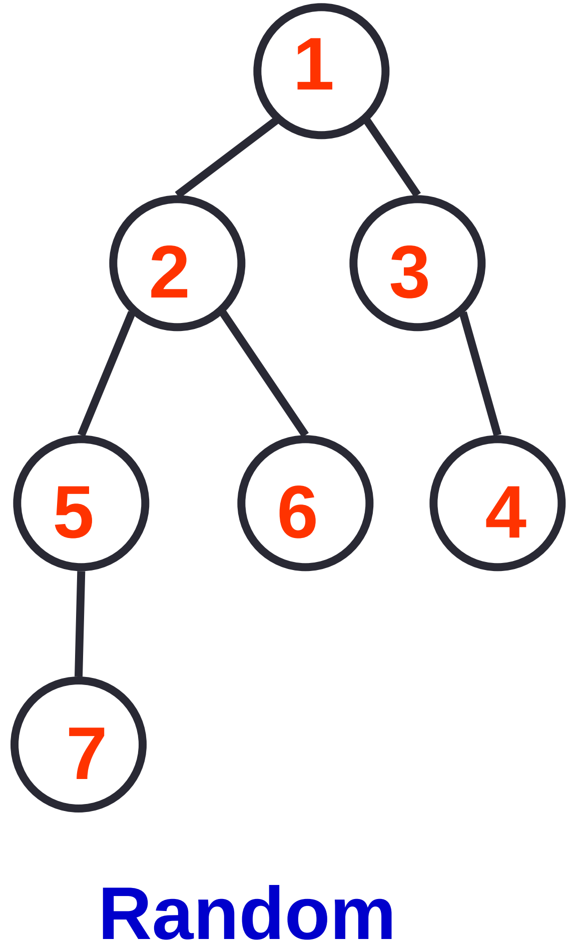


Breadth-first Tree Traversal

- Example traversals starting from 1





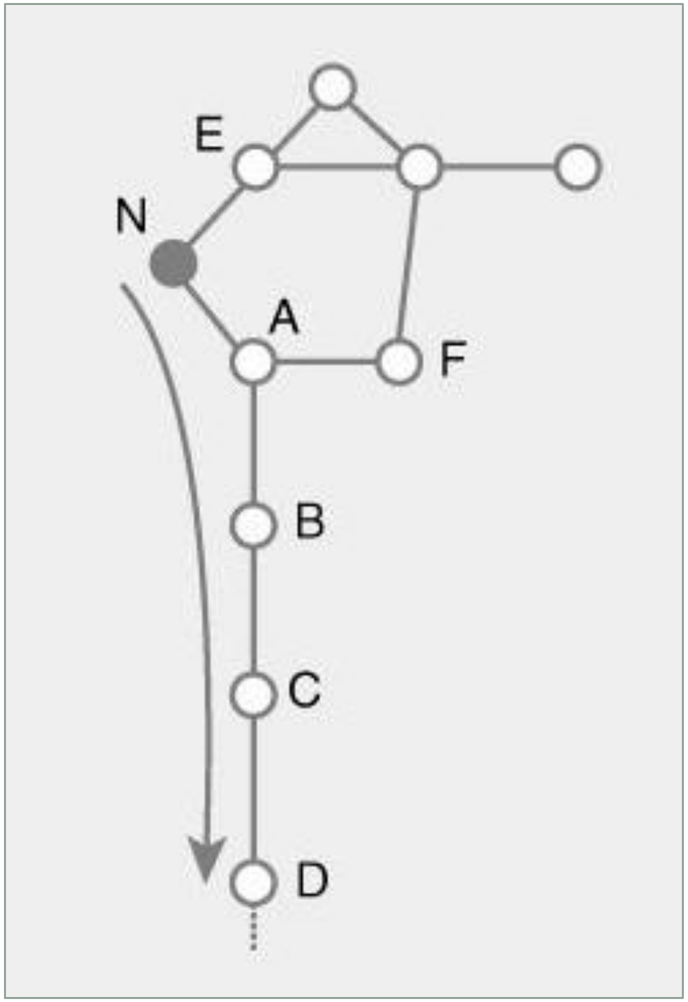


Traversals Orders

- Order of successors
  - For tree
    - Can order children nodes from left to right
  - For graph
    - Left to right doesn't make much sense
    - Each node just has a set of successors and predecessors; there is no order among edges
- For breadth first search
  - Visit all nodes at distance k from starting point
  - Before visiting any nodes at (minimum) distance k+1 from starting point

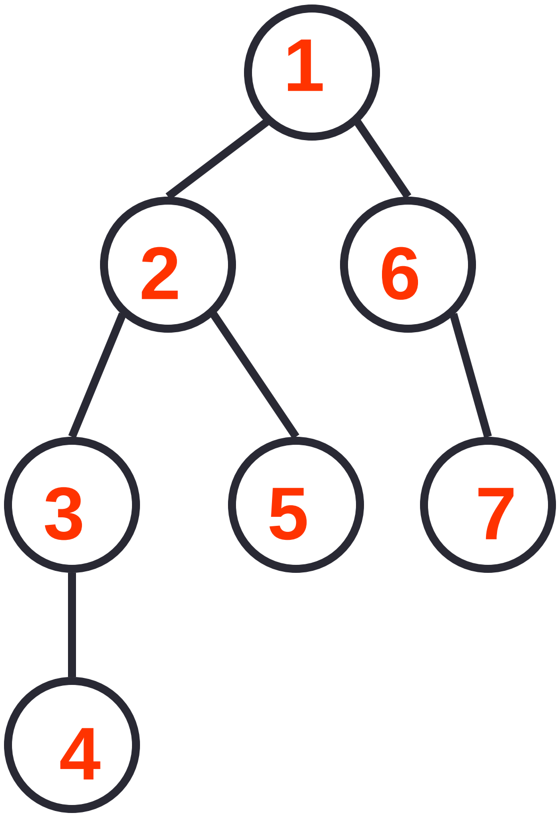
Depth-first Search (DFS)

- Approach
  - Visit all nodes on path first
  - Backtrack when path ends
  - Keep list of nodes to visit in a stack
- Example traversal
  1. N
  2. A
  3. B, C, D, …
  4. F…

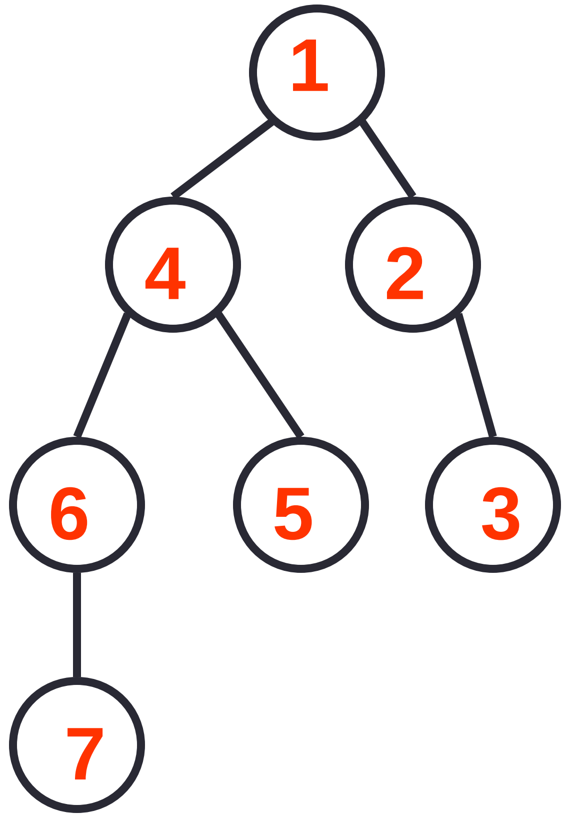


Depth-first Tree Traversal

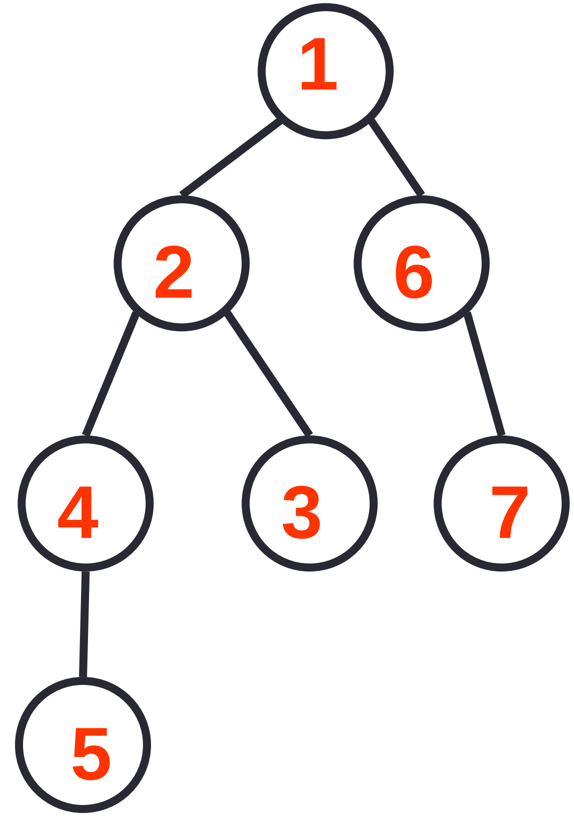
- Example traversals from 1 (preorder)



Left to right



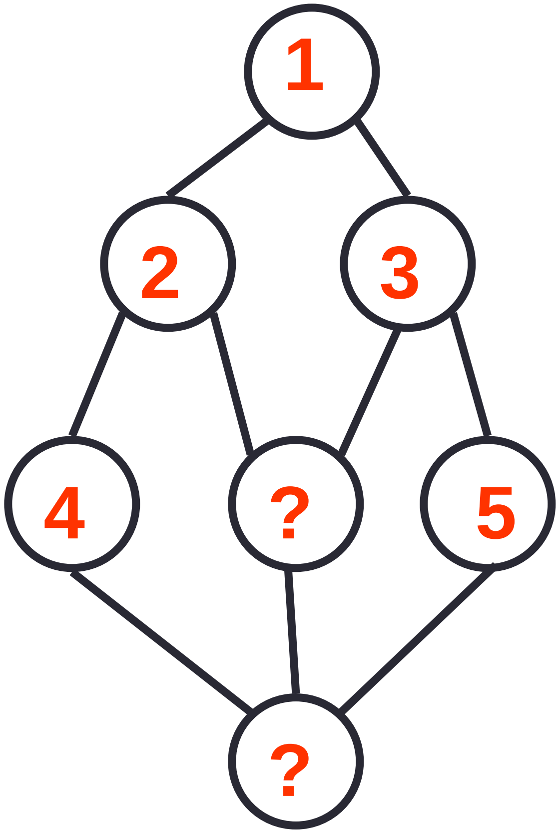
Right to left



Random

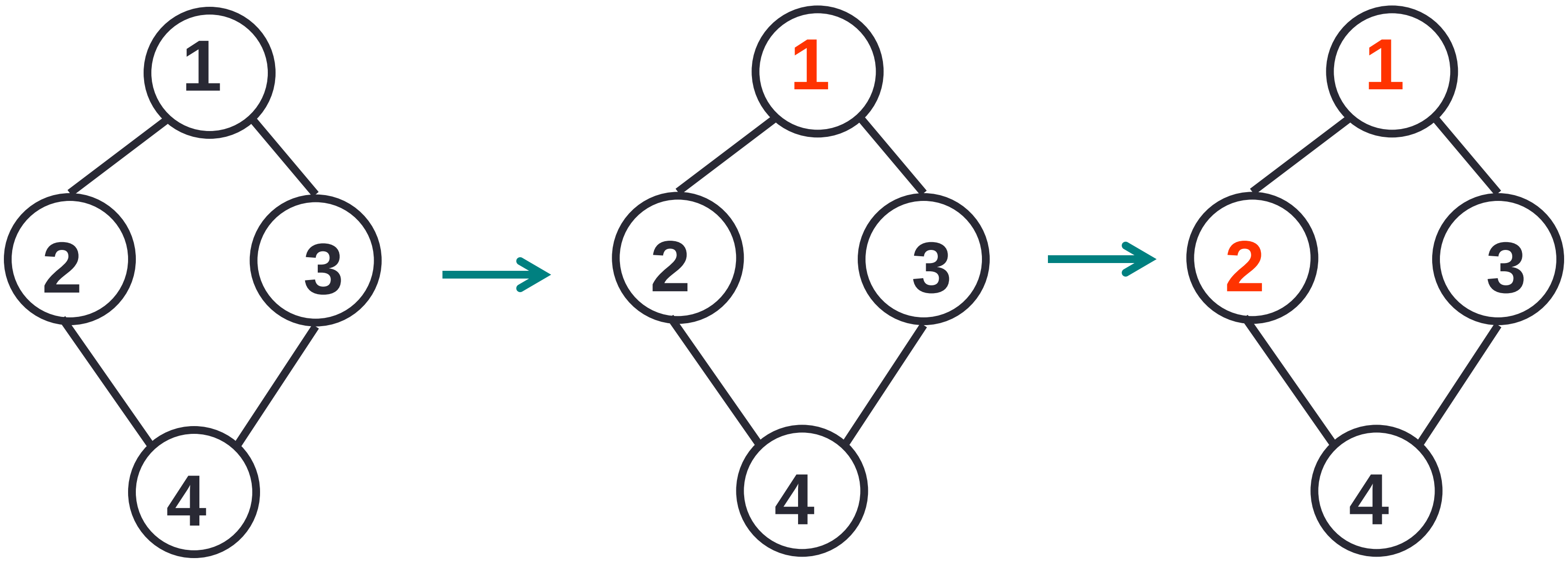
Traversal Algorithms

- Issue
  - How to avoid revisiting nodes
  - Infinite loop if cycles present
- Approaches
  - Record set of visited nodes
  - Mark nodes as visited



Traversal – Avoid Revisiting Nodes

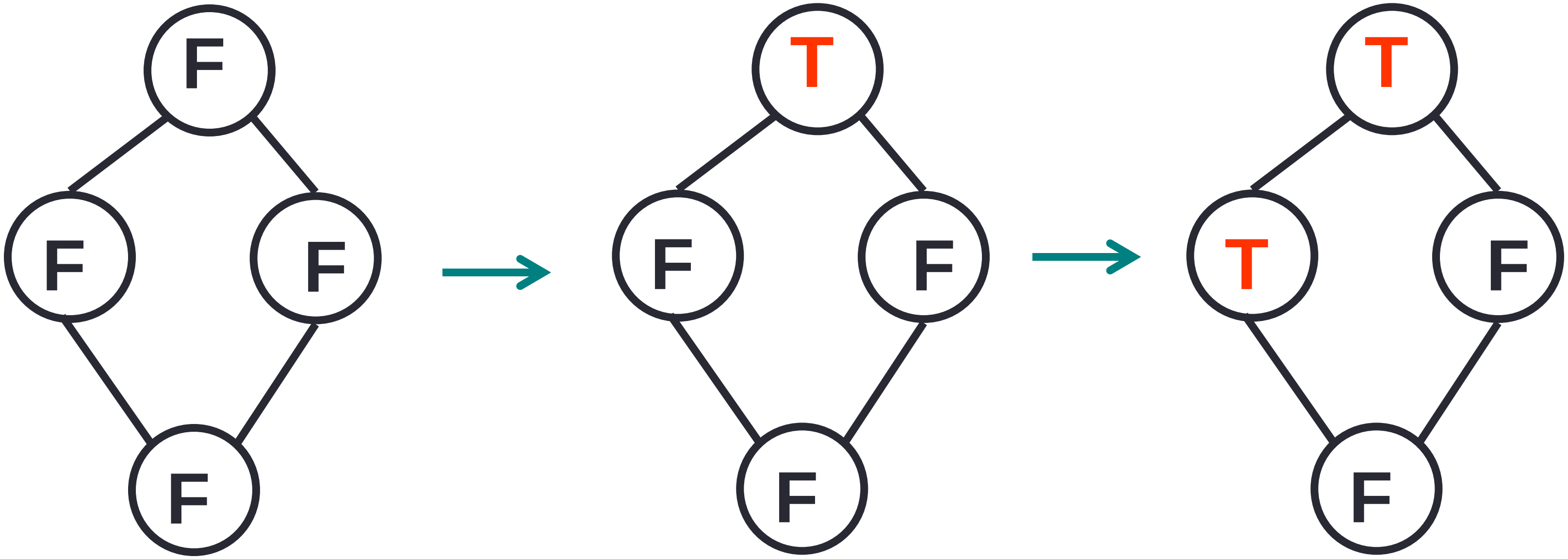
- Record set of visited nodes
  - Initialize { Visited } to empty set
  - Add to { Visited } as nodes is visited
  - Skip nodes already in { Visited }



V = ∅ **V = { 1 }** **V = { 1, 2 }**

Traversal – Avoid Revisiting Nodes

- Mark nodes as visited
  - Initialize tag on all nodes (to False)
  - Set tag (to True) as node is visited
  - Skip nodes with tag = True



Traversal Algorithm Using Sets

```
{ Visited } = ∅
{ Discovered } = { 1st node }
while ( { Discovered } ≠ ∅ )
    take node X out of { Discovered }
    if X not in { Visited }
        add X to { Visited }
        for each successor Y of X
            if ( Y is not in { Visited } )
                add Y to { Discovered }
```

Traversal Algorithm Using Tags

```
for all nodes X
      set X.tag = False
{ Discovered } = { 1st node }
while ( { Discovered } ≠ ∅ )
     take node X out of { Discovered }
     if (X.tag = False)
          set X.tag = True
          for each successor Y of X
               if (Y.tag = False)
                    add Y to { Discovered }
```

BFS vs. DFS Traversal

- Order nodes taken out of { Discovered } key
- Implement { Discovered } as Queue
  - First in, first out
  - Traverse nodes breadth first
- Implement { Discovered } as Stack
  - First in, last out
  - Traverse nodes depth first

BFS Traversal Algorithm

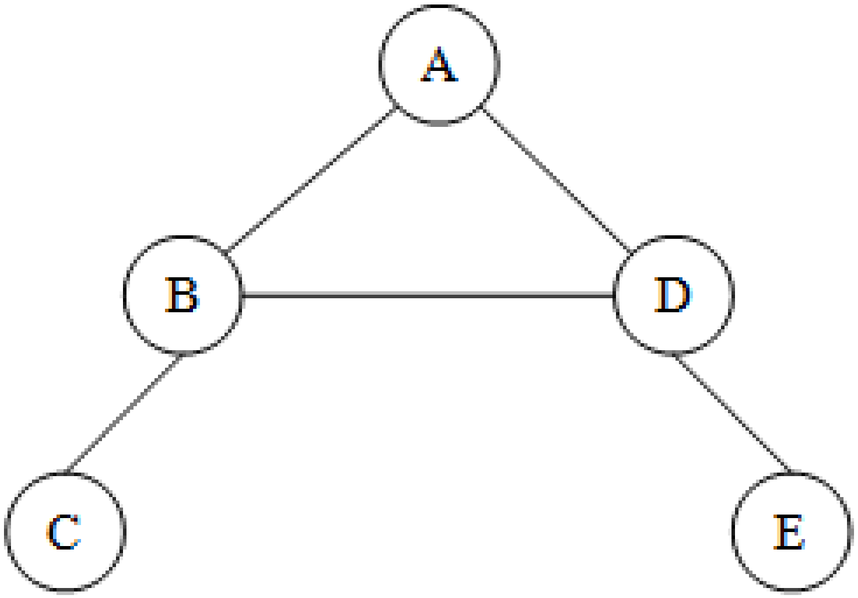
```
for all nodes X
     X.tag = False
put 1st node in Queue
while ( Queue not empty )
     take node X out of Queue
     if (X.tag = False)
          set X.tag = True
          for each successor Y of X
               if (Y.tag = False)
                    put Y in Queue
```

DFS Traversal Algorithm

```
for all nodes X
     X.tag = False
put 1st node in Stack
while (Stack not empty )
    pop X off Stack
    if (X.tag = False)
        set X.tag = True
        for each successor Y of X
            if (Y.tag = False)
                push Y onto Stack
```

Example

- Let’s do a BFS/DFS using the following graph (start vertex A)



Recursive Graph Traversal

- Can traverse graph using recursive algorithm
  - Recursively visit successors

- Approach

```
Visit ( X )
    for each successor Y of X
        Visit ( Y )
```

- Implicit call stack & backtracking
  - Results in depth-first traversal

Recursive DFS Algorithm

```
Traverse( )
    for all nodes X
        set X.tag = False
    Visit ( 1st node )
Visit ( X )
    set X.tag = True
    for each successor Y of X
        if (Y.tag = False)
            Visit ( Y )
```