On the Science Behind Computing (Part I)

Samir Khuller *

1 Mathematical Induction

Induction is a rather elegant way of proving a lot of mathematical theorems, and in addition, it is a key ingredient to establishing the correctness of many algorithms so its useful to know the basic principle behind induction.

Say we want to prove something like the following: For all positive integers N

$$\sum_{i=1}^{N} i = \frac{N(N+1)}{2}$$

If we are not as clever as Gauss (this is a funny story), then how can we prove something like this? One option is to at least check for small values of N that this claim is true. For example, for $N = 1$ the left side is just 1. The right side is $\frac{1 \cdot 2}{2} = 1$. So at least we know that this claim holds for $N = 1$. However this does not mean it will hold for any integer N. We can easily make up general forumlas that only hold for specific values (for example $2N - 1 = N$ for $N = 1$, but does not hold in general).

We can also check it for $N = 2$ quite easily since both sides are exactly 3. How do we confirm that it is true for any integer N? The secret lies in proving the following: suppose it holds for $N = k$, then we shall prove that it hold for $N = (k + 1)$. Now once we do that, if we can prove it hold for $N = 1$ then it follows that it holds for $N = 2$. But if it holds for $N = 2$, then it must hold for $N = 3$ etc. In this way we can see that it holds for all integers N.

Suppose it hold for $N = k$. Then $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$. We need to prove that $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$. But the left hand side is $\sum_{i=1}^{k} i + (k+1)$. Since we can assume that the sum of the first k integers is $\frac{k(k+1)}{2}$, we get $\frac{k(k+1)}{2} + (k + 1) = \frac{(k+1)(k+2)}{2}$. Simply adding gives the answer (the right hand side).

Note that here we were given the formula that we needed to establish. So induction is an easy way to confirm something, if we know what we are looking for. If we do not know the formula, then it takes some time to come up with the "correct" guess – and this takes years of practice. Verifying that a given algorithm works is a lot easier than coming up with the algorithm – its a similar situation.

*Department of Computer Science, University of Maryland, College Park, MD 20742. E-mail : samir@cs.umd.edu.