Spring 2012 **CMSC 858T: Homework 1** Samir Khuller

Due in class: Due date: Feb 22.

For this homework, assume that bipartite weighted matching and network flow can be solved in polynomial time. If you are not familiar with these concepts then come and talk to me!

(1) We have a periodic scheduling problem. There are n tasks. Each task takes one unit of time to perform. The requirement is that task i should be scheduled once in each time period p_i.

For example if task i, has period p_i. We must schedule it in each time-window of the form $[p_i \cdot j + 1, \ldots, p_i \cdot (j + 1)]$ for $j = 0....$ Let $L = lcm(p_1, p_2, \ldots, p_n)$. Moreover suppose that $\sum_i \frac{1}{p_i} \leq 1$. Show how to find a periodic schedule for the first L time slots. If the above condition is not satisfied, there is no periodic schedule. Why?

(2) Prove that we can greedily schedule n jobs on m identical machines and get an approximation factor of $\frac{4}{3}$ for the makespan. The jobs are scheduled in decreasing size order. Moreover, is this bound tight? In other words, is there an example where the cost of the schedule can be as bad as $\frac{4}{3}$ times the optimal solution cost?

(3) Consider the problem of scheduling n jobs on m unrelated machines. Suppose we simply wish to minimize the total (sum) of the completion times of the jobs, how can we do this in polynomial time?

(4) The rank of job j is defined as $\frac{p_j}{w_j}$ where p_j is the processing time of the job and w_j is its weight. Prove that to minimize the weighted completion time of a set of n jobs on one machine, we should order them by non-decreasing rank order.