

CMSC 132:

OBJECT-ORIENTED PROGRAMMING II



Advanced Tree Structures

Department of Computer Science
University of Maryland, College Park

IMPORTANT

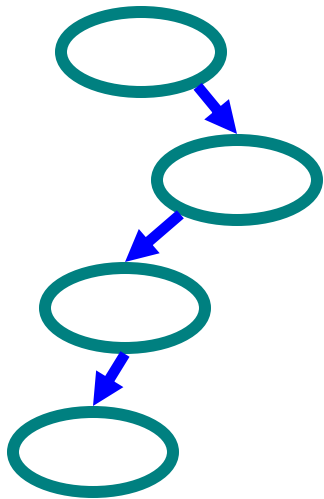
- Make sure you check your e-mails every day and the messages we post on the class announcements. It is your responsibility to check them so you are aware of important information/deadlines.
- Final exam information is available on the class web page. Please complete course evaluations 😊
 - <https://courseevalum.umd.edu/>
- Double-check all your scores in grades server are correct
- Save your projects (CVS repository will disappear after class is over)

Overview

- Binary trees
 - Balance
 - Rotation
- Multi-way trees
 - Search
 - Insert
- Indexed tries

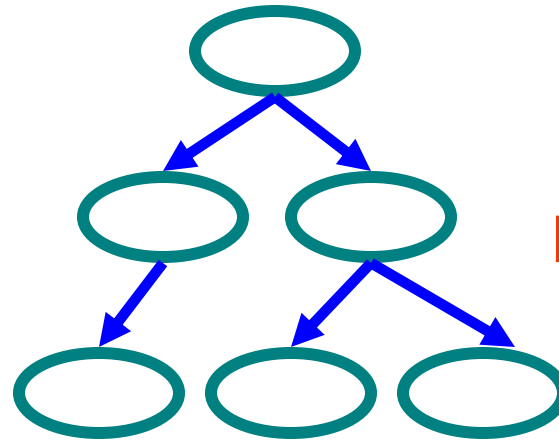
Tree Balance

- Degenerate
 - Worst case
 - Search in $O(n)$ time



**Degenerate
binary tree**

- Balanced
 - Average case
 - Search in $O(\log(n))$ time



**Balanced
binary tree**

Tree Balance

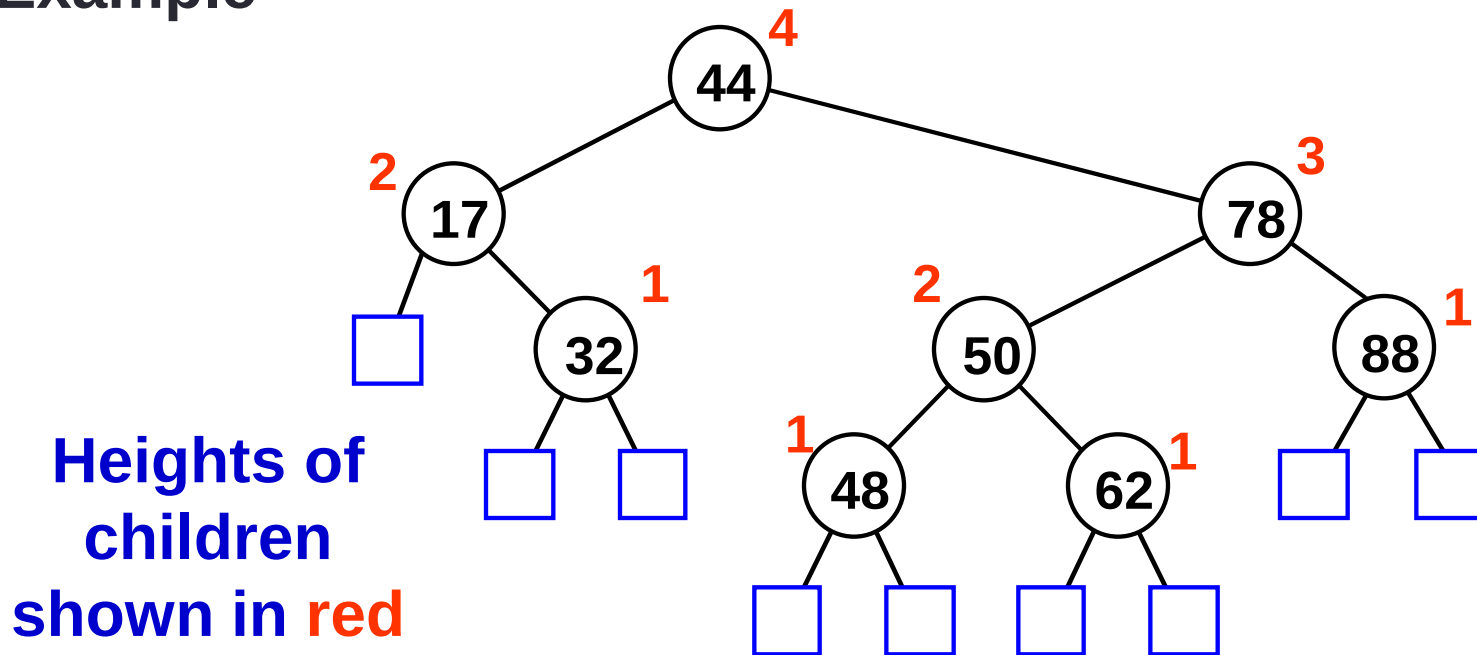
- **Question**
 - Can we keep tree (mostly) balanced?
- **Self-balancing binary search trees**
 - AVL trees
 - Red-black trees
- **Approach**
 - Select invariant (that keeps tree balanced)
 - Fix tree after each insertion / deletion
 - Maintain invariant using **rotations**
 - Provides operations with $O(\log(n))$ worst case

AVL Trees

- **Properties**

- Binary search tree
- Heights of children for node **differ by at most 1**

- **Example**



AVL Trees

- **History**

- Discovered in 1962 by two Russian mathematicians, Adelson-Velskii & Landis

- **Algorithm**

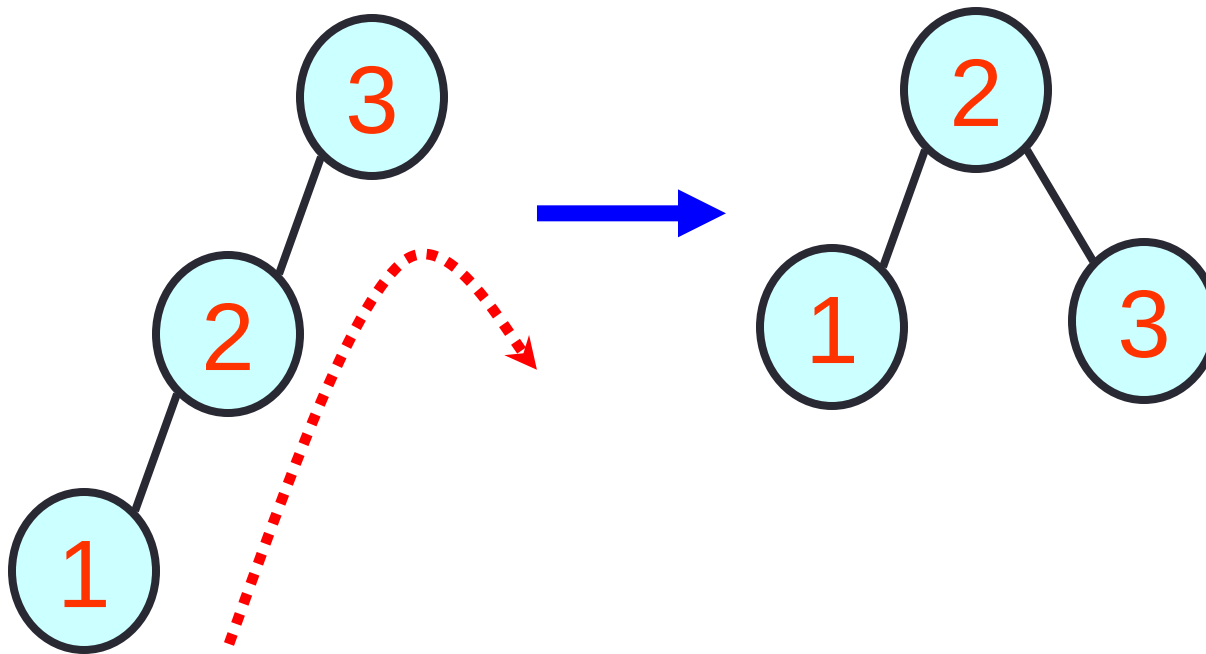
1. Find / insert / delete as a binary search tree
2. After each insertion / deletion
 1. If height of children differ by more than 1
 2. **Rotate** children until subtrees are balanced
 3. Repeat check for parent (until root reached)

Tree Rotations

- **Changes shape of tree**
 - Rotation moves one node up in the tree and one node down
 - Height is decreased by moving larger sub-trees up and smaller sub-trees down
- **Types**
 - Single rotation
 - Left
 - Right
 - Double rotation
 - Left-right
 - Right-left

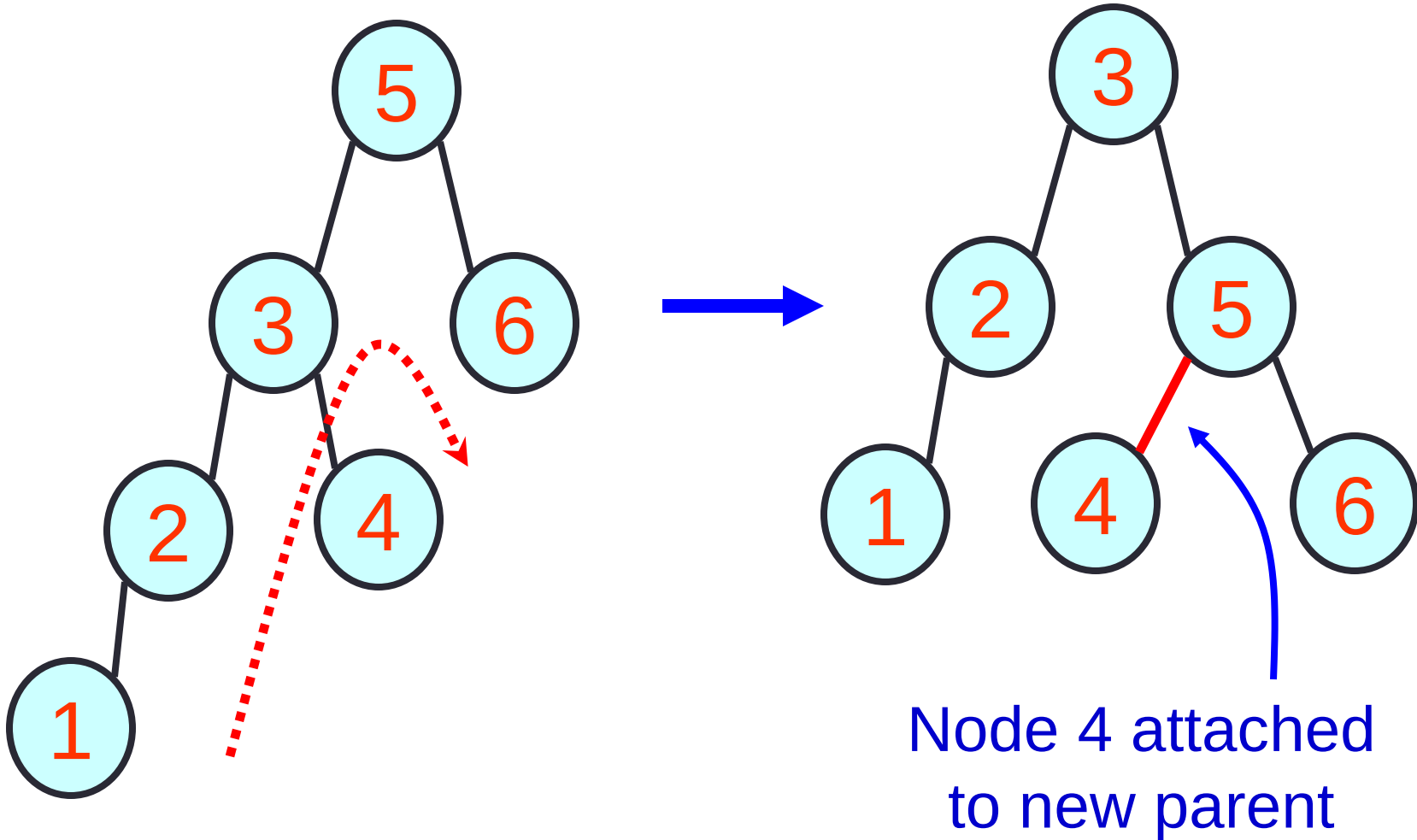
Tree Rotation Example

- Single right rotation



Tree Rotation Example

- Single right rotation

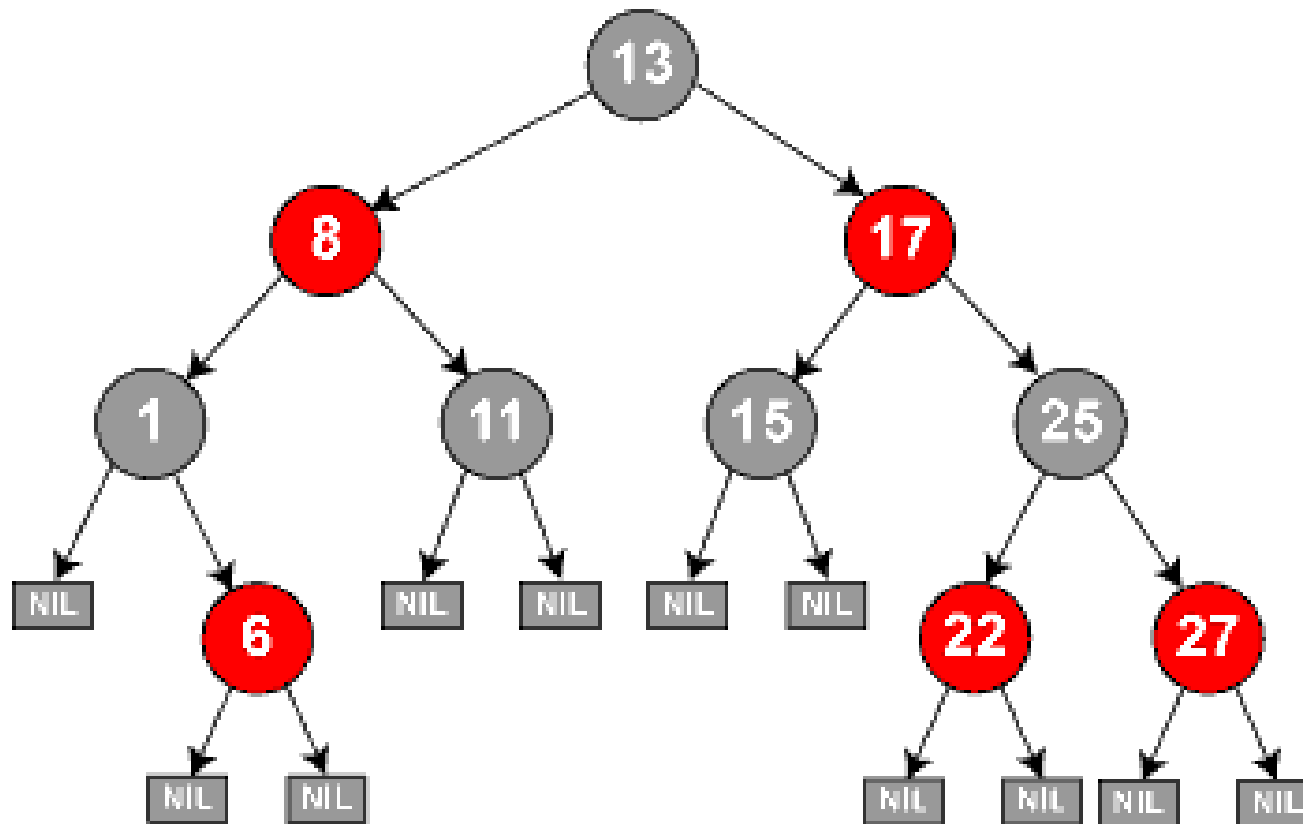


Red-black Trees

- **History**
 - Discovered in 1972 by Rudolf Bayer
- **Algorithm**
 - Insert / delete may require complicated bookkeeping & rotations
- **Java collections**
 - TreeMap and TreeSet use red-black trees
- **Properties**
 - Binary search tree
 - Every node is red or black
 - The root is black
 - Every leaf is black
 - All children of red nodes are black
 - For each leaf, same # of black nodes on path to root
- **Characteristics**
 - Properties ensures no leaf is twice as far from root as another leaf

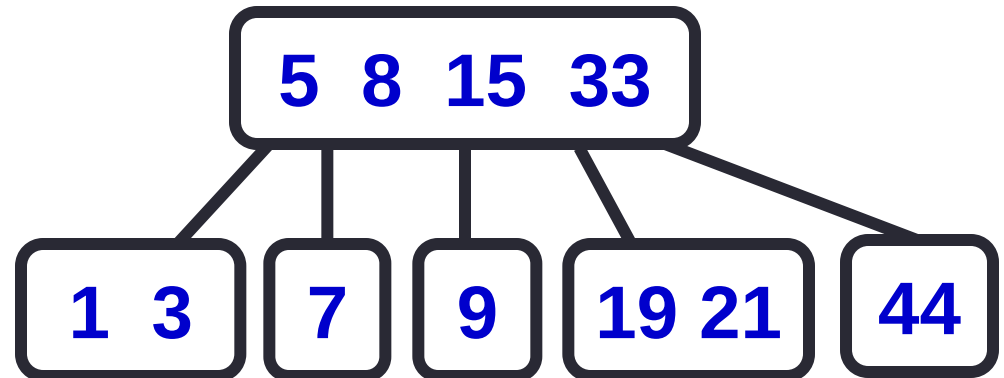
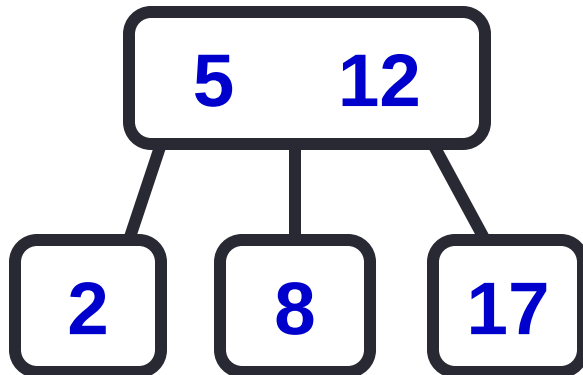
Red-black Trees

- Example



Multi-way Search Trees

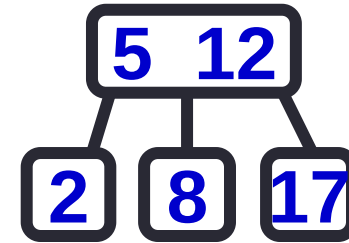
- **Properties**
 - Generalization of binary search tree
 - Node contains $1 \dots k$ keys (in sorted order)
 - Node contains $2 \dots k+1$ children
 - Keys in j^{th} child $< j^{\text{th}}$ key $<$ keys in $(j+1)^{\text{th}}$ child
- **Examples**



Types of Multi-way Search Trees

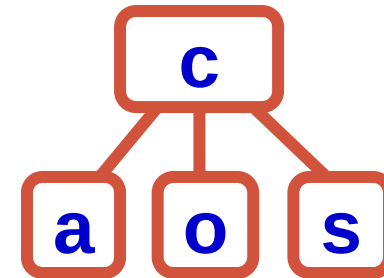
- **2-3 Tree**

- Internal nodes have 2 or 3 children



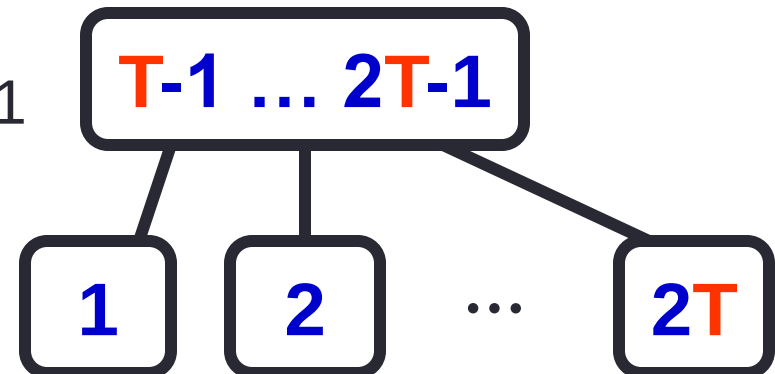
- **Indexed Search Tree (trie)**

- Internal nodes have up to 26 children (for strings)



- **B-Tree**

- T = minimum degree
- Non-root internal nodes have $T-1$ to $2T-1$ children
- All leaves have same depth



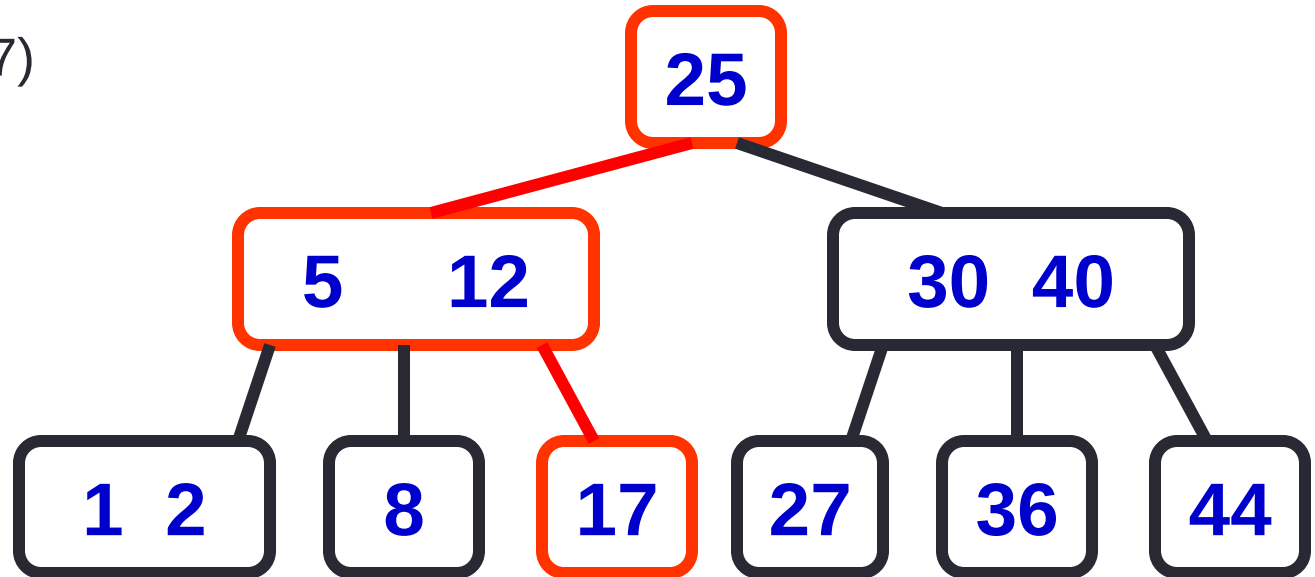
Multi-way Search Trees

- **Search algorithm**

1. Compare key x to $1 \dots k$ keys in node
2. If $x = \text{some key}$ then return node
3. Else if ($x < \text{key } j$) search child j
4. Else if ($x > \text{all keys}$) search child $k+1$

- **Example**

- Search(17)

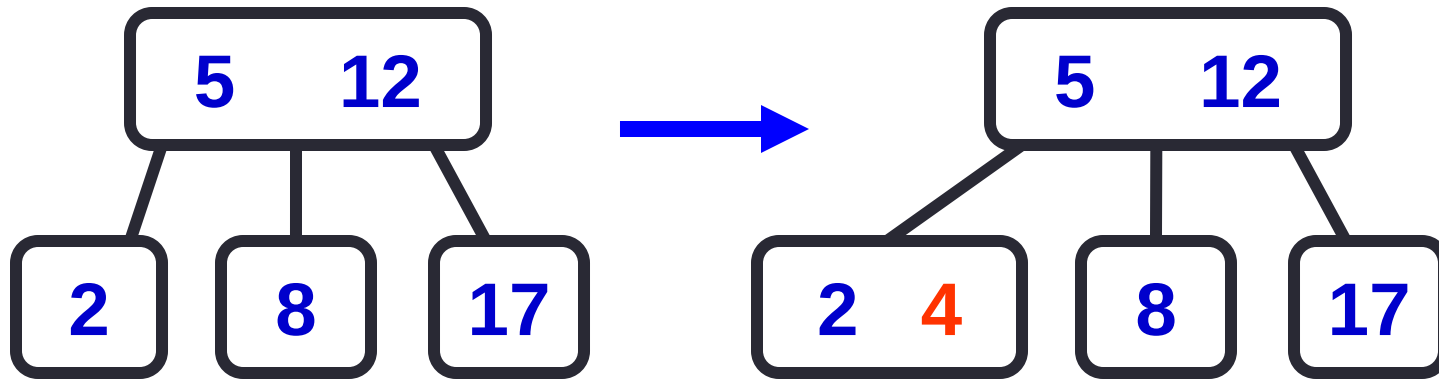


Multi-way Search Trees

- Insert algorithm
 1. Search key x to find node n
 2. If (n not full) insert x in n
 3. Else if (n is full)
 - a) Split n into two nodes
 - b) Move middle key from n to n 's parent
 - c) Insert x in n
 - d) Recursively split n 's parent(s) if necessary

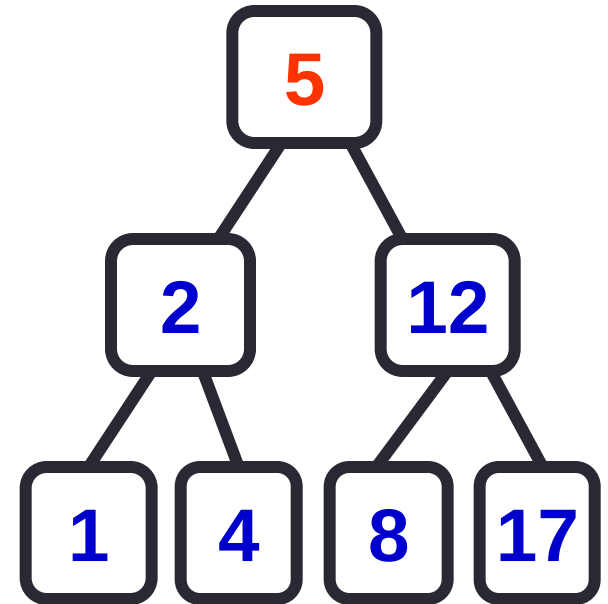
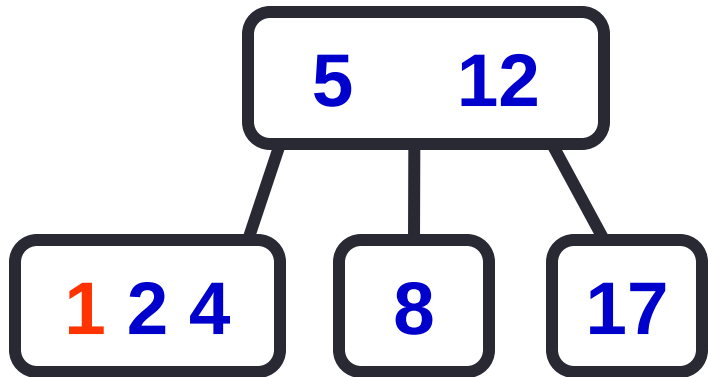
Multi-way Search Trees

- Insert Example (for 2-3 tree)
 - Insert(4)

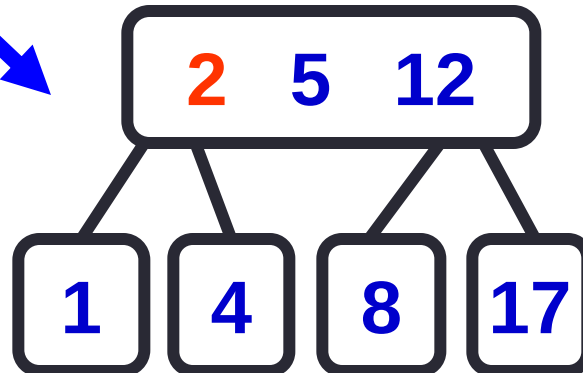


Multi-way Search Trees

- Insert Example (for 2-3 tree)
 - Insert(1)



Split node



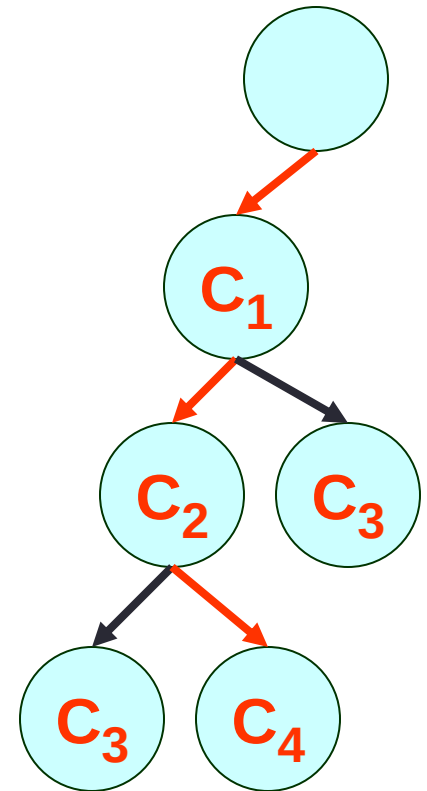
Split parent

B-Trees

- **Characteristics**
 - Height of tree is $O(\log_T(n))$
 - Reduces number of nodes accessed
 - Wasted space for non-full nodes
- **Popular for large databases (indices)**
 - 1 node = 1 disk block
 - Reduces number of disk blocks read

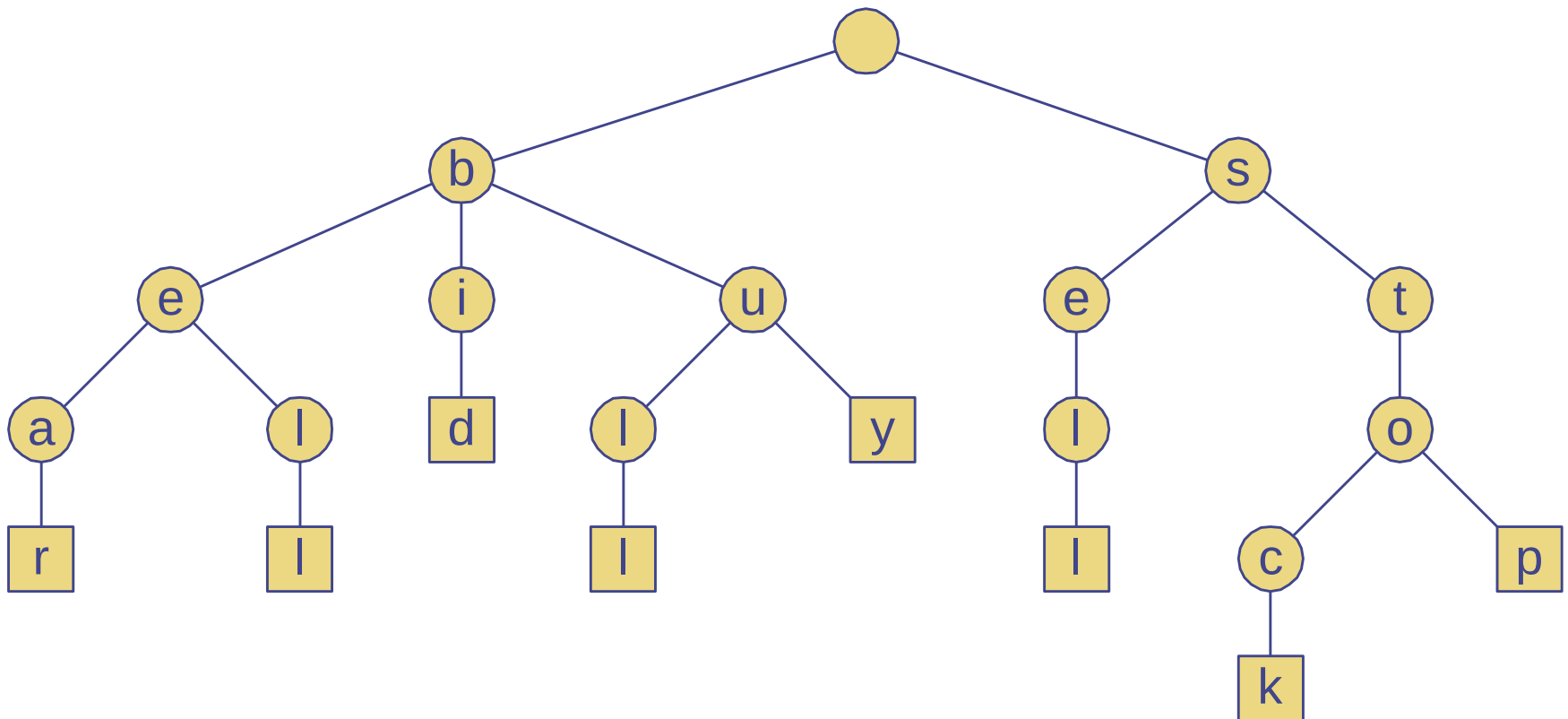
Indexed Search Tree (Trie)

- Special case of tree
- Applicable when
 - Key **C** can be decomposed into a sequence of subkeys $C_1, C_2, \dots C_n$
 - Redundancy exists between subkeys
- Approach
 - Store subkey at each node
 - Path through trie yields full key



Standard Trie Example

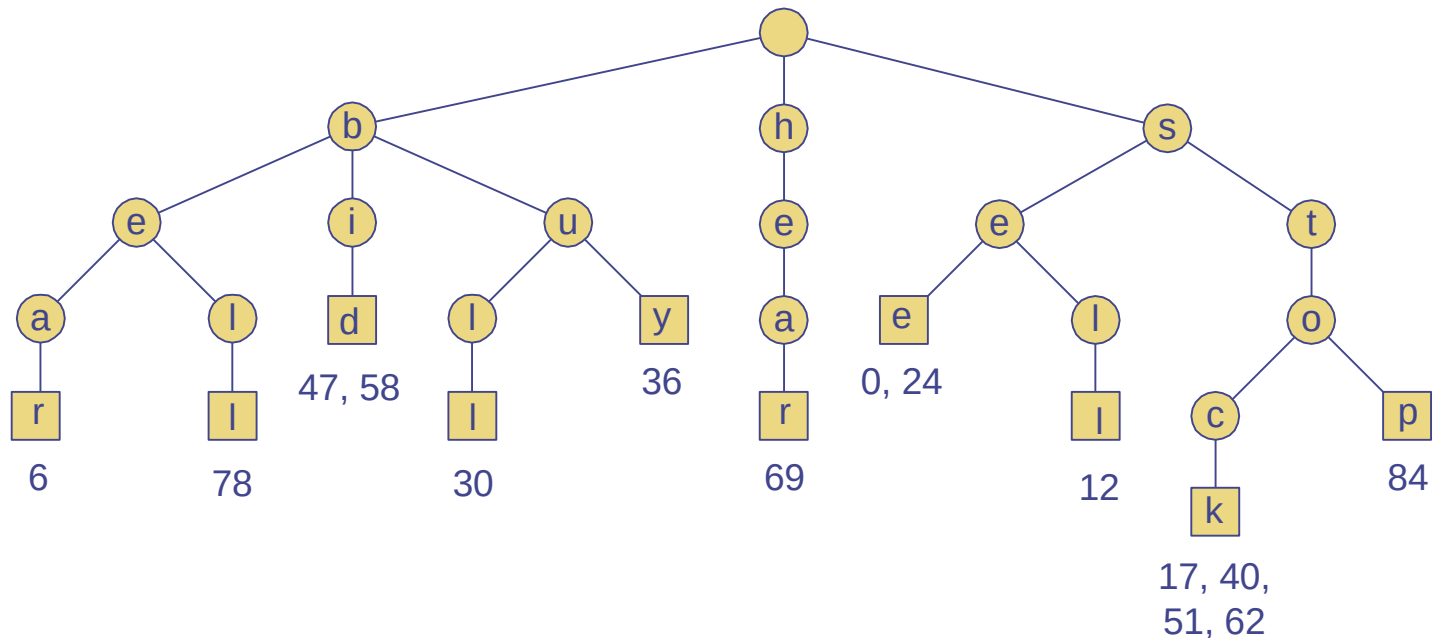
- For strings
 - { bear, bell, bid, bull, buy, sell, stock, stop }



Word Matching Trie

- Insert words into trie
- Each leaf stores occurrences of word in the text

s	e	e		a		b	e	a	r	?		s	e	l	l		s	t	o	c	k	!		
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	
s	e	e		a		b	u	l	l	?		b	u	y			s	t	o	c	k	!		
24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46		
b	i	d		s	t	o	c	k	!		b	i	d		s	t	o	c	k	!				
47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68			
h	e	a	r		t	h	e		b	e	l	l	?		s	t	o	p	!					
69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88					



Compressed Trie

- **Observation**

- Internal node v of T is redundant if v has one child and is not the root

- **Approach**

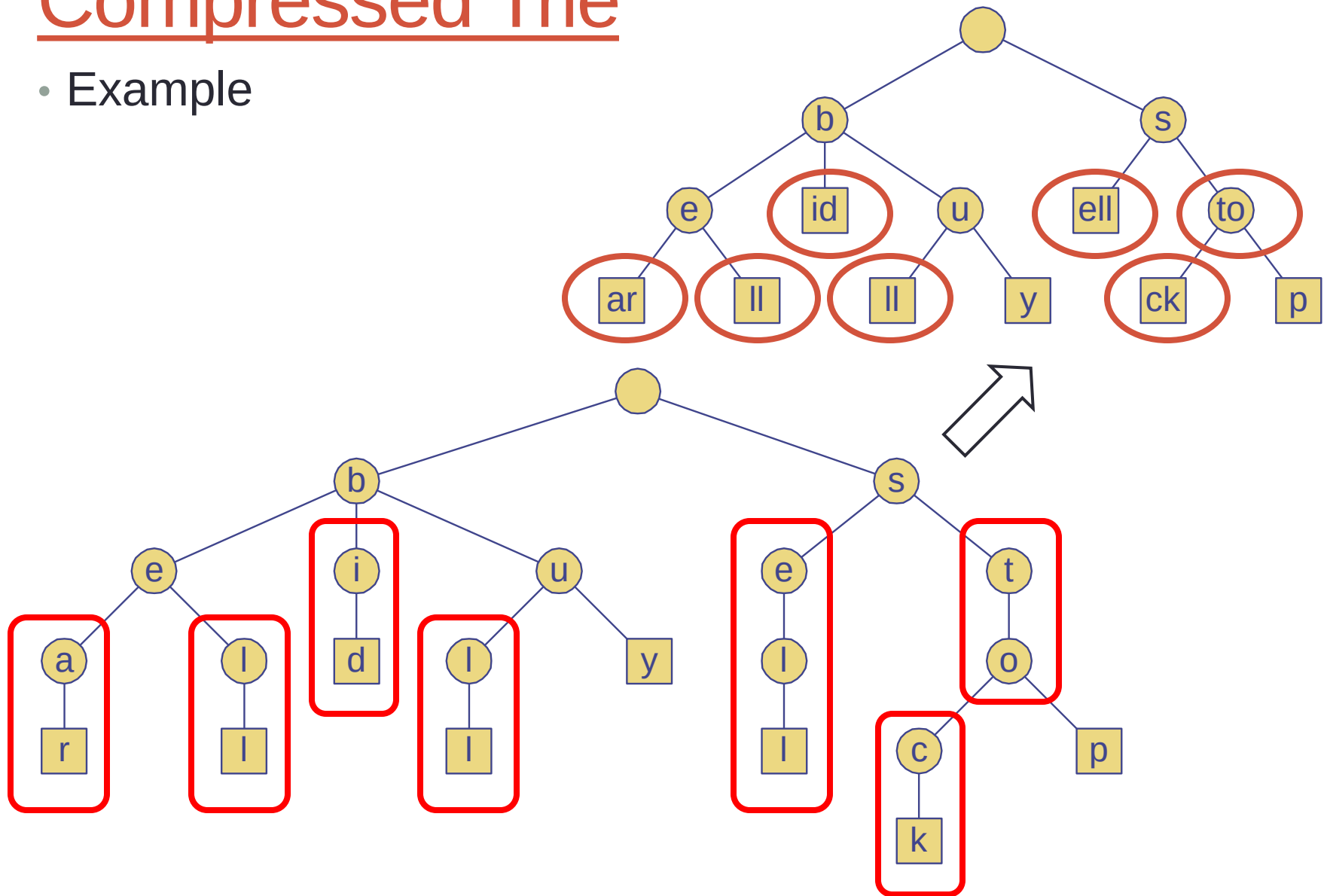
- A chain of redundant nodes can be compressed
 - Replace chain with single node
 - Include concatenation of labels from chain

- **Result**

- Internal nodes have at least 2 children
- Some nodes have multiple characters

Compressed Trie

- Example



Tries and Web Search Engines

- **Search engine index**
 - Collection of all searchable words
 - Stored in compressed trie
- **Each leaf of trie**
 - Associated with a word
 - List of pages (URLs) containing that word
 - Called occurrence list
- **Trie is kept in memory (fast)**
- **Occurrence lists kept in external memory**
 - Ranked by relevance