CMSC 132:
OBJECT-ORIENTED PROGRAMMING II

Trees & Binary Search Trees

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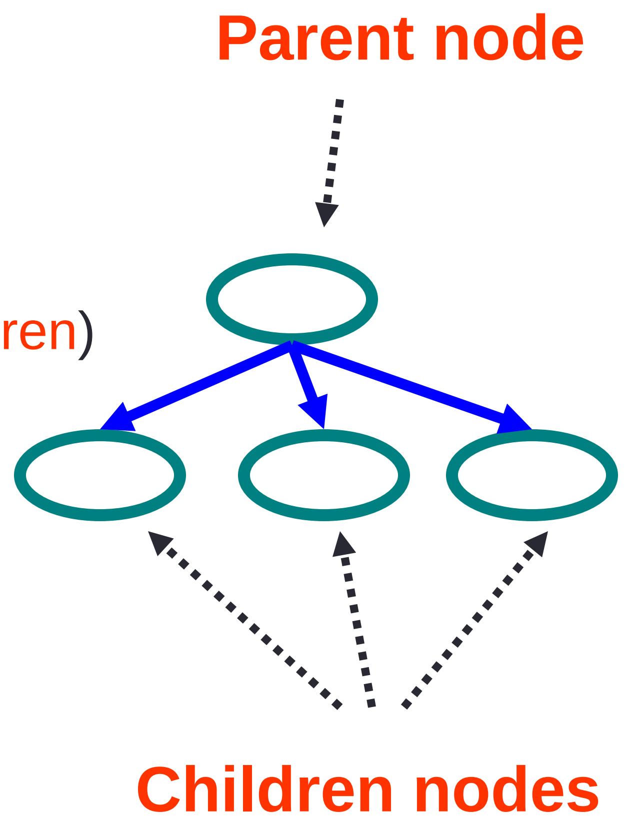
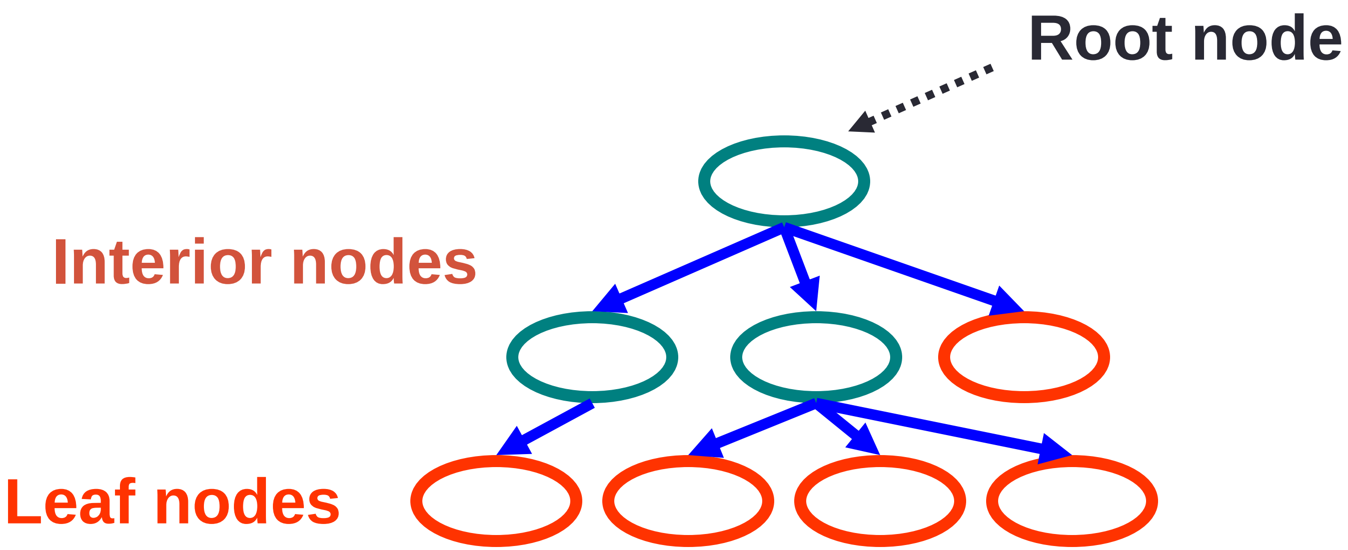
Trees

- Trees are hierarchical data structures
  - One-to-many relationship between elements
- Tree node / element
  - Contains data
  - Referred to by only 1 (parent) node
  - Contains links to any number of (children) nodes



Trees

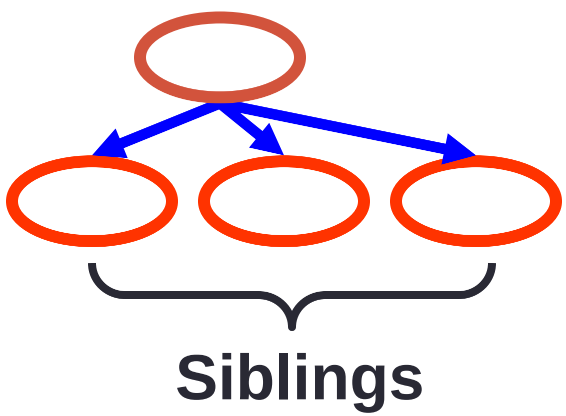
- Terminology
  - Root ⇒ node with no parent
  - Leaf ⇒ all nodes with no children
  - Interior ⇒ all nodes with children

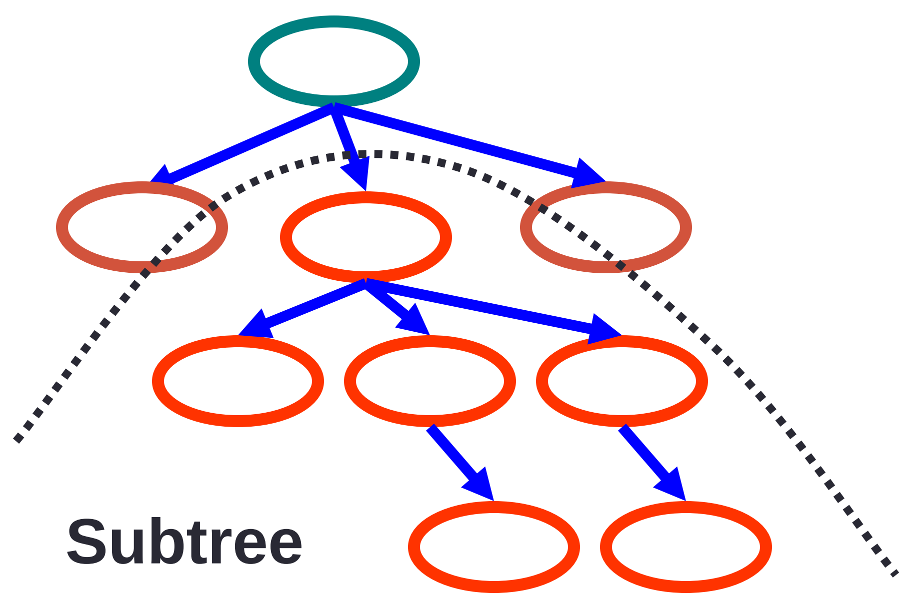


Trees

- Terminology
  - Sibling ⇒ node with same parent
  - Descendent ⇒ children nodes & their descendents
  - Subtree ⇒ portion of tree that is a tree by itself

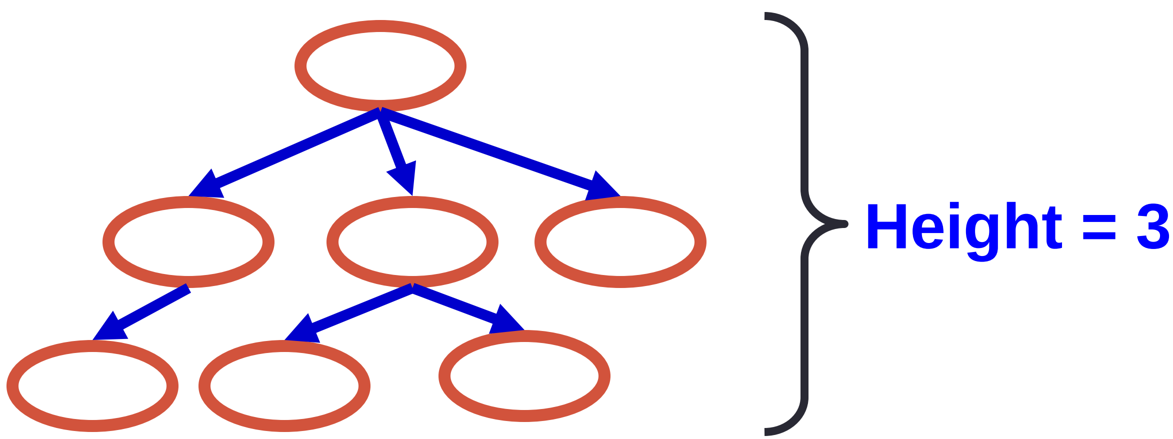
    ⇒ a node and its descendents





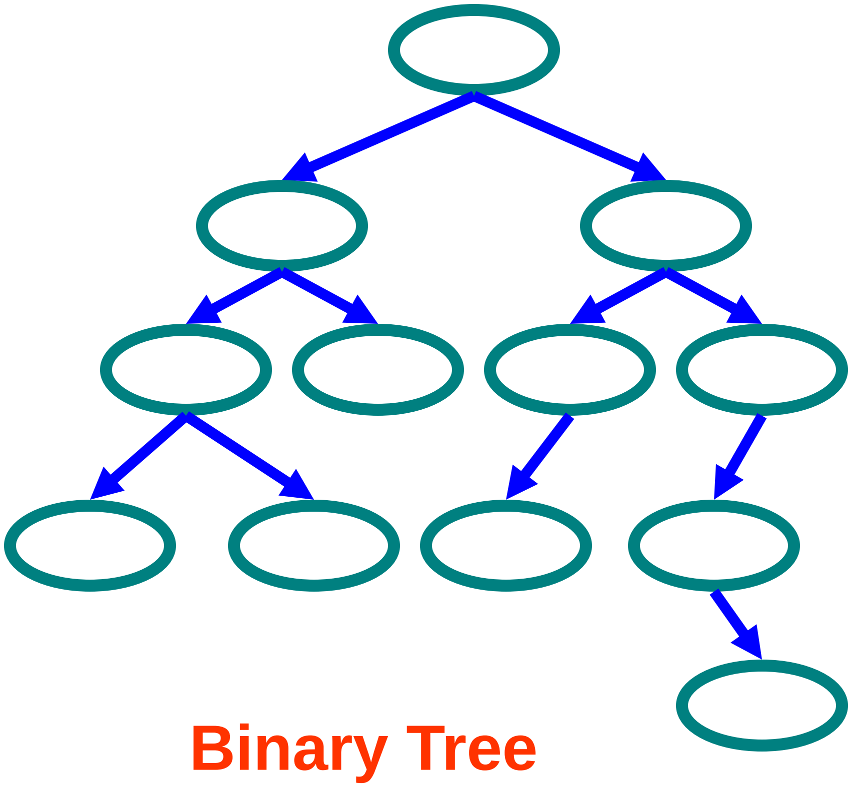
Trees

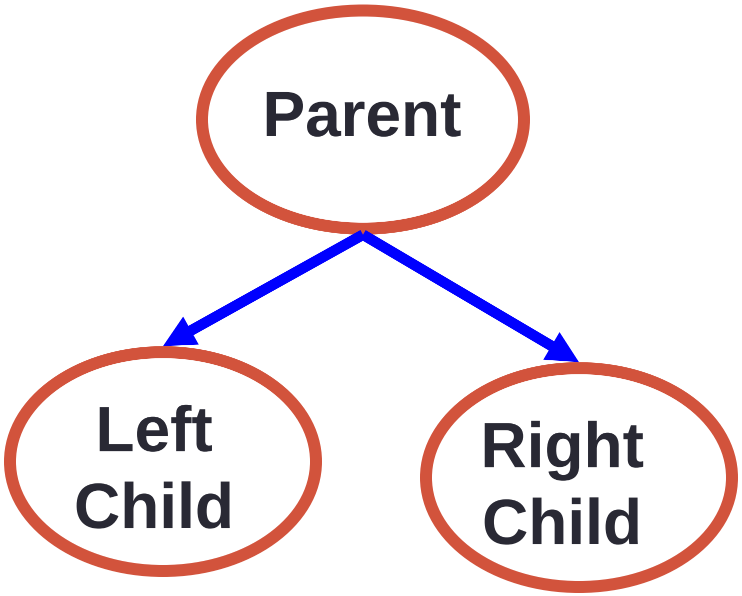
- Terminology
  - Level ⇒ is a measure of a node's distance from root
  - Definition of level
    - If node is the root of the tree, its level is 1
    - Else, the node's level is 1 + its parent's level
  - Height (depth) ⇒ max level of any node in tree



Binary Trees

- Binary tree
  - Tree with 0–2 children per node
    - Left & right child / subtree





Tree Traversal

- Often we want to
  - Find all nodes in tree
  - Determine their relationship
- Can do this by
  - Walking through the tree in a prescribed order
  - Visiting the nodes as they are encountered
- Process is called tree traversal

Tree Traversal

- Goal
  - Visit every node in binary tree
- Approaches
  - **Breadth first** $\Rightarrow$ closer nodes first
  - **Depth first**
    - Preorder $\Rightarrow$ parent, left child, right child
    - Inorder $\Rightarrow$ left child, parent, right child
    - Postorder $\Rightarrow$ left child, right child, parent

    NOTE: left visited before right

Tree Traversal Methods

- Pre-order
  1. Visit node // first
  2. Recursively visit left subtree
  3. Recursively visit right subtree
- In-order
  1. Recursively visit left subtree
  2. Visit node // second
  3. Recursively right subtree
- Post-order
  1. Recursively visit left subtree
  2. Recursively visit right subtree
  3. Visit node // last

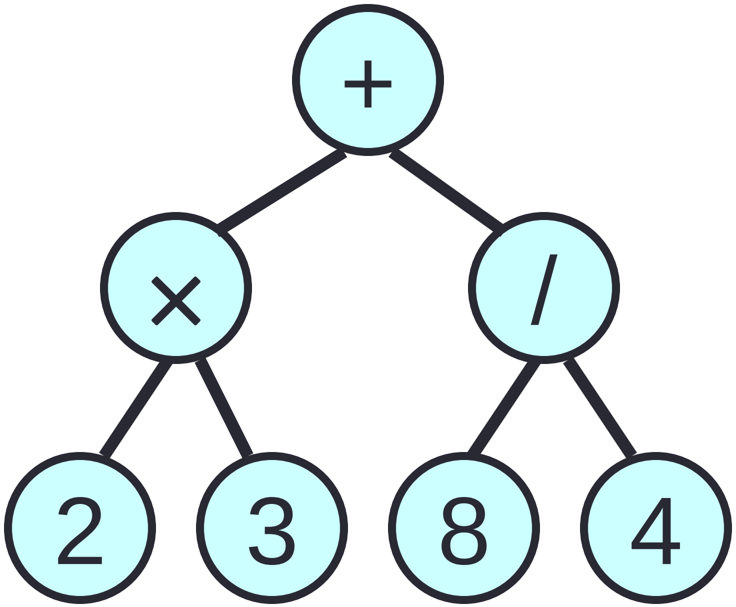
Tree Traversal Methods

- Breadth-first

```
BFS(Node n) {
   Queue Q = new Queue();
   Q.enqueue(n);              // insert node into Q
   while ( !Q.empty()) {
      n = Q.dequeue();        // remove next node
      if ( !n.isEmpty()) {
         visit(n);            // visit node
         Q.enqueue(n.Left()); // insert left subtree in Q
         Q.enqueue(n.Right()); // insert right subtree in Q
} }
```

Tree Traversal Examples

- Breadth-first
  - + × / 2 3 8 4
- Pre-order (prefix)
  - + × 2 3 / 8 4
- In-order (infix)
  - 2 × 3 + 8 / 4
- Post-order (postfix)
  - 2 3 × 8 4 / +



Expression tree

Binary Tree Implementation

- **Choice #1:** Using a class to represent a Node

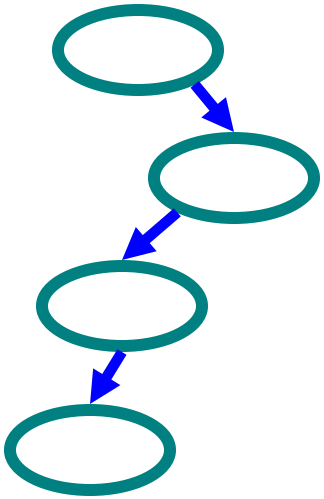
```
Class Node {
    KeyType key;
    Node left, right;    // null if empty
}

Node root = null; // Empty Tree
```

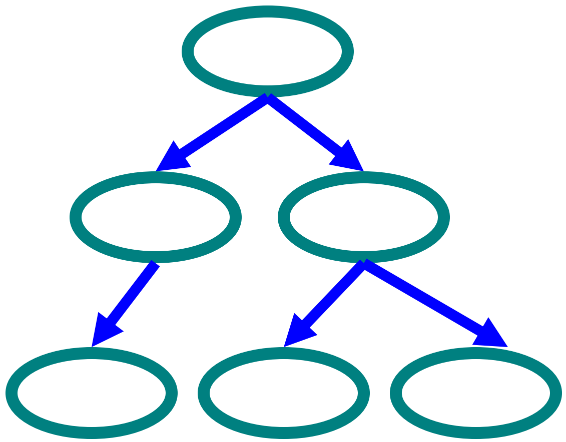
- **Choice #2:** Using a Polymorphic Binary Tree
  - We will talk about this implementation later on

Types of Binary Trees

- **Degenerate**
  - Mostly 1 child / node
  - Height = O(n)
  - Similar to linear list
- **Balanced**
  - Mostly 2 child / node
  - Height = O(log(n))
  - 2^{Height} - 1 = n (# of nodes)
  - Useful for searches



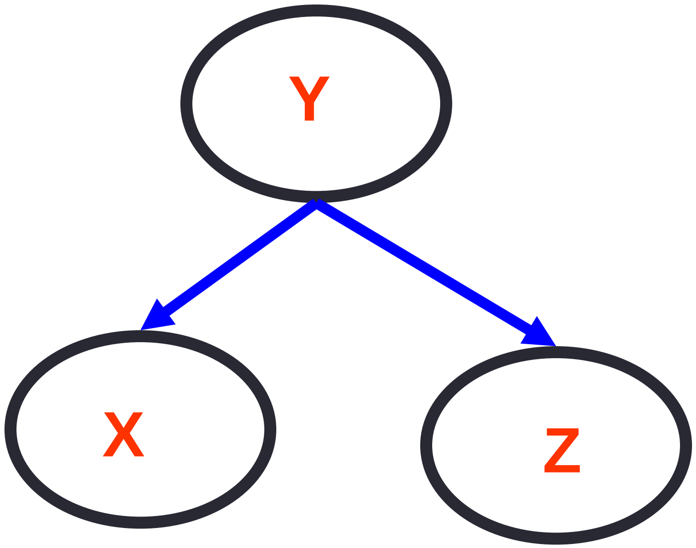
Degenerate binary tree



Balanced binary tree

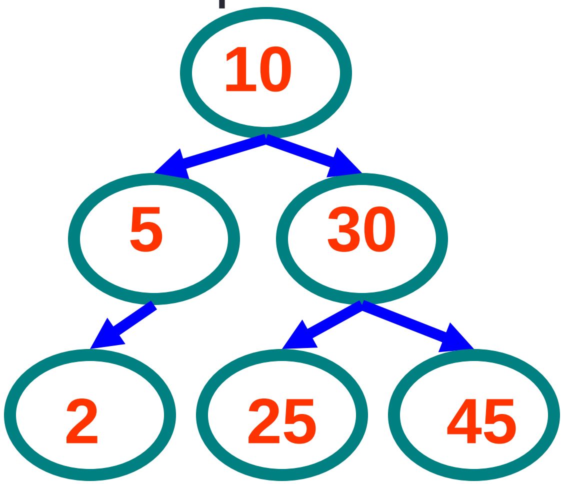
Binary Search Trees

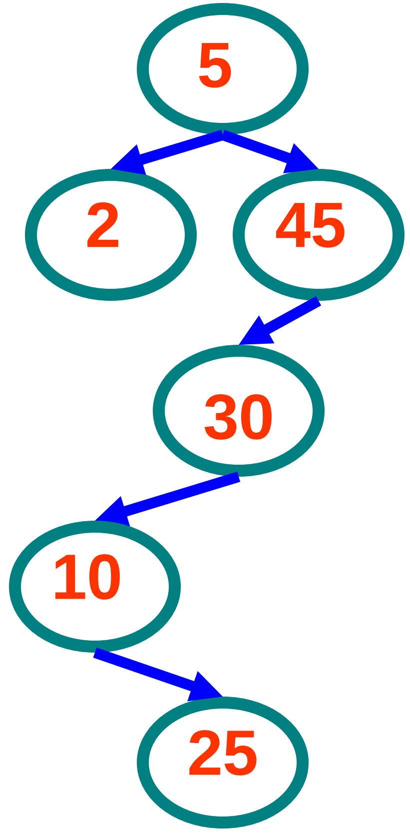
- Key property
  - Value at node
    - Smaller values in left subtree
    - Larger values in right subtree
  - Example
    - Y > X
    - Y < Z



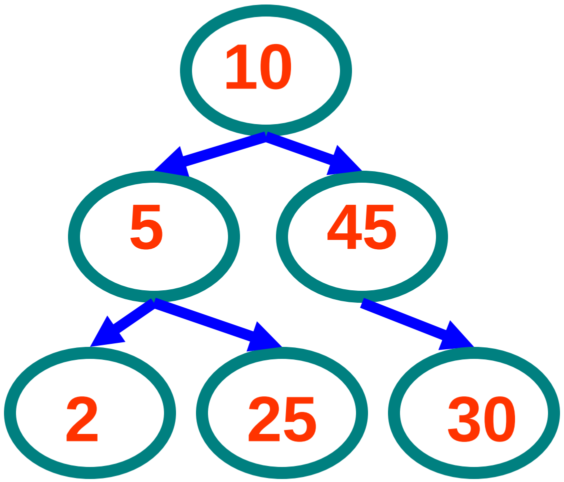
Binary Search Trees

- Examples





Binary search trees

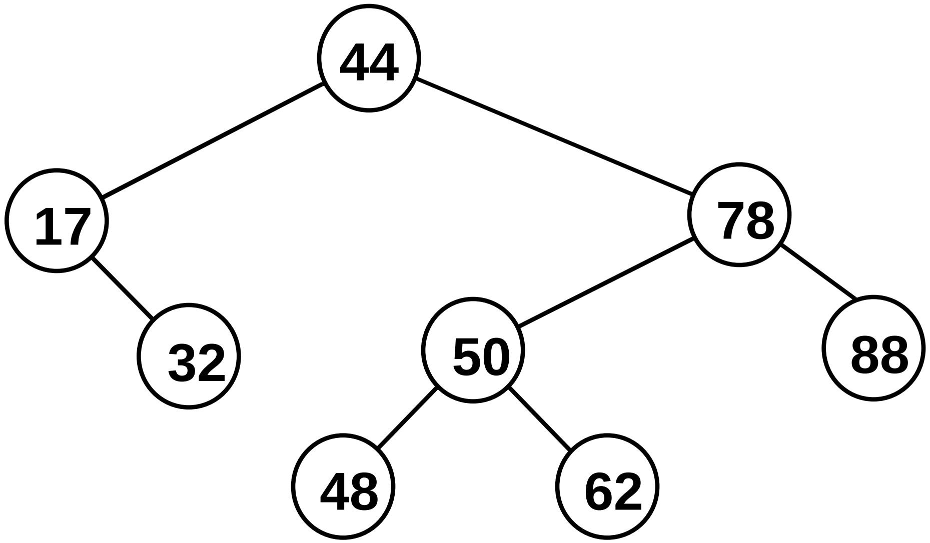


Non-binary search tree

Tree Traversal Examples

- In-order
  - 17, 32, 44, 48, 50, 62, 78, 88

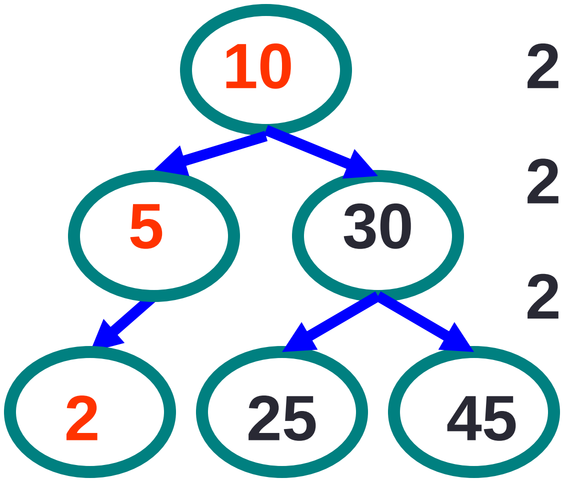
Sorted order!



Binary search tree

Example Binary Searches

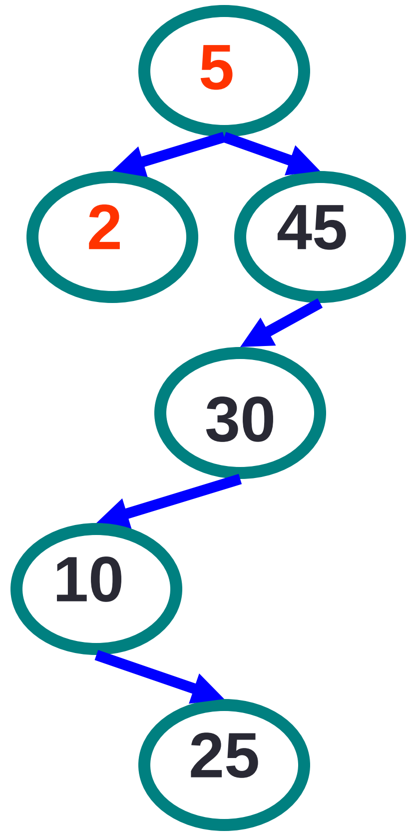
- Find (2)



2 < 10, left

2 < 5, left

2 = 2, found

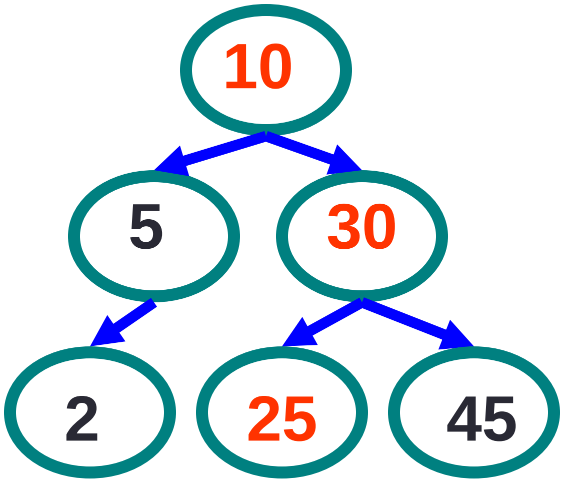


2 < 5, left

2 = 2, found

Example Binary Searches

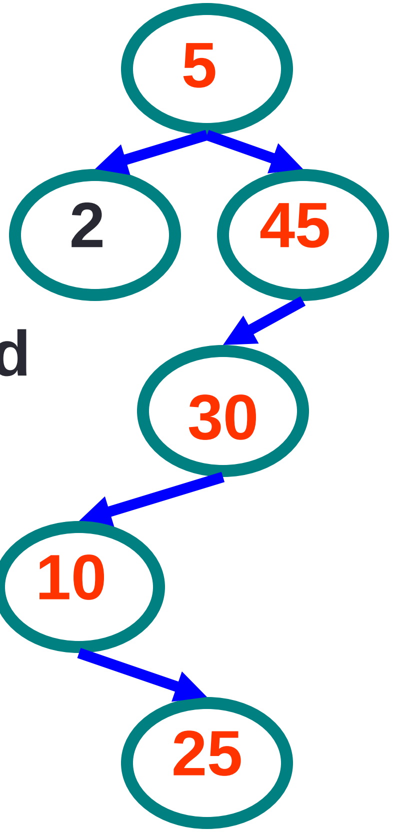
- Find (25)



25 > 10, right

25 < 30, left

25 = 25, found



25 > 5, right

25 < 45, left

25 < 30, left

25 > 10, right

25 = 25, found

Binary Search Properties

- Time of search
  - Proportional to height of tree
  - Balanced binary tree
    - O(log(n)) time
  - Degenerate tree
    - O(n) time
    - Like searching linked list / unsorted array
- Requires
  - Ability to compare key values

Binary Search Tree Construction

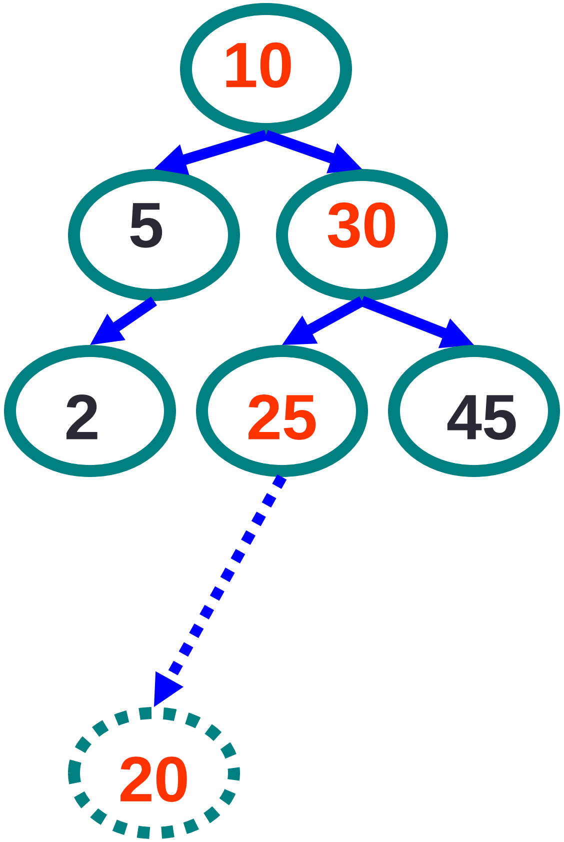
- How to build & maintain binary trees?
  - Insertion
  - Deletion
- Maintain key property (invariant)
  - Smaller values in left subtree
  - Larger values in right subtree

Binary Search Tree – Insertion

- Algorithm
  1. Perform search for value X
  2. Search will end at node Y (if X not in tree)
  3. If X < Y, insert new leaf X as new left subtree for Y
  4. If X > Y, insert new leaf X as new right subtree for Y
- Observations
  - O(log(n)) operation for balanced tree
  - Insertions may unbalance tree

Example Insertion

- Insert (20)



20 > 10, right

20 < 30, left

20 < 25, left

Insert 20 on left

Binary Search Tree – Deletion

- Algorithm
  1. Perform search for value X
  2. If X is a leaf, delete X
  3. Else // must delete internal node

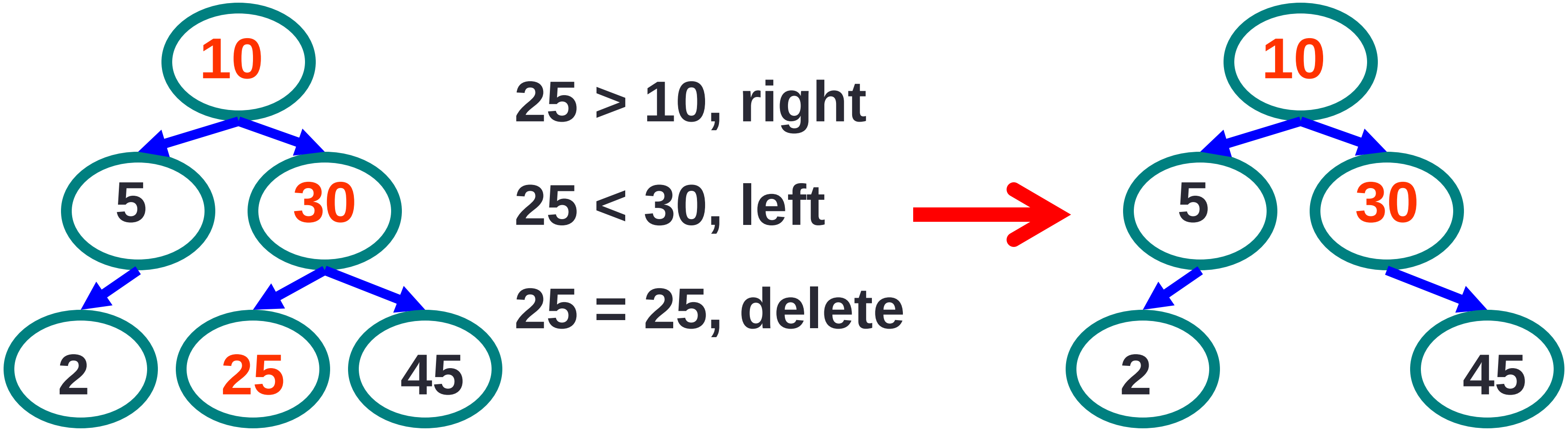
     a) Replace with largest value Y on left subtree

     OR smallest value Z on right subtree

     b) Delete replacement value (Y or Z) from subtree
- Observation
  - O(log(n)) operation for balanced tree
  - Deletions may unbalance tree

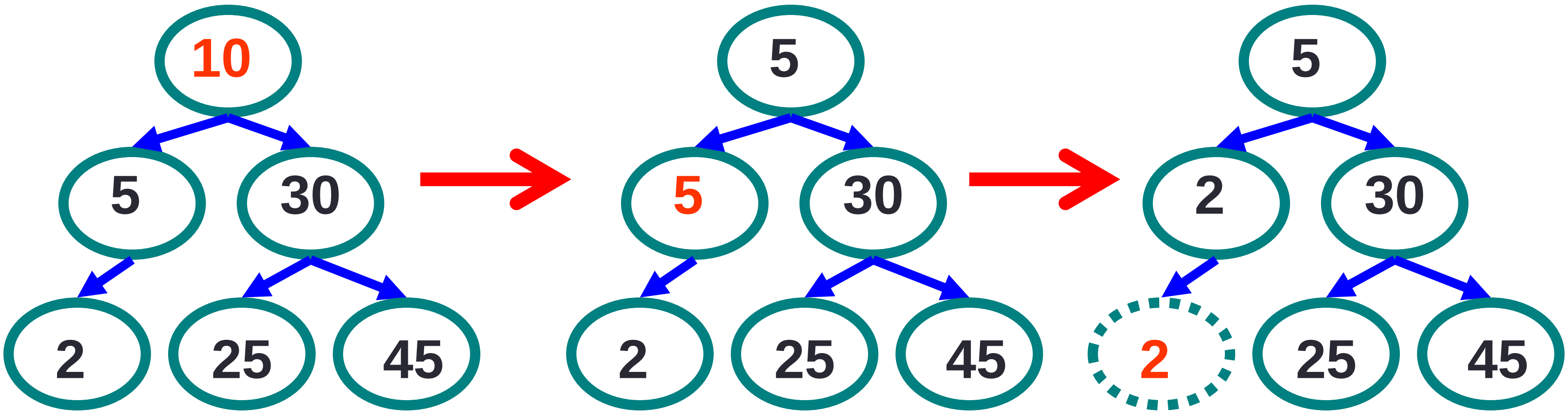
Example Deletion (Leaf)

- Delete (25)



Example Deletion (Internal Node)

- Delete (10)



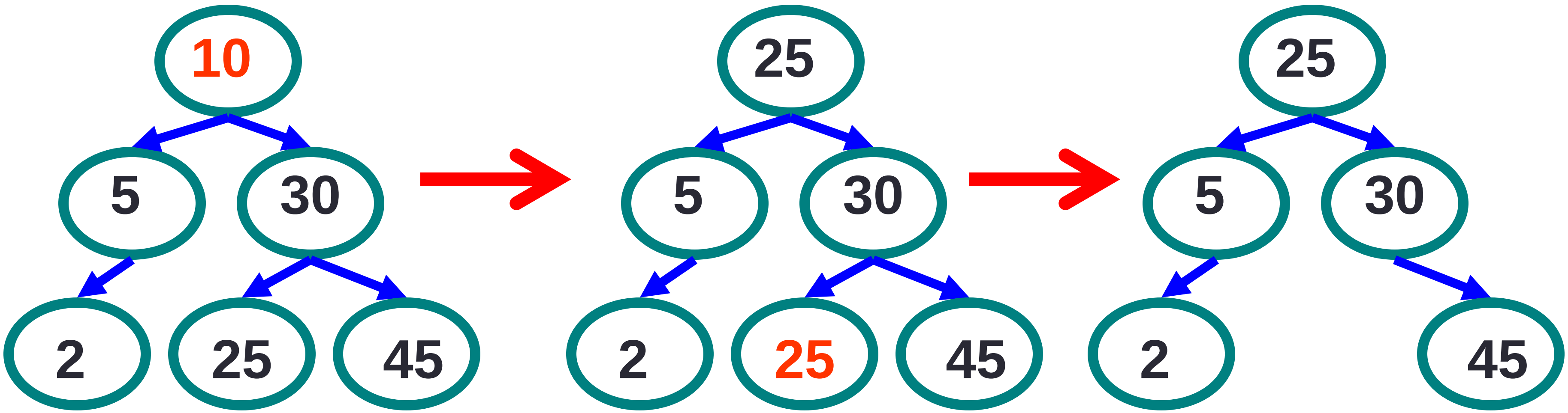
Replacing 10 with largest value in left subtree

Replacing 5 with largest value in left subtree

Deleting leaf

Example Deletion (Internal Node)

- Delete (10)



Replacing 10 with smallest value in right subtree

Deleting leaf

Resulting tree

Building Maps w/ Search Trees

- Binary Search trees often used to implement maps
  - Each non-empty node contains
    - Key
    - Value
    - Left and right child

- Need to be able to compare keys
  - Generic type <K extends Comparable<K>>
    - Denotes any type K that can be compared to K’s

BST (Binary Search Tree) Implementation

- Implementing Tree using traditional approach
- Based on the BST definition below let's see how to implement typical BST Operations (constructor, add, print, find, isEmpty, isFull, size, height, etc.)

```
public class BinarySearchTree <K extends Comparable<K>, V> {
    private class Node {
        private K key;
        private V data;
        private Node left, right;
        public Node(K key, V data) {
            this.key = key;
            this.data = data;
        }
    }
    private Node root;
}
```

- **See code distribution:** LectureBinaryTreeCode.zip

BST Testing

- How can we test the correctness of BST Methods?
- What is the best approach?