CMSC 132: OBJECT-ORIENTED PROGRAMMING II

Heaps & Priority Queues

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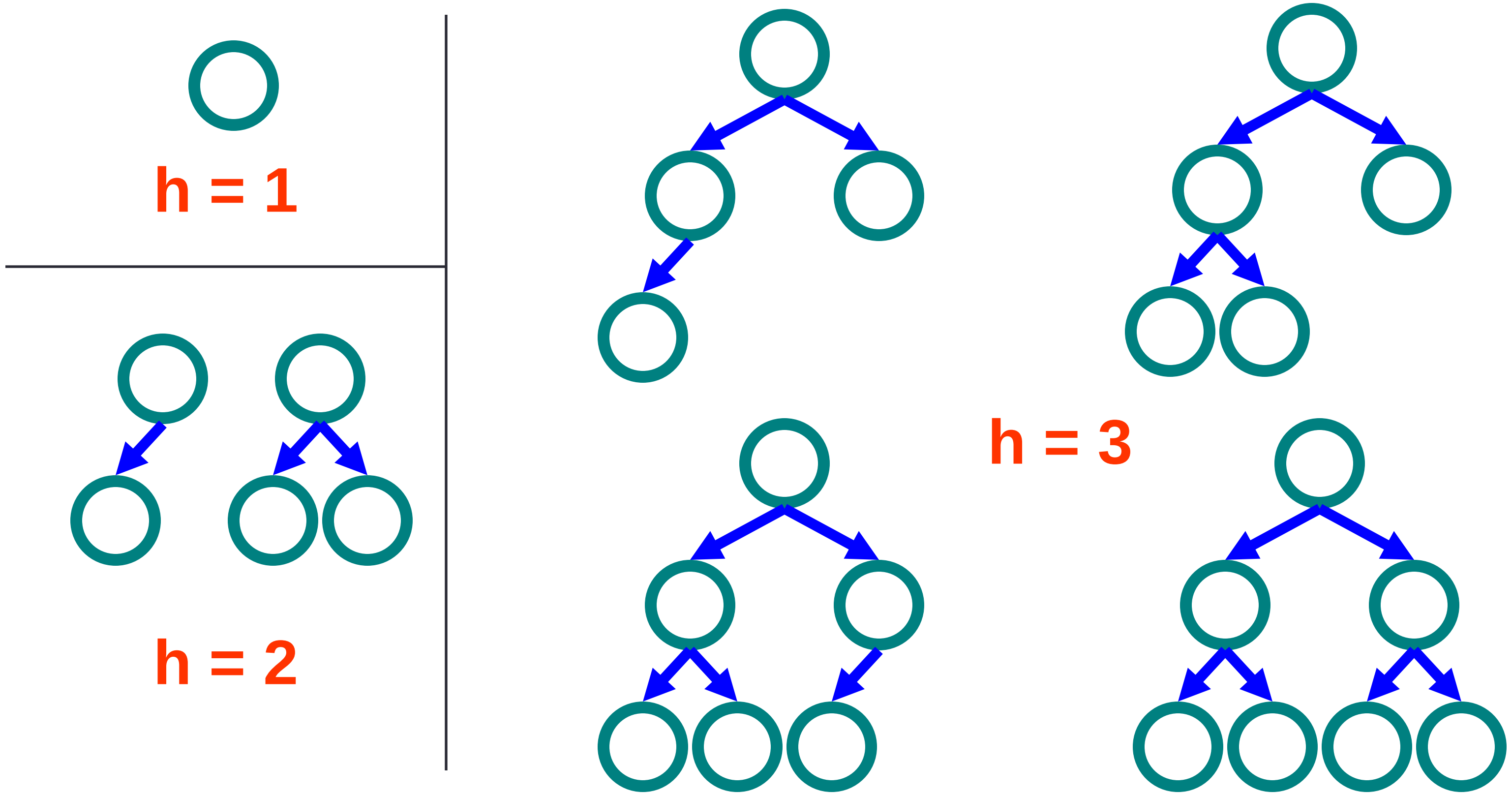
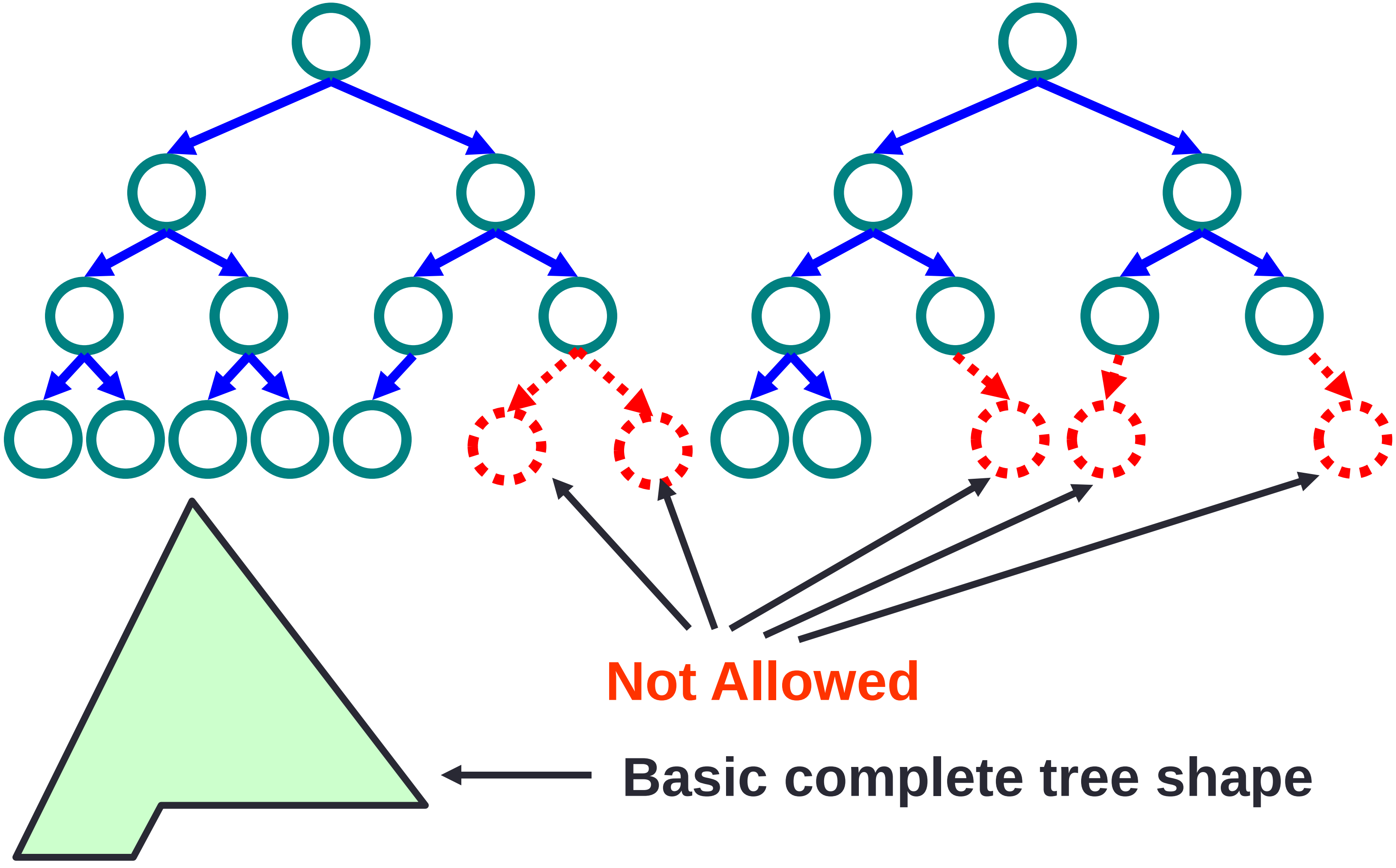
Complete Binary Trees

- An binary tree (height h) where
  - Perfect tree to level h-1
  - Leaves at level h are as far left as possible

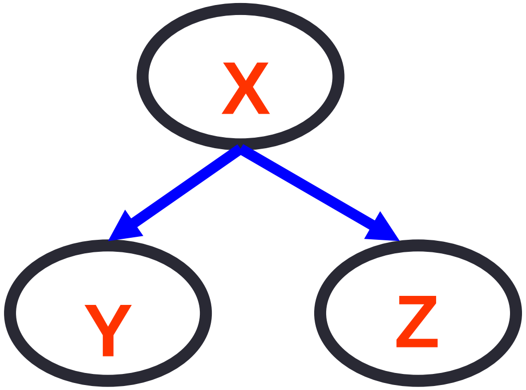


Complete Binary Trees

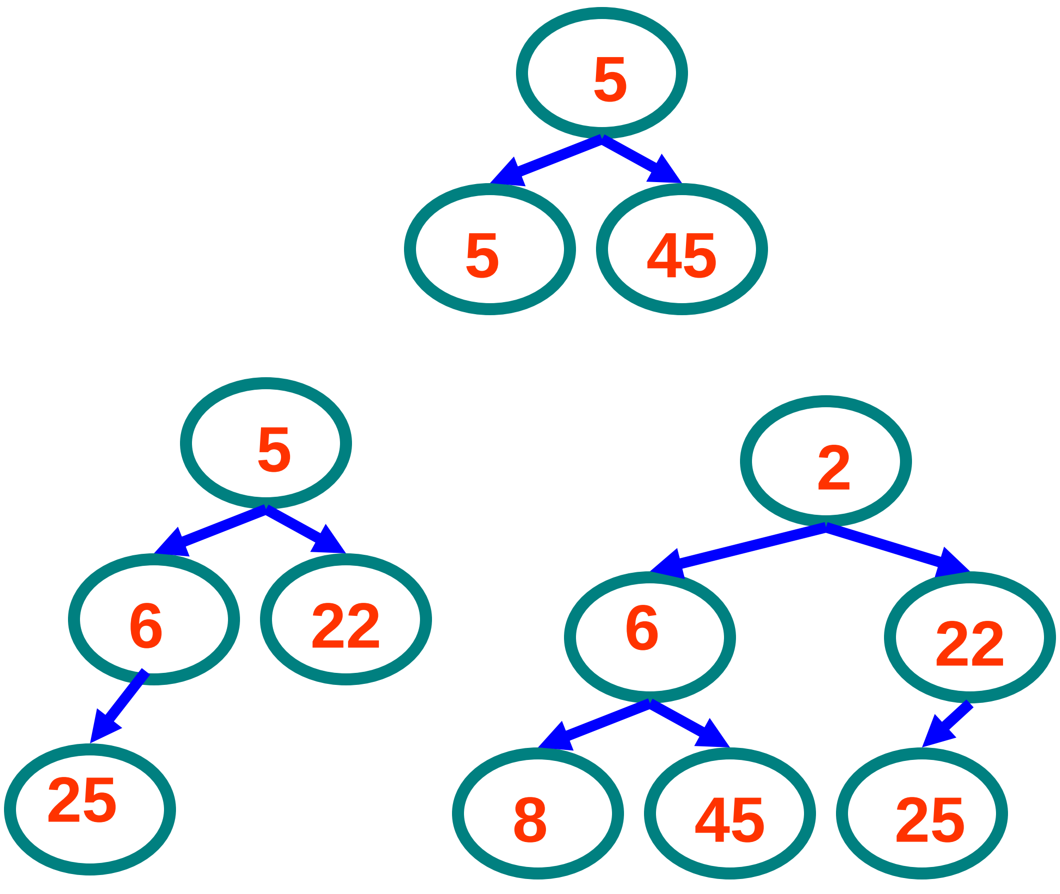


Heaps

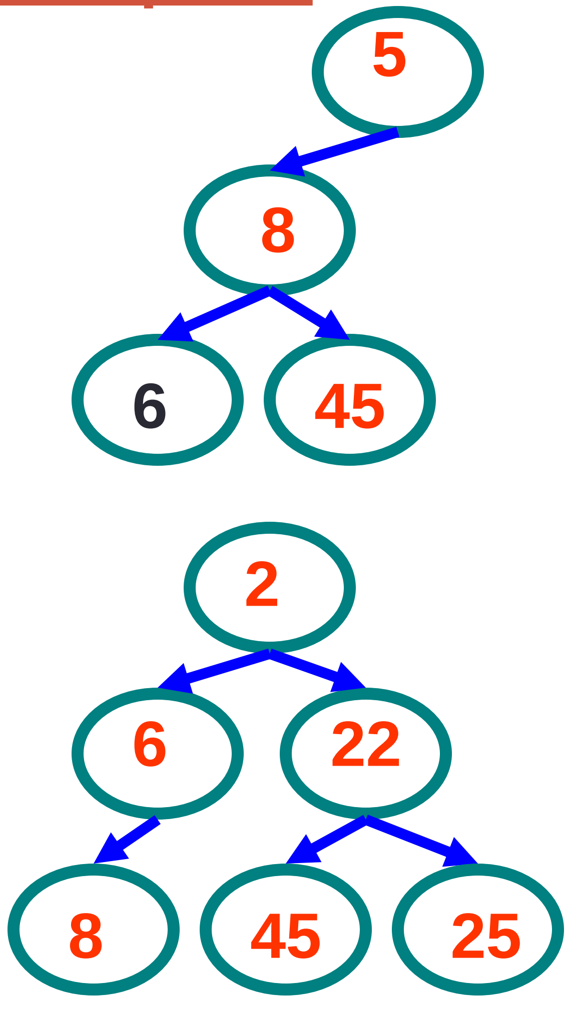
- **Two key properties**
  - Complete binary tree (shape property)
  - Value at node (value property)
    - **Minheap**
      - smaller than or equal to values in subtrees ($X \leq Y, X \leq Z$)
    - **Maxheap**
      - larger than or equal to values in subtrees ($X \geq Y, X \geq Z$)
- We will use minheap in our discussion



Heap (min) & Non-heap Examples



Heaps



Non-heaps

Heap Properties

- Heaps are **balanced trees**
    - Height = $\log_2(n) = O(\log(n))$
- Can find smallest/largest element easily
    - Always at top of heap!
    - Heap can track either min or max, but not both

Heap

- Key operations
  - Insert (X)
  - getSmallest ( )

- Key applications
  - Heapsort
  - Priority queue

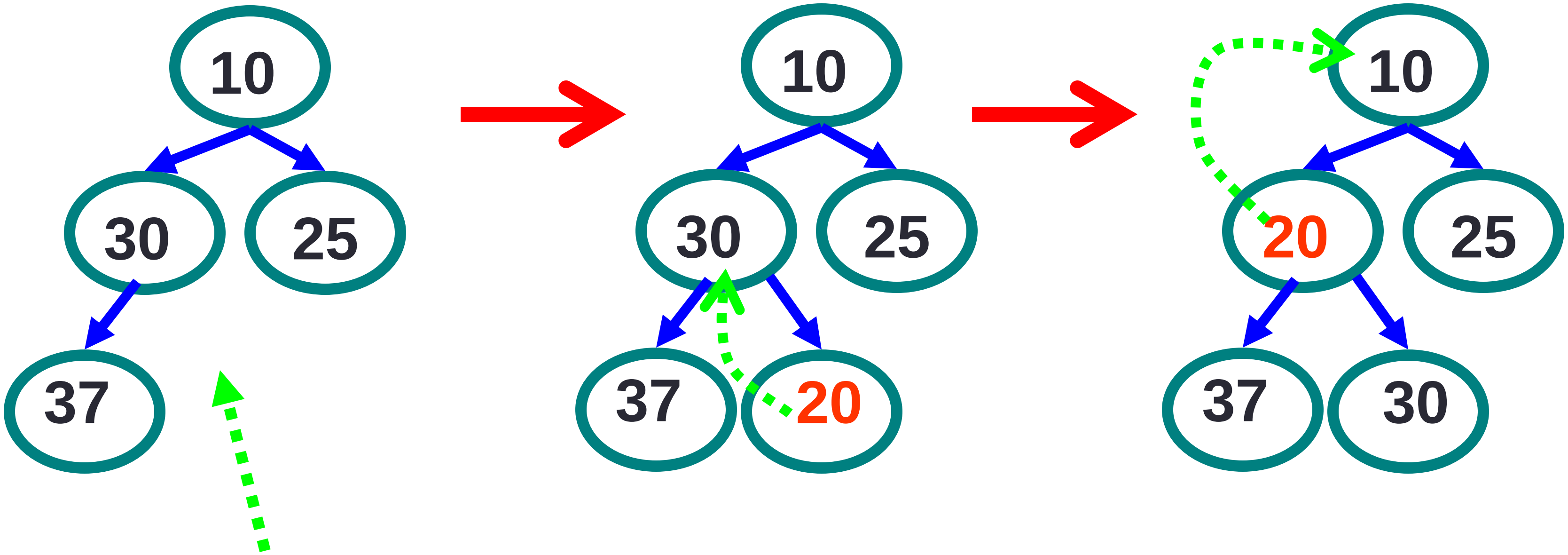
Heap Operations – Insert(X)

- Algorithm
  - Add X to end of tree
  - While (X < parent)
    Swap X with parent // X bubbles up tree

- Complexity
  - # of swaps proportional to height of tree
  - O(log(n))

Heap Insert Example

- Insert (20)



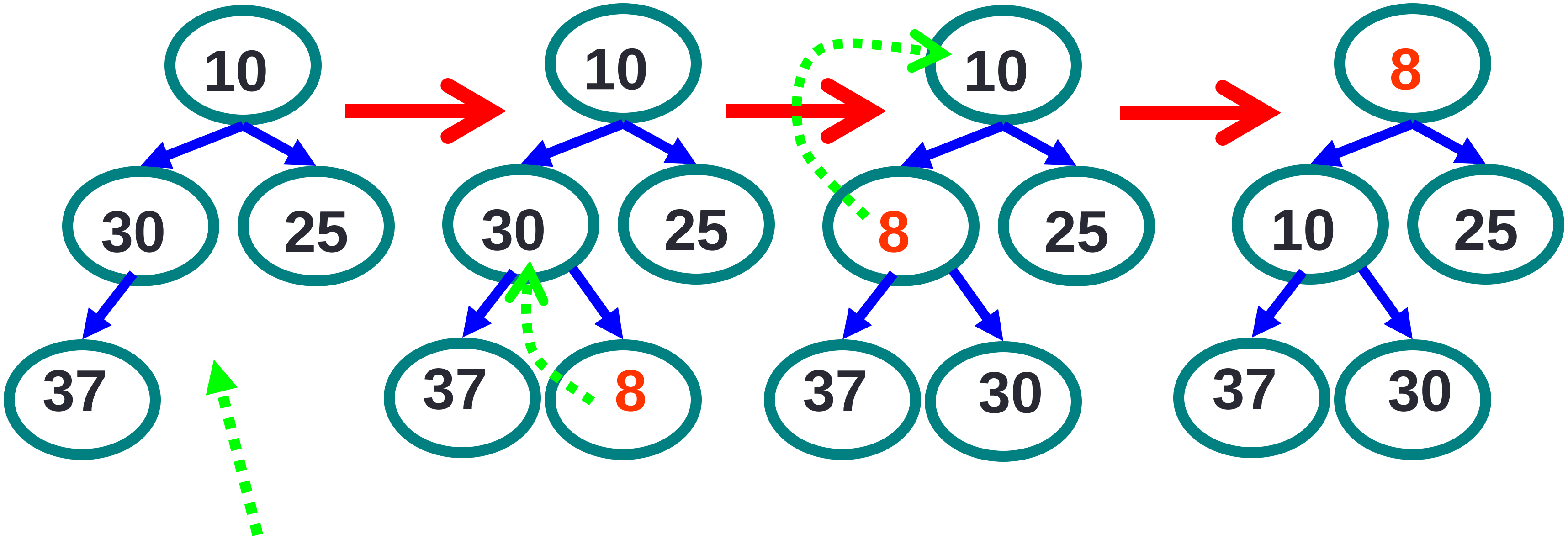
1) Insert to end of tree

2) Compare to parent, swap if parent key larger

3) Insert complete

Heap Insert Example

- Insert (8)



1) Insert to end of tree

2) Compare to parent, swap if parent key larger

3) Insert complete

Heap Operation – getSmallest()

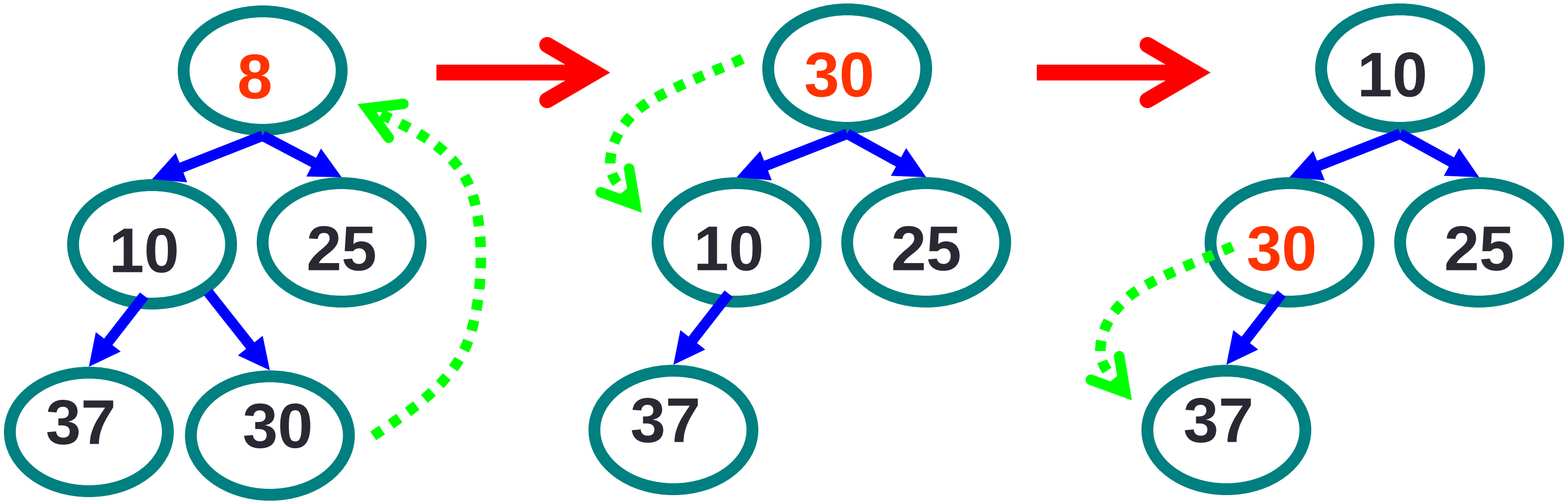
- Algorithm
  - Get smallest node at root
  - Replace root with X (rightmost node) at end of tree
  - While (X > child)

    Swap X with smallest child   // X drops down tree
  - Return smallest node

- Complexity
  - # swaps proportional to height of tree
  - O(log(n))

Heap GetSmallest Example

- getSmallest ()



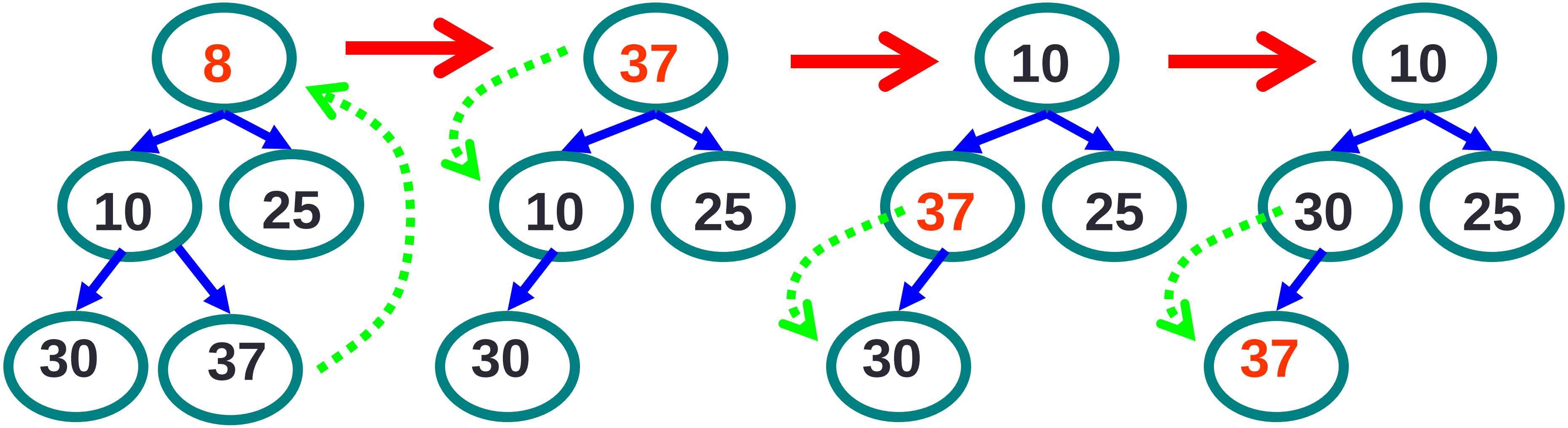
1) Replace root with end of tree

2) Compare node to children, if larger swap with smallest child

3) Repeat swap if needed

Heap GetSmallest Example

- getSmallest ()



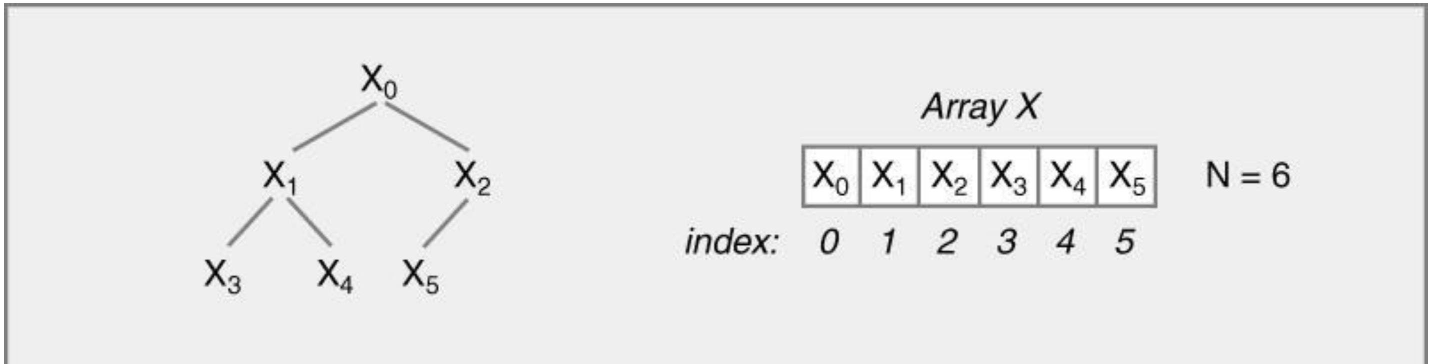
1) Replace root with end of tree

2) Compare node to children, if larger swap with smallest child

3) Repeat swap if needed

Heap Implementation

- Can implement heap as array
  - Store nodes in array elements
  - Assign location (index) for elements using formula
- Observations
  - Compact representation
  - Edges are implicit (no storage required)
  - Works well for complete trees (no wasted space)

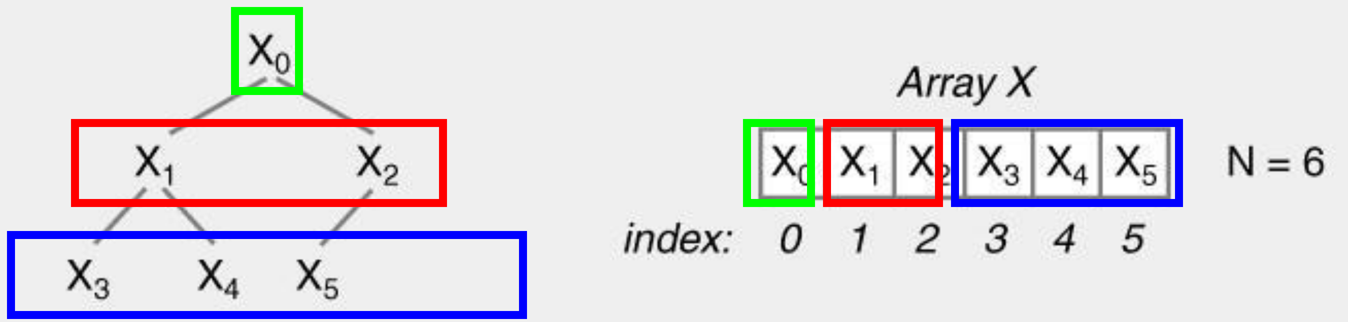


(a) Heap represented as a tree

(b) Heap represented as an array

Heap Implementation

- ⌊ ⌋ → floor (e.g., 1.7 → 1, 2 → 2)
- Calculating node locations
  - Array index i starts at 0
  - Parent(i) = ⌊ (i – 1) / 2 ⌋
  - LeftChild(i) = 2 × i +1
  - RightChild(i) = 2 × i +2



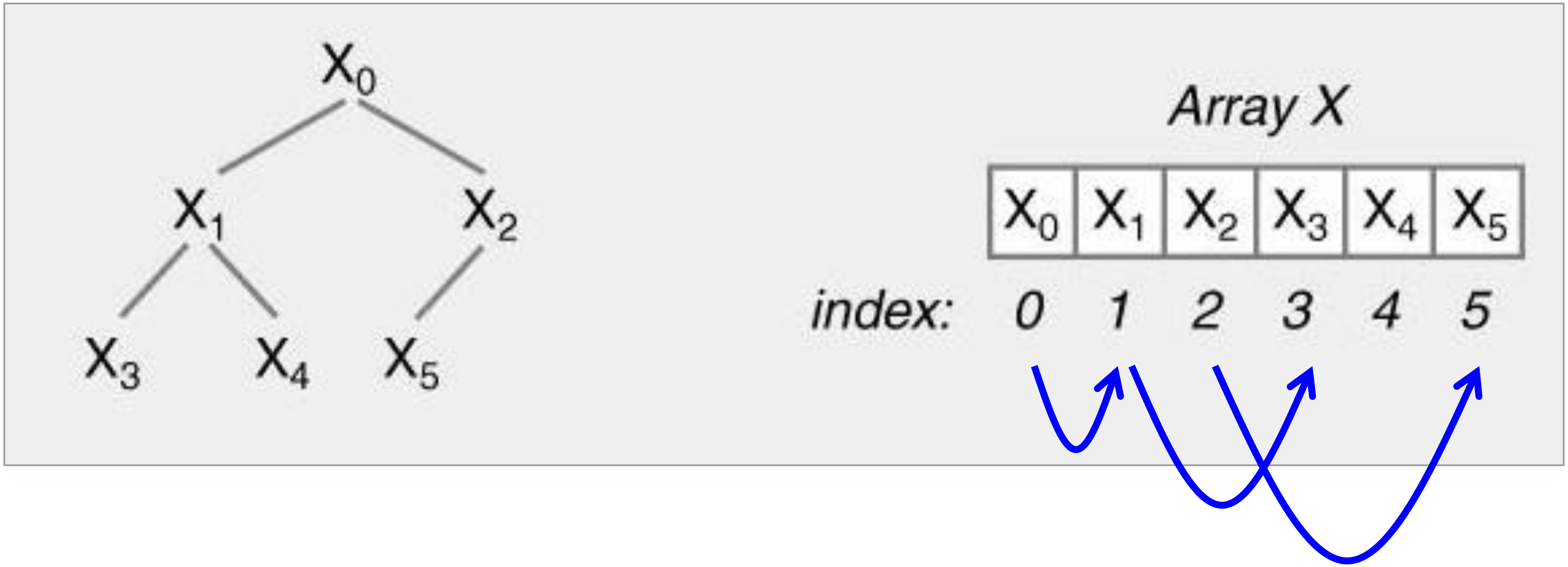
(a) Heap represented as a tree (b) Heap represented as an array

Heap Implementation

- Example
  - Parent(1) = ⌊ (1 – 1) / 2 ⌋ = ⌊ 0 / 2 ⌋ = 0
  - Parent(2) = ⌊ (2 – 1) / 2 ⌋ = ⌊ 1 / 2 ⌋ = 0
  - Parent(3) = ⌊ (3 – 1) / 2 ⌋ = ⌊ 2 / 2 ⌋ = 1
  - Parent(4) = ⌊ (4 – 1) / 2 ⌋ = ⌊ 3 / 2 ⌋ = 1
  - Parent(5) = ⌊ (5 – 1) / 2 ⌋ = ⌊ 4 / 2 ⌋ = 2

Heap Implementation

- Example
  - LeftChild(0) = 2 × 0 +1 = 1
  - LeftChild(1) = 2 × 1 +1 = 3
  - LeftChild(2) = 2 × 2 +1 = 5



Heap Implementation

- Example
  - RightChild(0) = 2 × 0 +2 = 2
  - RightChild(1) = 2 × 1 +2 = 4

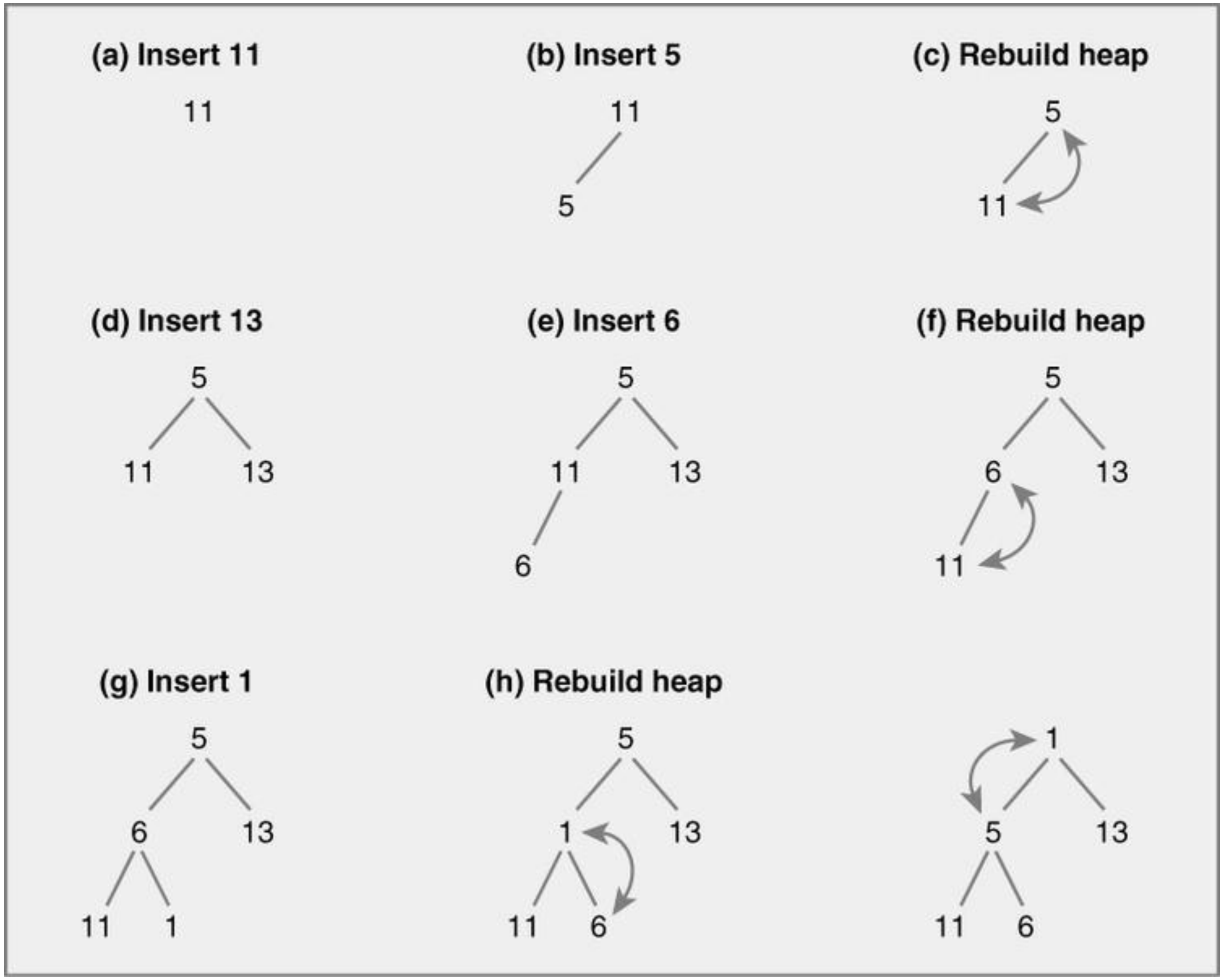
Heap Application – Heapsort

- Use heaps to sort values
  - Heap keeps track of smallest/largest element in heap
- Algorithm
  1. Create heap
  2. Insert values in heap
  3. Remove values from heap (in ascending/descending order)
- Complexity
  - O(nlog(n))

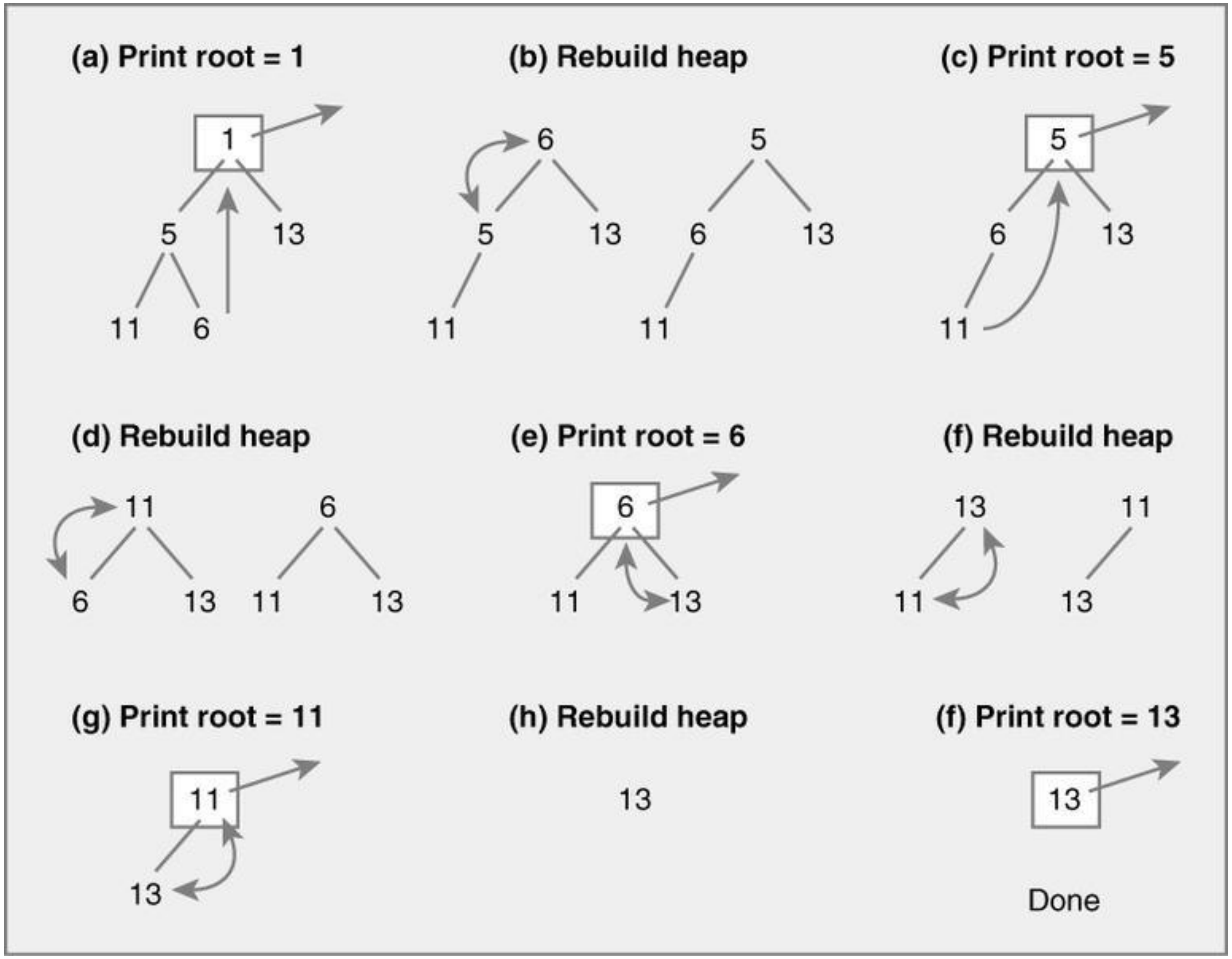
Heapsort Example

- Input
  - 11, 5, 13, 6, 1
- View heap during insert, removal
  - As tree
  - As array

Heapsort – Insert Values



Heapsort – Remove Values



Heapsort – Insert in to Array 1

- Input
  - 11, 5, 13, 6, 1

Index =	0	1	2	3	4
Insert 11	11				

Heapsort – Insert in to Array 2

- Input
  - 11, 5, 13, 6, 1

Index =	0	1	2	3	4
Insert 5	11	5			
Swap	5	11			

Heapsort – Insert in to Array 3

- Input
  - 11, 5, 13, 6, 1

Index =	0	1	2	3	4
Insert 13	5	11	13		

Heapsort – Insert in to Array 4

- Input
  - 11, 5, 13, 6, 1

Index =	0	1	2	3	4
Insert 6	5	11	13	6	
Swap	5	6	13	11	

...

Heapsort – Remove from Array 1

- Input
  - 11, 5, 13, 6, 1

Index =	**0**	**1**	**2**	**3**	**4**
Remove root	**1**	**5**	**13**	**11**	**6**
Replace	**6**	**5**	**13**	**11**	
Swap w/ child	**5**	**6**	**13**	**11**	

Heapsort – Remove from Array 2

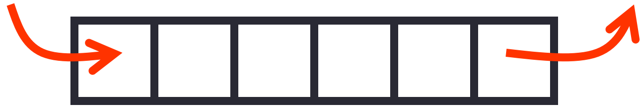
- Input
  - 11, 5, 13, 6, 1

Index =	**0**	**1**	**2**	**3**	**4**
Remove root	**5**	**6**	**13**	**11**	
Replace	**11**	**6**	**13**		
Swap w/ child	**6**	**11**	**13**		

Heap Application – Priority Queue

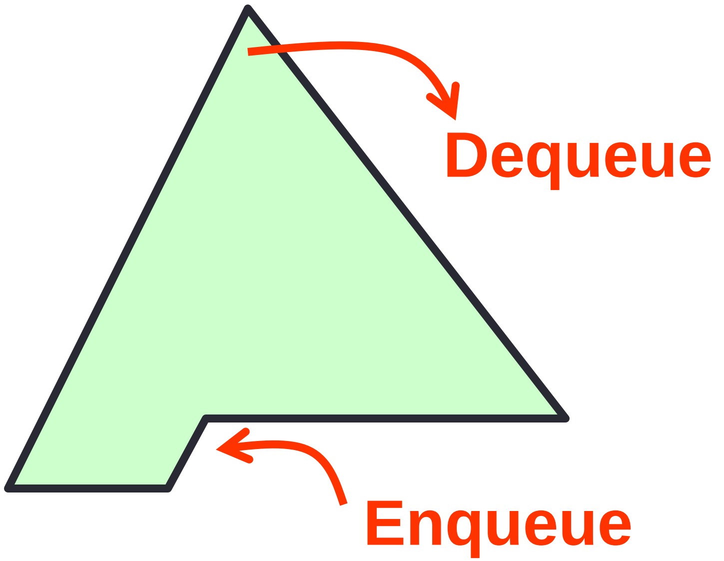
- Queue
    - Linear data structure
    - First-in First-out (FIFO)
    - Implement as array / linked list

Enqueue **Dequeue**



Heap Application – Priority Queue

- Priority queue
  - Elements are assigned priority value
  - Higher priority elements are taken out first
  - Implement as heap
    - Enqueue $\Rightarrow$ insert( )
    - Dequeue $\Rightarrow$ getSmallest( )



Priority Queue

- Properties
  - Lower value = higher priority
  - Heap keeps highest priority items in front

- Complexity
  - Enqueue $\Rightarrow$ insert( ) = O(log(n))
  - Dequeue $\Rightarrow$ getSmallest( ) = O(log(n))
  - For any heap

Heap vs. Binary Search Tree

- Binary search tree
  - Keeps values in sorted order
  - Find any value
    - O(log(n)) for balanced tree
    - O(n) for degenerate tree (worst case)
- Heap
  - Keeps smaller values in front
  - Find minimum value
    - O(log(n)) for any heap

About Heap Implementation

- Implementing a heap (**Video I**/ **Video II**) This videos illustrates the process a programmer (Prof. Bill Pugh in this case) goes through while implementing code. This video was filmed in Dr. Bill Pugh's lecture. Keep in mind that in this video some bugs might be present in the implementation as the testing phase has not been completed yet.