Problem 1. Use mathematical induction to show that

\[(a) \quad \sum_{i=1}^{n} i(i + 1) = \frac{n(n + 1)(n + 2)}{3} \quad (b) \quad \sum_{i=0}^{n} 2^i = 2^{n+1} - 1\]

Problem 2. See bottom of this sheet.

(a) Assume \(b^x = a\). What is \(x\) (in terms of \(a\) and \(b\))? 
(b) Using only part (a), show that \(\log_c(ab) = \log_c a + \log_c b\). 
(c) Show that \(a^{\log_b n} = n^{\log_b a}\)

Problem 3. Differentiate the following functions:

(a) \(\ln(x^2 + 5)\) 
(b) \(\text{lg}(x^2 + 5)\) 
(c) \(\frac{1}{\ln(x^2 + 5)}\)

Problem 4. Integrate the following functions:

(a) \(\frac{1}{x}\) 
(b) \(\frac{1}{3x+7}\) 
(c) \(\ln x\) [HINT: Use integration by parts.] 
(d) \(x \ln x\) [HINT: Use integration by parts.] 
(e) \(x \text{lg} x\)

\[
\begin{align*}
\text{lg} \ n & = \ \log_2 \ n \\
\ln \ n & = \ \log_e \ n \\
\text{lg}^k \ n & = \ (\text{lg} \ n)^k \\
\text{lg} \ \text{lg} \ n & = \ \text{lg} (\text{lg} \ n)
\end{align*}
\]

For all real \(a > 0\), \(b > 0\), \(c > 0\), and \(n\),

\[
\begin{align*}
\ a & = b^{\log_b a} \\
\log_c (ab) & = \log_c a + \log_c b \\
\log_b a^n & = n \log_b a \\
\log_b a & = \frac{\log_c a}{\log_c b} \\
\log_b (1/a) & = - \log_b a \\
\log_b a & = \frac{1}{\log_a b} \\
a^{\log_b n} & = n^{\log_b a}
\end{align*}
\]